

# Mini-Lecture 1.1

## Tips for Success in Mathematics

### Learning Objectives:

1. Get ready for this course.
2. Understand some general tips for success.
3. Understand how to use this text.
4. Get help as soon as you need it.
5. Learn to prepare for and take exams.
6. Develop good time management.

### Examples:

1. Getting ready for this course.
  - a) Positive attitude
  - b) Allow adequate time for class arrival
  - c) Bring all required materials
2. Understanding some general tips for success.
  - a) Find a contact person
  - b) Choose to attend all classes
  - c) Do your homework
  - d) Check your work and learn from mistakes
  - e) Seek help when needed
  - f) Stay organized
  - g) Ask questions
  - h) Hand in all assignments on time
3. Understanding how to use the text.
  - a) Each example in every section has a Practice Problem associated with it.
  - b) Refer to the Lecture Video CD's and the Test Prep Video CD's.
  - c) Review the meaning of icons used in text.
  - d) At beginning of each section, a list of icons shows availability of support materials.
  - e) Each chapter ends with Chapter Highlights, Reviews, Practice Tests, and Cumulative Reviews.
4. Get help as soon as you need it.
  - a) Try your instructor, a tutoring center, a math lab, or you may want to form a study group with fellow classmates.
5. Learning to prepare for and take exams.
  - a) Review previous homework assignments, class notes, quizzes, etc.
  - b) Read Chapter Highlights to review concepts and definitions.
  - c) Practice working out exercises in the end-of-the-chapter Review and Test.
  - d) When taking a test, read directions and problems carefully.
  - e) Pace yourself. Use all available time. Check your work and answers.
6. Good time management.
  - a) Make a list of all weekly commitments with estimated time needed.
  - b) Be sure to schedule study time. Don't forget eating, sleeping, and relaxing!

### Teaching Notes:

- Most developmental students have a high anxiety level with mathematics.
- Many developmental students are hesitant to ask questions and seek extra help.
- Be sure to include your individual expectations. Keep your expectations clear and concise.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

## Mini-Lecture 1.2

### Symbols and Sets of Numbers

#### Learning Objectives:

1. Use a number line to order numbers.
2. Translate sentences into mathematical statements.
3. Identify natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.
4. Find the absolute value of a real number.

#### Examples:

1. Insert  $<$ ,  $>$ , or  $=$  in the space between the paired numbers to make each statement true.

a)  $2 \underline{\quad} 8$                       b)  $41 \underline{\quad} 14$                       c)  $2.12 \underline{\quad} -2.12$                       d)  $\frac{3}{7} \underline{\quad} \frac{9}{21}$

Determine whether each statement is true or false.

e)  $15 \leq 20$                       f)  $3.002 \geq 3.202$                       g)  $\frac{14}{18} \neq \frac{7}{9}$                       h)  $\frac{6}{7} \geq \frac{11}{14}$

2. Translate each sentence into a mathematical statement.

a) Negative eleven is less than  
or equal to negative four.                      b) Fourteen is greater than one

3. Tell which set or sets each number belongs to: natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.

a) 5                      b) -3                      c)  $\frac{8}{3}$                       d)  $\sqrt{5}$                       e) 0

4. Find each absolute value.

a)  $|6.2|$                       b)  $|-14|$                       c)  $\left|-\frac{2}{9}\right|$                       d)  $|0.03|$                       e)  $|0|$

#### Teaching Notes:

- Some students need to be reminded to read the expression and inequality symbols from left to right.
- Some students are not familiar with  $<$  or  $>$  and need to be told to point the symbol at the smaller value.
- Many students need to see value plotted on a number line to visually understand less than or greater than.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a)  $<$ ; 1b)  $>$ ; 1c)  $>$ ; 1d)  $=$ ; 1e) false; 1f) false; 1g) false; 1h) false; 2a)  $-11 \leq -4$ ; 2b)  $14 > 1$ ; 3a) N, W, ., Rat., Real; 3b) I, Rat., Real; 3c) Rat., Real; 3d) Irr., Real; 3e) W, I, Real; 4a) 6.2; 4b) 14; 4c) 2/9; 4d) 0.03; 4e) 0

# Mini-Lecture 1.3

## Fractions

### Learning Objectives:

1. Write fractions in simplest form.
2. Multiply and divide fractions.
3. Add and subtract fractions.

### Examples:

1. Simplify by dividing the numerator by the denominator.

a)  $\frac{3}{3}$                       b)  $\frac{40}{4}$                       c)  $\frac{16}{1}$                       d)  $\frac{0}{7}$                       e)  $\frac{12}{0}$

2. Write each fraction as an equivalent fraction with the given denominator.

a)  $\frac{3}{10}$  with a denominator of 40                      b)  $\frac{4}{7}$  with a denominator of 56

3. Simplify the following fractions.

a)  $\frac{5}{10}$                       b)  $\frac{9}{15}$                       c)  $\frac{88}{66}$                       d)  $\frac{300}{550}$

4. Multiply or divide as indicated.

a)  $\frac{1}{3} \cdot \frac{5}{7}$                       b)  $\frac{28}{6} \cdot \frac{8}{21}$                       c)  $\frac{6}{19} \div \frac{9}{13}$                       d)  $\frac{9}{14} \div \frac{3}{10}$

5. Add or subtract as indicated.

a)  $\frac{1}{8} + \frac{3}{8}$                       b)  $\frac{1}{6} + \frac{7}{15}$                       c)  $\frac{5}{9} - \frac{1}{12}$                       d)  $\frac{7}{8} - \frac{5}{6}$

6. Perform the indicated operations on mixed numbers.

a)  $13\frac{2}{3} + 6\frac{5}{8}$                       b)  $5 - 2\frac{3}{7}$                       c)  $2\frac{1}{3} \cdot 6\frac{3}{7}$                       d)  $3\frac{4}{5} \div 1\frac{2}{5}$

### Teaching Notes:

- Some students may need a visual review of the meaning of a fraction (i.e. pie drawing).
- Most students have experience with fractions but may have forgotten the procedures.
- Stress that all fractions must be written in simplified form.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 1; 1b) 10; 1c) 16; 1d) 0; 1e) undefined; 2a) 12/40; 2b) 32/56; 3a) 1/2; 3b) 3/5; 3c) 4/3; 3d) 6/11; 4a) 5/21; 4b) 16/9; 4c) 26/57; 4d) 15/7; 5a) 1/2; 5b) 19/30; 5c) 17/36; 5d) 1/24; 6a) 20 7/21; 6b) 2 4/7; 6c) 15; 6d) 19/7;

## Mini-Lecture 1.4

### Introduction to Variable Expressions and Equations

#### Learning Objectives

1. Define and use exponents and the order of operations.
2. Evaluate algebraic expressions, given replacement values for variables.
3. Determine whether a number is a solution of a given equation.
4. Translate phrases into expressions and sentences into equations.

#### Examples:

1. Evaluate.

a)  $2^3$                       b)  $1^7$                       c)  $\left(\frac{6}{7}\right)^2$                       d)  $(0.3)^3$

Using order of operation, simplify each expression.

e)  $7 + 3 \cdot 2$                       f)  $25 - 3^2 \cdot 2$                       g)  $6[-5 + 6(-3 + 8)]$                       h)  $\frac{20(-1) - (-4)(-3)}{2[-12 \div (-3 - 3)]}$

2. Evaluate each expression when  $x = 3$ ,  $y = 2$ , and  $z = 6$ .

a)  $x + y + z$                       b)  $3x - z$                       c)  $|5x - 2z|$                       d)  $\frac{5z}{x} - \frac{3y^2}{z}$

3. Determine whether the given number is a solution of the given equation.

a)  $x - 12 = 15$ ; 27                      b)  $12 + y = 29$ ; 7                      c)  $\frac{3}{4}x = \frac{15}{20}$ ; 5                      d)  $y = 3y + 2$ ; 0

4. Write each phrase as an algebraic expression.

a) The sum of a number and thirteen                      b) The quotient of forty-two and a number

Write each sentence as an equation.

c) The product of one-third and a number is nine                      d) A number added to twelve is fourteen.

#### Teaching Notes:

- Be sure to identify base and exponent when working with exponential notation.
- Most students find order of operations challenging.
- Many students will confuse expression and equation. Be sure students understand that you simplify an expression, but solve an equation.
- Many students have problems with translating sentences into equations.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 8; 1b) 1; 1c) 36/49; 1d) 0.027; 1e) 13; 1f) 7; 1g) 150; 1h) -8; 2a) 11; 2b) 3; 2c) 3; 2d) 8; 3a) true; 3b) false; 3c) false; 3d) false; 4a)  $x + 13$ ; 4b)  $42/x$ ; 4c)  $1/3 x = 9$ ; 4d)  $12 + x = 14$

## Mini-Lecture 1.5

### Adding Real Numbers

#### Learning Objectives:

1. Add real numbers with the same sign.
2. Add real numbers with unlike signs.
3. Solve problems that involve addition of real numbers.
4. Find the opposite of a number.

#### Examples:

1. Add the following real numbers with the same sign.

a)  $8 + 11$                       b)  $(-3) + (-15)$                       c)  $(-14) + (-35)$                       d)  $\left(-\frac{3}{5}\right) + \left(-\frac{1}{2}\right)$

2. Add the following real numbers with different signs.

a)  $(-9) + 5$                       b)  $16 + (-25)$                       c)  $(-15.3) + 27.03$                       d)  $\left(\frac{1}{2}\right) + \left(-\frac{5}{8}\right)$

Mixed exercise of addition of signed numbers.

e)  $-7 + (-23)$                       f)  $-42 + 38$                       g)  $53 + (-22)$                       h)  $\left(-\frac{5}{12}\right) + \left(\frac{3}{8}\right)$

3. Solve each of the following.

a) At the beginning of a chemistry experiment, Amy measured the temperature of a liquid to be  $-5^{\circ}\text{C}$ . During the experiment, the temperature rose  $14^{\circ}\text{C}$ . What was the liquid's temperature at the end of the experiment?

b) A local restaurant reported net incomes of  $-\$1,397$ ,  $-\$2,042$ , and  $-\$809$  for the past three months. What was its total net income for the three months?

4. Find the additive inverse or opposite.

a) 8                      b) -9                      c) 0                      d)  $|-17|$

#### Teaching Notes:

- Some students will need to see addition performed on a number line.
- Some students will need instruction with inputting negative numbers into a calculator.
- Review the definition of absolute value.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a) 19; 1b) -18; 1c) -49; 1d) -11/10; 2a) -4; 2b) -9; 2c) 11.73; 2d) -1/8; 2e) -30; 2f) -4; 2g) 31; 2h) -1/24; 3a)  $9^{\circ}\text{C}$ ; 3b)  $-\$4,248$ ; 4a) -8; 4b) 9; 4c) 0; 4d) -17

# Mini-Lecture 1.6

## Subtracting Real Numbers

### Learning Objectives:

1. Subtract real numbers.
2. Add and subtract real numbers.
3. Evaluate algebraic expressions using real numbers
4. Solve problems that involve subtraction of real numbers.

### Examples:

1. Subtract.

a)  $-8 - 4$

b)  $11 - 18$

c)  $-15 - (-10)$

d)  $-12 - 12$

e)  $22 - (-13)$

f)  $-132 - (-207)$

g)  $1.3 - (3.8)$

h)  $\frac{15}{7} - \left(-\frac{9}{14}\right)$

2. Simplify each expression.

a)  $-3 - (-4) - 5 + (-2)$

b)  $7 - 10 - 8 + (-7)$

c)  $-2 + |-3 - 5| - 3^2$

3. Evaluate each expression when  $x = -3$ ,  $y = -7$ , and  $z = 9$

a)  $x - y$

b)  $\frac{10 - x}{y - 2}$

c)  $|x| + |y| - |z|$

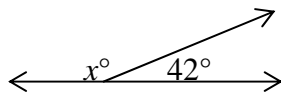
d)  $x^2 - y$

4. Solve:

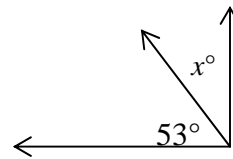
- a) In a game of cards, Alicia won 11 chips, lost 6 chips, won 3 chips, lost 14 chips, and won 1 chip. What was her final count of chips?

Find the complementary or supplementary angle.

b)



c)



### Teaching Notes:

- Remind students to always change subtraction to addition and “add the opposite”.
- Some students forget to change the sign of the second value after changing to addition.
- Encourage students to take the time to write the steps:  $3 - (-2) = 3 + (+2) = 5$
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a) -12; 1b) -7; 1c) -5; 1d) -24; 1e) 35; 1f) 75; 1g) -2.5; 1h) 39/14; 2a) -6, 2b) -18, 2c) -3; 3a) 4; 3b) -13/9; 3c) 1; 3d) 16; 4a) -5; 4b) 138°; 4c) 37°

# Mini-Lecture 1.7

## Multiplying and Dividing Real Numbers

### Learning Objectives

1. Multiply and divide real numbers.
2. Evaluate algebraic expressions using real numbers.

### Examples

1. Multiply the real numbers.

a)  $-6(5)$                       b)  $(-11)(-3)$                       c)  $-\frac{3}{5}\left(\frac{10}{21}\right)$                       d)  $2(-5)(-1)(-3)$

Find the reciprocal of the real number.

e)  $\frac{3}{7}$                       f)  $5$                       g)  $-\frac{5}{21}$                       h)  $0.3$

Divide the real numbers.

i)  $\frac{27}{-3}$                       j)  $-90 \div (-5)$                       k)  $-\frac{1}{2} \div \left(-\frac{8}{15}\right)$                       l)  $\frac{-22}{0}$

2. Evaluate each expression.

a)  $2x - y^2$ , when  $x = 4, y = -3$                       b)  $\frac{-2-x}{y-5}$ , when  $x = -4, y = 6$

c)  $\frac{-6x-4y}{-2z+3-(-10)}$  when  $x = 5, y = -1, z = 0$                       d)  $-8^2$

e)  $(-7)^2$                       f)  $-1^8$                       g)  $(-1)^{87}$

### Teaching Notes:

- Most students find multiplying and dividing real numbers relatively easy.
- Many students confuse  $\frac{0}{5} = 0$  and  $\frac{5}{0} = \text{undefined}$ .
- Many students have difficulty with the fact that  $-5^2 \neq (-5)^2$
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) -30; 1b) 33; 1c) -2/7; 1d) -30; 1e) 7/3; 1f) 1/5; 1g) -21/5; 1h) 10/3; 1i) -9; 1j) 18;  
1k) 15/16; 1l) undefined; 2a) -1, 2b) 2, 2c) -2, 2d) -64, 2e) 49, 2f) -1, 2g) -1

# Mini-Lecture 1.8

## Properties of Real Numbers

### Learning Objectives:

1. Use the commutative and associative properties.
2. Use the distributive property.
3. Use the identity and inverse properties.

### Examples:

1. Use the commutative property of addition or multiplication to complete each statement.

a)  $3 + y = \underline{\hspace{2cm}}$     b)  $a + (-9) = \underline{\hspace{2cm}}$     c)  $-10 \cdot x = \underline{\hspace{2cm}}$     d)  $s \cdot t = \underline{\hspace{2cm}}$

Use the associative property of addition or multiplication to complete each statement.

e)  $(3 + x) + y = \underline{\hspace{2cm}}$     f)  $-2 \cdot (5x) = \underline{\hspace{2cm}}$

Use the commutative and associative properties to simplify each expression.

g)  $12 + (4 + x)$     h)  $-7(5x)$     i)  $\left(-\frac{1}{3} + x\right) + \frac{5}{12}$     j)  $0.13(-1.2y)$

2. Use the distributive property to write each expression without parentheses. Then simplify the result, if possible.

a)  $8(x + y)$     b)  $-3(7x - 9)$     c)  $-2(-6y - 10)$     d)  $6(4x - 3y - 9)$

Use the distributive property to write each sum as a product.

e)  $6 \cdot x + 6 \cdot y$     f)  $13 \cdot x + 13 \cdot 4$     g)  $(-2)x + (-2)y$     h)  $\frac{1}{3}a + \frac{1}{3} \cdot 6$

3. Name the property that is illustrated by each true statement.

a)  $0 + 11 = 11$     b)  $3 \cdot \frac{1}{3} = 1$     c)  $5 + (-5) = 0$     d)  $12 \cdot 1 = 12$

### Teaching Notes:

- Many students use the Properties of Real Numbers without realizing that they are using these properties.
- Some students, when using the distributive property, forget to multiply the second term.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a)  $y+3$ ; 1b)  $-9+a$ ; 1c)  $x \cdot -10$ ; 1d)  $t \cdot s$ ; 1e)  $3 + (x + y)$ ; 1f)  $(-2 \cdot 5)x$ ; 1g)  $16 + x$ ; 1h)  $-35x$ ;  
1i)  $1/12 + x$ ; 1j)  $-0.156y$ ; 2a)  $8x + 8y$ ; 2b)  $-21x + 27$ ; 2c)  $12y + 20$ ; 2d)  $24x - 18y - 54$ ; 2e)  $6(x + y)$ ;  
2f)  $13(x + 4)$ ; 2g)  $-2(x + y)$ ; 2h)  $1/3(a + 6)$ ; 3a) addition property of zero; 3b) inverse property of multiplication; 3c) inverse property of addition; 3d) multiplication property of one



# Mini-Lecture 2.1

## Simplifying Algebraic Expressions

### Learning Objectives:

1. Identify terms, like terms, and unlike terms.
2. Combine like terms.
3. Use the distributive property to remove parentheses.
4. Write word phrases as algebraic expressions.

### Examples

1. Identify the numerical coefficient of each term.

a)  $9x$

b)  $-3y$

c)  $-x$

d)  $2.7x^2y$

Indicate whether the terms in each list are like or unlike.

e)  $6x, -3x$

f)  $-xy^2, -x^2y$

g)  $5ab, -\frac{1}{2}ba$

h)  $2x^3yz^2, -x^3yz^3$

2. Simplify each expression by combining any like terms.

a)  $7x - 2x + 4$

b)  $-9y + 2 - 1 + 6 + y - 7$

c)  $1.6x^5 + 0.9x^2 - 0.3x^5$

3. Simplify each expression. Use the distributive property to remove any parentheses.

a)  $3(x + 6)$

b)  $-(-5m + 6n - 2p)$

c)  $\frac{1}{3}(6x - 9)$

Remove parentheses and simplify each expression.

d)  $14(2x + 6) - 4$

e)  $10a - 5 - 2(a - 3)$

f)  $3(2x - 5) - (x + 7)$

4. Write each phrase as an algebraic expression. Simplify if possible.

a) Add  $-4y + 3$  to  $6y - 9$

b) Subtract  $2x - 1$  from  $3x + 7$

c) Triple a number, decreased by six

d) Six times the sum of a number and two

### Teaching Notes:

- Students will need repeated practice with identifying terms and like terms.
- Some students do not know that a variable without a numerical coefficient actually has a coefficient of 1.
- Some students will forget to distribute the minus sign in 3b), 3e), and 3f). Some students might need to write a 1 in front of the parentheses in 3b) and 3f).
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 9; 1b) -3; 1c) -1; 1d) 2.7; 1e) like; 1f) unlike; 1g) like; 1h) unlike; 2a)  $5x+4$ ; 2b)  $-8y$ ;  
2c)  $1.3x^5+0.9x^2$ ; 3a)  $3x+18$ ; 3b)  $5m-6n+2p$ ; 3c)  $2x-3$ ; 3d)  $28x+80$ ; 3e)  $8a+1$ ; 3f)  $5x-22$ ;  
4a)  $(-4y+3) + (6y-9) = 2y-6$ ; 4b)  $(3x+7) - (2x-1) = x + 8$ ; 4c)  $3x-6$ ; 4d)  $6(x + 2)$

## Mini-Lecture 2.2

### The Addition Property of Equality

#### Learning Objectives:

1. Define linear equations and use the addition property of equality to solve linear equations
2. Write word phrases as algebraic expressions.

#### Examples:

1. Solve each equation. Check each solution.

a)  $y - 6 = 18$                       b)  $-18 = t - 5$                       c)  $8.1 + y = 13.9$                       d)  $a + \frac{2}{3} = -\frac{3}{4}$

Solve each equation. If possible, be sure to first simplify each side of the equation. Check each solution.

e)  $5(y + 2) = 6(y - 3)$

f)  $10x = 4x + 9 + 5x$

g)  $-8z + 5 + 6z = -3z + 10$

h)  $-5x + 4 + 6x = 15 - 28$

i)  $-\frac{1}{6}x - \frac{1}{3} = \frac{5}{6}x + \frac{1}{2}$

j)  $-14.9 + 4a - 2.7 + 2a = 5.1 + 7a + 1.5$

2. Write each algebraic expression described.

a) Two numbers have a sum of 72. If one number is  $z$ , express the other number in terms of  $z$ .

b) During a recent marathon, Tom ran 8 more miles than Judy ran. If Judy ran  $x$  miles, how many miles did Tom run?

c) On a recent car trip, Raymond drove  $x$  miles on day one. On day two, he drove 170 miles more than he did on day one. How many miles, in terms of  $x$ , did Raymond drive for both days combined?

#### Teaching Notes:

- Some students need a quick review of “like terms”.
- Advise students to write out each step until they have mastered this concept. Avoid shortcuts!
- Some students need to be taught how to work a problem in sequential order showing each step.
- Encourage students to take their time and organize their work. This will help when the problems become more complex.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 24; 1b) -13; 1c) 5.8; 1d) -17/12; 1e) 28; 1f) 9; 1g) 5; 1h) -17; 1i) -5/6; 1j) 11;  
2a)  $72 - z$ ; 2b)  $x + 8$ ; 2c)  $2x + 170$

## Mini-Lecture 2.3

### The Multiplication Property of Equality

#### Learning Objectives:

1. Use the multiplication property of equality to solve linear equations.
2. Use both the addition and multiplication properties of equality to solve linear equations.
3. Write word phrases as algebraic expressions.

#### Examples:

1. Use the multiplication property of equality to solve the following linear equations. Check each solution.
  - a)  $-8x = -24$
  - b)  $7x = 0$
  - c)  $-z = 19$
  - d)  $3x = -22$
  - e)  $\frac{2}{5}a = 12$
  - f)  $\frac{y}{-11} = 2.5$
  - g)  $\frac{-3}{8}b = 0$
  - h)  $-10.2 = -3.4c$
2. Use the addition property of equality and the multiplication property of equality to solve the following linear equations. Check each solution.
  - a)  $5x + 6 = 46$
  - b)  $\frac{a}{9} - 7 = 11$
  - c)  $-24 = -3x - 9$
  - d)  $\frac{1}{3}y - \frac{1}{3} = -6$
  - e)  $-5.8z + 1.9 = -32.5 - 1.5z$
  - f)  $8y + 7 = 6 - 2y - 10y$
  - g)  $4(4x - 1) = (-8) - (-24)$
3. Write each algebraic expression described. Simplify if possible.
  - a) If  $z$  represents the first of two consecutive even integers, express the sum of the two integers in terms of  $z$ .
  - b) If  $x$  represents the first of three consecutive even integers, express the sum of the first and third integer in terms of  $x$ .
  - c) Houses on one side of a street are all numbered using consecutive odd integers. If the first house on the street is numbered  $x$ , write an expression in  $x$  for the sum of five house numbers in a row.

#### Teaching Notes:

- Review “like terms” with students.
- Many students do not combine like terms before using one of the properties.
- Encourage students to always take the time to check their solution.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 3; 1b) 0; 1c) -19; 1d) -22/3; 1e) 30; 1f) -27.5; 1g) 0; 1h) 3; 2a) 8; 2b) 162; 2c) 5; 2d) -17; 2e) 8; 2f) -1/20; 2g) 5/4; 3a)  $2z+2$ ; 3b)  $2x+4$ ; 3c)  $5x+20$

## Mini-Lecture 2.4

### Solving Linear Equations

#### Learning Objectives:

1. Apply a general strategy for solving a linear equation.
2. Solve equations containing fractions.
3. Solve equations containing decimals.
4. Recognize identities and equations with no solution.

#### Examples:

1. Solve the following linear equations.

a)  $6a - (5a - 1) = 4$

b)  $4(3b - 1) = 16$

c)  $4z = 8(2z + 9)$

d)  $2(x + 8) = 3(x - 5)$

e)  $3(2a - 3) = 5(a + 4)$

f)  $12(4c - 2) = 3c - 4$

2. Solve each equation containing fractions.

a)  $\frac{y}{6} - 4 = 1$

b)  $\frac{1}{4}x - \frac{3}{8}x = 5$

c)  $\frac{-6x + 5}{4} + 1 = -\frac{5x}{4}$

Solve each equation containing decimals.

d)  $0.05x + 0.06(x - 1,500) = 570$

e)  $0.4(x + 7) - 0.1(3x + 6) = -0.8$

3. Solve each equation. Indicate if it is an identity or an equation with no solution.

a)  $6(z + 7) = 6z + 42$

b)  $3 + 12x - 1 = 8x + 4x - 1$

c)  $\frac{x}{3} - 3 = \frac{2x}{6} + 1$

#### Teaching Notes:

- Refer students to the beginning of this section in the textbook for steps: To Solve Linear Equations in One Variable.
- Most students find solving equations with fractions or decimals difficult.
- Common error: When multiplying equations with fractions by the LCD, some students multiply only the terms with fractions instead of all terms.
- Common error: When solving equations with decimals and parentheses (examples 2d and 2e), some students multiply terms both inside parentheses and outside parentheses by a power of 10.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 3; 1b) 5/3; 1c) -6; 1d) 31; 1e) 29; 1f) 4/9; 2a) 30; 2b) -40; 2c) 9; 2d) 6,000 2e) -30; 3a) identify; 3b) no solution; 3c) no solution

## Mini-Lecture 2.5

### An Introduction to Problem Solving

#### Learning Objectives:

#### Apply the steps for problem solving as we

1. Solve problems involving direct translation.
2. Solve problems involving relationships among unknown quantities.
3. Solve problems involving consecutive integers.

#### Examples:

1. Solve.
  - a) Eight is added to a number and the sum is doubled, the result is  $-11$  less than the number. Find the number.
  - b) Three times the difference of a number and 2 is equal to 8 subtracted from twice a number. Find the integers.
2. Solve.
  - a) A college graduating class is made up of 450 students. There are 206 more girls than boys. How many boys are in the class?
  - b) A 22-ft pipe is cut into two pieces. The shorter piece is 7 feet shorter than the longer piece. What is the length of the longer piece?
  - c) A triangle has three angles, A, B, and C. Angle C is  $18^\circ$  greater than angle B. Angle A is 4 times angle B. What is the measure of each angle?  
(Hint: The sum of the angles of a triangle is  $180^\circ$ ).
3. Solve.
  - a) The room numbers of two adjacent hotel rooms are two consecutive odd numbers. If their sum is 1380, find the hotel room numbers.
  - b) When you open a book, the left and right page numbers are two consecutive natural numbers. The sum of their page numbers is 349. What is the number of the page that comes first?

#### Teaching Notes:

- Many students find application problems challenging.
- Encourage students, whenever possible, to draw diagrams, charts, etc.
- Encourage students to use algebra to solve a problem even though they may be able to solve without it.
- Refer students to *General Strategy for Problem Solving* section 2.4.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $-27$ ; 1b) 21, 63; 2a) 122 boys; 2b) 14.5 feet; 2c)  $A=108^\circ$ ,  $B=27^\circ$ ,  $C=45^\circ$ ; 3a) 689, 691;  
3b) 174

## Mini-Lecture 2.6

### Formulas and Problem Solving

#### Learning Objectives:

1. Use formulas to solve problems.
2. Solve a formula or equation for one of its variables.

#### Examples:

1. Substitute the given values into each given formula and solve for the unknown variable, If necessary, round to one decimal place.
  - a) Distance Formula  
 $d = rt; t = 9, d = 63$
  - b) Perimeter of a rectangle  
 $P = 2l + 2w; P = 32, w = 7$
  - c) Volume of a pyramid  
 $V = \frac{1}{3}Bh; V = 40, h = 8$
  - d) Simple interest  
 $I = prt; I = 23, p = 230, r = 0.02$
  - e) Convert the record high temperature of 102°F to Celsius. ( $F = \frac{9}{5}C + 32$ )
  - f) You have decided to fence an area of your backyard for your dog. The length of the area is 1 meter less than twice the width. If the perimeter of the area is 70 meters, find the length and width of the rectangular area.
  - g) For the holidays, Chris and Alicia drove 476 miles. They left their house at 7 a.m. and arrived at their destination at 4 p.m. They stopped for 1 hour to rest and re-fuel. What was their average rate of speed?
2. Solve each formula for the specified variable.
  - a) Area of a triangle  
 $A = \frac{1}{2}bh$  for  $b$
  - b) Perimeter of a triangle  
 $P = s_1 + s_2 + s_3$  for  $s_3$
  - c) Surface area of a special rectangular box  
 $S = 4lw + 2wh$  for  $l$
  - d) Circumference of a circle  
 $C = 2\pi r$  for  $r$

#### Teaching Notes:

- Most students will only need algebra reminders when working with a formula given values.
- Refer students to *Solving Equations for a Specified Variable* chart in the textbook.
- Most students have problems with applications. Refer them back to section 2.4 and the *General Strategy for Problem Solving* in the textbook.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 7; 1b) 9; 1c) 15; 1d) 5; 1e) 38.9°C; 1f)  $l=23, w=12$ ; 1g) 59.5 mph; 2a)  $b = \frac{2A}{h}$ ;

2b)  $s_3 = P - s_1 - s_2$ ; 2c)  $\frac{S - 2wh}{4w}$ ; 2d)  $r = \frac{C}{2\pi}$

## Mini-Lecture 2.7

### Percent and Mixture Problem Solving

#### Learning Objectives:

1. Solve percent equations.
2. Solve discount and mark-up problems.
3. Solve percent increase and percent decrease problems.
4. Solve mixture problems.

#### Examples:

1. Find each number described.
 

a) 5% of 300 is what number?	b) 207 is 90% of what number?
c) 15 is 1% of what number?	d) What percent of 350 is 420?
  
2. Solve the following discount and mark-up problems. If needed, round answers to the nearest cent.
  - a) A “Going-Out-Of-Business” sale advertised a 75% discount on all merchandise. Find the discount and the sale price of an item originally priced at \$130.
  - b) Recently, an anniversary dinner cost \$145.23 excluding tax. Find the total cost if a 15% tip is added to the cost.
  
3. Solve the following percent increase and decrease problems.
  - a) The number of minutes on a cell phone bill went from 1200 minutes in March to 1600 minutes in April. Find the percent increase. Round to the nearest whole percent.
  - b) In 2004, a college campus had 8,900 students enrolled. In 2005, the same college campus had 7,600 students enrolled. Find the percent decrease. Round to the nearest whole percent.
  - c) Find the original price of a pair of boots if the sale price is \$120 after a 20% discount.
  
4. How much pure acid should be mixed with 4 gallons of a 30% acid solution in order to get a 80% acid solution? Use the following table to model the situation.

	Number of Gallons · Acid Strength = Amount of Acid		
Pure Acid			
30% Acid Solution			
80% Acid Solution Needed			

#### Teaching Notes:

- Most students find problem solving challenging. Encourage students to make a list of all appropriate formulas.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a) 15; 1b) 230; 1c) 1500; 1d) 120%; 2a) discount - \$97.50, sale price - \$32.50; 2b) \$167.01; 3a) 33%; 3b) 15%; 3c) \$150; 4) 10 gallons

## Mini-Lecture 2.8

### Further Problem Solving

#### Learning Objectives:

1. Solve problems involving distance.
2. Solve problems involving money.
3. Solve problems involving interest.

#### Examples:

1. How long will it take a car traveling 60 miles per hour to overtake an activity bus traveling 45-miles per hour if the activity bus left 2 hours before the car?

	<i>r</i>	<i>D</i>	<i>t</i>
<b>Car</b>	60 mph	60x	x
<b>Activity Bus</b>	45 mph	45(x + 2)	x + 2

2. A collection of dimes and quarters and nickels are emptied from a drink machine. There were four times as many dimes as quarters, and there were ten less nickels than there were quarters. If the value of the coins was \$19.50, find the number of quarters, the number of dimes, and the number of nickels.

	Number	Value of each	Total value	
<b>Quarters</b>	x	0.25	0.25x	40 @ 0.25=\$10.00
<b>Dimes</b>	2x	0.10	0.10(2x)	80 @ 0.10=\$8.00
<b>Nickels</b>	x - 10	0.05	0.05(x - 10)	30 @ 0.05=\$1.50
<b>Entire Collection</b>			\$19.50	\$19.50

3. Jeff received a year end bonus of \$80,000. He invested some of this money at 8% and the rest at 10%. If his yearly earned income was \$7,300, how much did Jeff invest at 10%? Use the following table to model the situation.

	Principal	Rate	Time	= Interest
8% Fund	x	0.08	1	0.08x
10% Fund	80,000 - x	0.1	1	0.01(80,000 - x)
<b>Total</b>	80,000			7,300

#### Teaching Notes:

- Most students find problem solving challenging. Encourage students to make a list of all appropriate formulas.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1) 6 hours; 2) Number of Quarters = 40, Number of dimes = 80, number of nickels = 30; 3) \$45,000



## Mini-Lecture 2.9

### Solving Linear Inequalities

#### Learning Objectives:

1. Define linear inequality in one variable, graph solution sets on a number line, and use interval notation.
2. Solve linear inequalities.
3. Solve compound inequalities.
4. Solve inequality applications.
5. Key Vocabulary: *inequality*,  $<$ ,  $\leq$ ,  $>$ ,  $\geq$ , *addition property of inequality*, *multiplication property of inequality*, *at least*, *no less than*, *at most*, *no more than*, *is less than*, *is greater than*.

#### Examples:

1. Graph each inequality on a number line and write it in interval notation.  
a)  $x \geq -5$                       b)  $y < 7$                       c)  $-\frac{3}{2} \geq m$                       d)  $x > -\frac{2}{5}$
2. Using the addition property of inequality, solve each inequality. Graph the solution set and write it in interval notation.

a)  $x + 7 \leq 12$                       b)  $x - 10 > -3$                       c)  $-4z - 2 > -5z + 1$                       d)  $18 - 2x \leq -3x + 24$

Using the multiplication property of inequality, solve each inequality. Graph the solution set and write it in interval notation.

e)  $-8 \geq \frac{x}{3}$                       f)  $3x < 73$                       g)  $0 < \frac{y}{8}$                       h)  $-\frac{3}{5}z \leq 9$

Using both properties, solve each inequality.

i)  $3(3x - 16) < 12(x - 2)$                       j)  $-18(z - 2) \geq -21z + 24$                       k)  $\frac{8}{21}(x + 2) > \frac{1}{7}(x + 3)$

3. Solve each inequality. Graph the solution set and write it in interval notation.

a)  $-5 < t \leq 0$                       b)  $-12 \leq 2x < -8$                       c)  $3 \leq 4x - 9 \leq 7$

4. Solve the following.

a) Eight more than twice a number is less than negative twelve. Find all numbers that make this statement true.

b) One side of a triangle is six times as long as another side and the third side is 8 inches long. If the perimeter can be no more than 106 inches, find the maximum lengths of the other two sides.

#### Teaching Notes:

- Remind students to reverse the direction of the inequality symbol when multiplying or dividing by a negative number.
- Suggest students keep the coefficient of the variable positive whenever possible.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) – 3c) graph answers at end of mini-lectures; 1a)  $[-5, \infty)$ ; 1b)  $(\infty, 7)$ ; 1c)  $[-3/2, \infty)$ ; 1d)  $(-2/5, \infty)$ ; 2a)  $(-\infty, 5]$ ; 2b)  $(7, \infty)$ ; 2c)  $(3, \infty)$ ; 2d)  $(-\infty, 6]$ ; 2e)  $(-\infty, -24]$ ; 2f)  $(-\infty, 24 \frac{1}{3})$ ; 2g)  $(0, \infty)$ ; 2h)  $[-15, \infty)$ ; 2i)  $(-8, \infty)$ ; 2j)  $[-4, \infty)$ ; 2k)  $(7/5, \infty)$ ; 3a)  $(-5, 0]$ ; 3b)  $[-6, 4)$ ; 3c)  $[3, 4]$ ; 4a)  $x < -10$ ; 4b) 14, 84

# Mini-Lecture 3.1

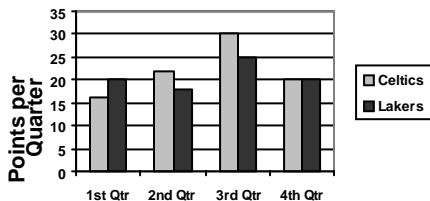
## Reading Graphs and the Rectangular Coordinate System

### Learning Objectives:

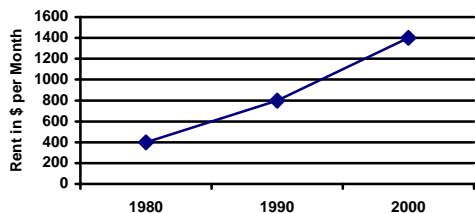
1. Read bar and line graphs.
2. Define the rectangular coordinate system and plot ordered pairs of numbers.
3. Graph paired data to create a scatter diagram.
4. Determine whether an ordered pair is a solution of an equation in two variables.
5. Find the missing coordinate of an ordered pair solution, given one coordinate of the pair.

### Examples:

1. a) The following bar graph shows points scored per quarter in a basketball game. Use the bar graph to find the final score.



- b) The following line graph shows the average monthly rent for people in Worcester. How much did monthly rent increase from 1980 – 2000.



2. Plot each order pair. State in which quadrant or on which axis each point lies.

- a) (-2, -5)                      b) (0, -4)                      c)  $(2\frac{2}{3}, 4\frac{1}{2})$                       d) (-1, 4)

3. The table gives a bookstore's net dollar sales for the period of 2000-2005.

Year (x)	2000	2001	2002	2003	2004	2005
Sales – in thousands (y)	19	22	21	23	25	26

- Write this paired data as a set of ordered pairs of the form (year, sales)
- In your own words, write the meaning of the order pair.
- Create a scatter diagram of the paired data.

4. Determine whether each ordered pair is a solution of the given linear equation.

- a)  $5x + y = 15$ ; (1,4), (2,5), (0, 15)                      b)  $x = \frac{1}{4}y$ ; (1,4), (8,2), (0,0)

5. Complete each ordered pair so that it is a solution of the given linear equation.

- a)  $x + 2y = 6$ ; (2,   ), (   , -3)                      b)  $y = \frac{1}{3}x - 2$ ; (6,   ), (   ,  $-\frac{1}{3}$ )

### Teaching Notes:

- Most students can read bar and line graphs successfully.
- Many students have trouble putting meaning to an ordered pair.
- Remind students that an ordered pair is (x, y) – alphabetical order.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a) 88 to 82; 1b) \$1000; 2a) III; 2b) y-axis; 2c) I; 2d) II; 3a)  $\{(2000,19), (2001,22), (2002, 21), (2003, 23), (2004, 25), (2005, 26)\}$ ; 3b) answers will vary; 4a) No, Yes, Yes; 4b) Yes, No, Yes;

5a) (2, 2), (12, -3); 5b) (6, 0), (5,  $-\frac{1}{3}$ )

## Mini-Lecture 3.2

### Graphing Linear Equations

#### Learning Objectives:

1. Identify linear equations.
2. Graph a linear equation by finding and plotting ordered pair solutions.

#### Examples:

1. For each equation, find three ordered pair solutions by completing the table. Then use the ordered pairs to graph the equation.

a)  $x - y = 2$

x	y
3	
	-2
-1	

b)  $y = -\frac{1}{3}x - 2$

x	y
6	
	-4
	0

c)  $y = \frac{2}{3}x$

x	y
-6	
	0
	$\frac{10}{3}$

d)  $y = -3$

x	y
2	
-1	
0	

Graph the following linear equations.

e)  $x + y = 0$

f)  $y = -2x - 1$

g)  $x - 2 = 0$

2. Solve: The value of a house ( $y$ ) increases in value  $x$  years after purchase by the formula  $y = 7,500x + 120,000$ .
  - a) Graph the linear equation
  - b) Complete the ordered pair  $(5, \quad)$
  - c) Write a sentence explaining the meaning of the ordered pair found in part b.

#### Teaching Notes:

- Problems 1a) – d) tend to pose the least amount of challenge.
- Some students become very confused when they can choose any value for  $x$  or  $y$  as a starting point for finding an ordered-pair solution.
- Many students do not understand problems 1d) and 1g) and must memorize the form for an equation for a horizontal or a vertical line.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers (for all graphs, see Mini-Lecture graphing answers at end of section): 1a) (3,1), (0, -2), (-1, -3); 1b) (6, -4), (6, -4), (-6, 0); 1c) (-6, -4), (0,0), (5,  $\frac{10}{3}$ ); 1d) (-2,-3), (-1, -3), (0, -3); 1e) – 1g) see graphing answers; 2a) see graphing answers; 2b) (5, \$157,500); 2c) After 5 years, the house's value increased to \$157,500.*

## Mini-Lecture 3.3

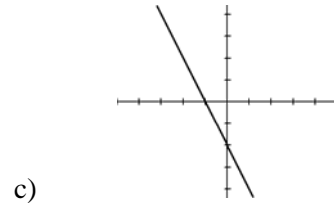
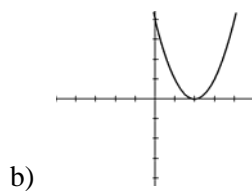
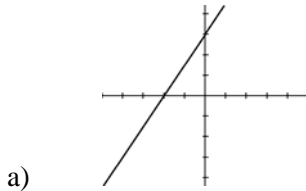
### Intercepts

#### Learning Objectives:

1. Identify intercepts of a graph.
2. Graph a linear equation by finding and plotting intercepts.
3. Identify and graph vertical and horizontal lines.

#### Examples:

1. Identify the intercepts.



2. Graph each linear equation by finding and plotting its intercepts.

a)  $x - y = 2$

b)  $x - y = -3$

c)  $2x + 4y = 8$

d)  $x - 3y = 0$

e)  $y = 3x + 3$

f)  $y = -2x - 4$

3. Identify the type of equation (horizontal or vertical line) and graph the equation.

a)  $x = -3$

b)  $y = 2$

c)  $x + 3 = 5$

#### Teaching Notes:

- Sometimes, students will list the intercepts as a single number; not an ordered pair. For example: x-intercept: 3, y-intercept: 4.
- Remind students that any time  $(0, 0)$  is a point on a graph, then that is both its x- and y-intercept.
- Some students confuse horizontal and vertical lines. For example: if  $x = -5$ , have students mentally graph the line that would intersect the x axis at  $-5$ . This line could only be vertical, not horizontal.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $(-2, 0), (0, 3)$ ; 1b)  $(2, 0), (0, 4)$ ; 1c)  $(-1, 0), (0, -2)$ ; 2a) – 2f) see mini-lecture graphing answers; 3a) vertical; 3b) horizontal; 3c) vertical.

## Mini-Lecture 3.4

### Slope and Rate of Change

#### Learning Objectives:

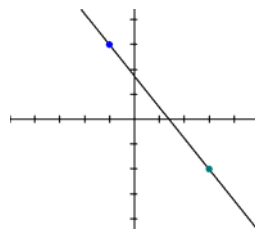
1. Find the slope of a line given two points of the line.
2. Find the slope of a line given its equation.
3. Find the slopes of horizontal and vertical lines.
4. Compare the slopes of parallel and perpendicular lines.
5. Slope as a rate of change.

#### Examples:

1. Find the slope of the line that passes through the given points.

a) (6, 5) and (1, 7)

b)



c) (-5, 0) and (0, -3)

2. Find the slope of each line.

a)  $x + y = 12$

b)  $3x + y = 8$

c)  $11x - 3y = 33$

d)  $9x + y = -12$

e)  $y + 5 = 0$

f)  $2x - 7 = 0$

3. Determine whether each pair of lines is parallel, perpendicular, or neither.

a)  $3x = 2y + 3$   
 $2x + 3y = 2$

b)  $x + 3y = 4$   
 $8x + 2y = 2$

c)  $9x = 16 - 3y$   
 $16 - 4y = 12x$

Find the slope of a line that is (a) parallel and (b) perpendicular to the line passing through each pair of points.

d) (-5, -5) and (-1, -1)

e) (-2, 10) and (5, -4)

4. An inclined ramp leading to a warehouse is to rise 16 inches for each horizontal distance of 17 feet. Write this slope as a grade. (Round to the nearest tenth of a percent, if necessary).

#### Teaching Notes:

- Many students confuse the change in y and change in x in the slope formula. Hint: if you can imagine a picnic table ( $\overline{X} \quad \overline{X}$ ), the x is on the bottom. If the y is on the bottom ( $\overline{Y} \quad \overline{Y}$ ), the picnic table will fall over!
- Remind students to “read” the slope of the line as it moves from left to right.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $-\frac{2}{5}$ ; 1b)  $\frac{5}{4}$ ; 1c)  $-\frac{3}{5}$ ; 2a) -1, 2b) -3; 2c)  $\frac{11}{3}$ ; 2d) -9; 2e) 0; 2f) undefined;

3a) perpendicular; 3b) neither; 3c) parallel; 3d) a: 1, b: -1; 3e) a: -2, b: 1/2; 4) 4/51 or 7.8%

## Mini-Lecture 3.5

### Equations of Lines

#### Learning Objectives

1. Use the slope-intercept form to write an equation of a line.
2. Use the slope-intercept form to graph a linear equation.
3. Use the point-slope form to find an equation of a line given its slope and a point on the line.
4. Use the point-slope form to find an equation of a line given two points on the line.
5. Find equations of vertical and horizontal lines.
6. Use the point-slope form to solve problems.

#### Examples:

1. Write an equation of the line with each given slope,  $m$ , and  $y$ -intercept,  $(0, b)$ .  
a)  $m = -9; b = 4$       b)  $m = -\frac{2}{3}; b = 7$       c)  $m = 0; b = \frac{1}{2}$       d)  $m = -\frac{5}{2}; b = \frac{31}{2}$
2. Use the slope-intercept form to graph each equation.  
a)  $y = \frac{1}{2}x - 3$       b)  $y = -\frac{1}{4}x + 2$       c)  $y = -4x$       d)  $5x + 2y = 10$
3. Find an equation of each line with the given slope that passes through the given point. Write the equation in the form  $Ax + By = C$ .  
a)  $m = 4; (10, 5)$       b)  $m = -\frac{7}{9}; (5, 2)$       c)  $m = -6; (-8, -10)$       d)  $m = \frac{1}{2}; (-4, 8)$
4. Find an equation of the line passing through each pair of points. Write the equation in the form  $Ax + By = C$ .  
a)  $(-7, -4)$  and  $(0, 5)$       b)  $(3, 7)$  and  $(-2, -6)$   
c)  $(9, -9)$  and  $(6, -5)$       d)  $\left(-\frac{1}{2}, \frac{3}{4}\right)$  and  $\left(-\frac{5}{3}, \frac{1}{3}\right)$
5. Find an equation of each line.  
a) Vertical line through  $(0, 5)$       b) Horizontal line through  $(4, 3)$
6. Solve.  
Assume the following describes a linear relationship. Write an equation in slope-intercept form. A faucet is used to add water to a large bottle that already contains some water. After it has been filling for 3 seconds, the gauge on the bottle indicates that it contains 10 ounces of water. After it has been filling for 20 seconds, the gauge indicates the bottle contains 24 ounces of water. Let  $y$  be the amount of water in the bottle  $x$  seconds after the faucet was turned on. Write a linear equation that models the amount of water in the bottle in terms of  $x$ .

#### Teaching Notes:

- Many students do not understand that you leave “ $x$  and  $y$ ” in the final equation.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $y = -9x + 4$ ; 1b)  $y = -\frac{2}{3}x + 7$ ; 1c)  $y = \frac{1}{2}$ ; 1d)  $y = -\frac{5}{2}x + \frac{31}{2}$ ; 2a)-2d) see mini-lecture graphing answers; 3a)  $4x - y = 35$ ; 3b)  $7x + 9y = 53$ ; 3c)  $6x + y = -58$ ; 3d)  $x - 2y = -20$ ; 4a)  $9x - 7y = -35$ ; 4b)  $13x - 5y = 4$ ; 4c)  $4x + 3y = 9$ ; 4d)  $5x - 14y = -13$ ; 5a)  $x = 0$ ; 5b)  $y = 3$ ; 6)  $14x - 17y = -128$

# Mini-Lecture 3.6

## Functions

### Learning Objectives:

1. Identify relations, domains, and ranges.
2. Identify functions.
3. Use the vertical line test.
4. Use function notation.

### Examples:

1. Find the domain and range of each relation.

a)  $\{(2,3),(0,0),(-1,-5),(-2,6)\}$

b)  $\{(3,1),(3,-2),(3,0),(3,6)\}$

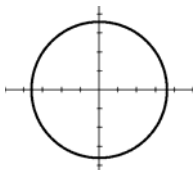
2. Determine whether each relation is also a function.

a)  $\{(10,5),(-3,-2),(2,-1),(6,5)\}$

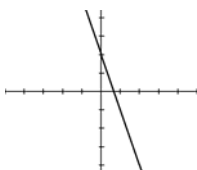
b)  $\{(3,5),(-3,5),(-3,0),(2,4)\}$

3. Determine whether each graph is the graph of a function.

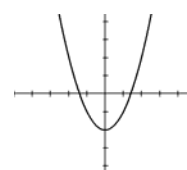
a)



b)



c)



4. For each function (a – c), find the value of the  $f(-3)$ ,  $f(2)$ , and  $f(0)$ .

a)  $f(x) = -\frac{1}{3}x - 5$

b)  $f(x) = 3x^2 - 2x - 2$

c)  $f(x) = |-3 - x|$

- d) If  $f(4) = 8$ , write a corresponding ordered-pair solution.

### Teaching Notes:

- At first, students find the definition of relation and function confusing.
- Many students have trouble determining if a relation is a function.
- Some students struggle with the concept of domain and range.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a)  $D: \{-2, -1, 0, 2\}$ ,  $R: \{-5, 0, 3, 6\}$ ; 1b)  $D: \{3\}$ ,  $R: \{-2, 0, 1, 6\}$ ; 2a) yes; 2b) no; 3a) no; 3b) yes; 3c) yes; 4a)  $-4, -\frac{17}{3}, -5$ ; 4b) 31, 6, -2; 4c) 0, 5, 3; 4d) (4, 8)

## Mini-Lecture 4.1

### Solving Systems of Linear Equations by Graphing

#### Learning Objectives:

1. Determine if an ordered pair is a solution of a system of equations in two variables.
2. Solve a system of linear equations by graphing.
3. Without graphing, determine the number of solutions of a system.

#### Examples:

1. Determine whether the ordered pair is a solution of the system of linear equations.

a)  $(-4, -5) \begin{cases} x + y = -9 \\ x - y = 1 \end{cases}$

b)  $(-5, -3) \begin{cases} 2x + y = -7 \\ 3x + 2y = -9 \end{cases}$

c)  $(2, -4) \begin{cases} 4x = 4 - y \\ 2x = -12 - 4y \end{cases}$

d)  $(-3, 1) \begin{cases} 3x = 10 - y \\ 4x = 15 - 3y \end{cases}$

2. Solve each system of linear equations by graphing. Note: All systems have a solution.

a)  $\begin{cases} 4x + y = -4 \\ 5x + 2y = -2 \end{cases}$

b)  $\begin{cases} 3x + 2y = 22 \\ 2x + 4y = 28 \end{cases}$

c)  $\begin{cases} x = 6 \\ \frac{1}{6}x - y = 1 \end{cases}$

d)  $\begin{cases} 2x + 5y = 32 \\ 3y = 20 - 2x \end{cases}$

3. Without graphing, determine the number of solutions of a system.  
Note: the systems have no solution or an infinite number of solutions.

a)  $\begin{cases} 4x - 16y = 12 \\ y = \frac{1}{4}x - \frac{3}{4} \end{cases}$

b)  $\begin{cases} -x = y \\ x = 6 - y \end{cases}$

#### Teaching Notes:

- Many students need to be reminded to use graph paper and be very neat with their graphing skills.
- Remind students to substitute their solution into the original equations to check their results.
- Many students get confused between consistent and inconsistent systems and its meaning for the solution.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) yes; 1b) no; 1c) yes; 1d) no; 2a)–2d) see mini-lecture graphing answers; 3a) infinite; 3b) no solution



## Mini-Lecture 4.2

### Solving Systems of Linear Equations by Substitution

#### Learning Objectives:

1. Use the substitution method to solve a system of linear equations.

#### Examples:

1. Solve each system of equations by the substitution method. Note: the following systems have one equation already solved for one variable.

a) 
$$\begin{cases} x + y = 9 \\ y = 2x \end{cases}$$

b) 
$$\begin{cases} x = y - 2 \\ x + y = 6 \end{cases}$$

Solve each system of equations by the substitution method.

c) 
$$\begin{cases} x + 6y = 2 \\ 4x + 5y = -11 \end{cases}$$

d) 
$$\begin{cases} x - 3y = 3 \\ -5x - 2y = 2 \end{cases}$$

e) 
$$\begin{cases} x - 4y = -1 \\ 6x - 3y = -6 \end{cases}$$

f) 
$$\begin{cases} 6x + 7y = 33 \\ 3x - 3y = -42 \end{cases}$$

g) 
$$\begin{cases} 4x - 3y = 30 + x \\ 4x = -(y + 2) + 3x \end{cases}$$

h) 
$$\begin{cases} x - y = -4 \\ \frac{1}{2}x + \frac{1}{2}y = -3 \end{cases}$$

i) 
$$\begin{cases} 4x + y = 11 \\ 12x + 3y = 33 \end{cases}$$

j) 
$$\begin{cases} -6x - 24y = -10 \\ 5x + 20y = 0 \end{cases}$$

#### Teaching Notes:

- Remind students to check their solution in the *original* equations.
- Many students write their final answer as  $x =$  a number and  $y =$  a number rather than an ordered pair  $(x, y)$ .
- Many students find working with fractional coefficients challenging.
- Refer students to the textbook's summary "To Solve a System of Two Linear Equations by the Substitution Method".
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) (3, 6); 1b) (2, 4); 1c) (-4, 1); 1d) (0, -1); 1e) (-1, 0); 1f) (-5, 9); 1g) (4, -6); 1h) (-5, -1); 1i) infinite; 1j) no solution.

## Mini-Lecture 4.3

### Solving Systems of Linear Equations by Addition

#### Learning Objectives:

1. Use the addition method to solve a system of linear equations.

#### Examples:

1. Solve each system of equations by the addition method.

$$\text{a) } \begin{cases} x + y = 5 \\ x - y = 11 \end{cases}$$

$$\text{b) } \begin{cases} -x + 4y = 28 \\ -6x - 4y = -56 \end{cases}$$

Solve each system of equation by the addition method. If a system contains fractions or decimals, you may want to first clear each equation of fractions or decimals.

$$\text{c) } \begin{cases} x + 5y = 49 \\ -7x + 4y = -31 \end{cases}$$

$$\text{d) } \begin{cases} x + 3y = 2 \\ 4x + 2y = 18 \end{cases}$$

$$\text{e) } \begin{cases} -2x - 7y = -6 \\ 5x - 3y = -26 \end{cases}$$

$$\text{f) } \begin{cases} 5x + 8y = 1 \\ 2x + 3y = 2 \end{cases}$$

$$\text{g) } \begin{cases} -x - 2y = -4 \\ 5x + 10y = 8 \end{cases}$$

$$\text{h) } \begin{cases} 4x - 6y = 1 \\ 20x - 30y = 3 \end{cases}$$

$$\text{i) } \begin{cases} 3x + \frac{1}{3}y = 10 \\ 2x + \frac{2}{3}y = 4 \end{cases}$$

$$\text{j) } \begin{cases} 3.5x + 0.3y = -18.7 \\ 0.7x + 0.9y = -7.1 \end{cases}$$

#### Teaching Notes:

- Encourage students to discuss which variable is the easiest to eliminate and what number an equation should be multiplied by to make the elimination possible.
- Remind students that there can be more than one way to solve a system.
- Remind students to check their solution.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) (8, -3); 1b) (4, 8); 1c) (9, 8); 1d) (5, -1); 1e) (-4, 2); 1f) (13, -8); 1g) no solution; 1h) no solution; 1i) (4, -6); 1j) (-5, -4)

## Mini-Lecture 4.4

### Systems of Linear Equations and Problem Solving

#### Learning Objectives:

1. Use a system of equations to solve problems.

#### Examples:

1. Solve .
  - a) **Finding Unknown Numbers:** The sum of two numbers is 7. Three times the first number equals 4 times the second number. Find the two numbers.
  - b) **Finding Unknown Numbers:** One number is four more than a second number. Two times the first number is 2 more than four times the second number.
  - c) **Solving a Problem about Prices:** Alicia purchased tickets to a local comedy club for 5 adults and 2 children. The total cost was \$161. The cost of a child's ticket was \$7 less than the cost of an adult's ticket. Find the price of an adult's ticket and a child's ticket.
  - d) **Solving a Problem about Prices:** Allison throws loose change found in the laundry into a container. After one month, she finds it contains all nickels and dimes. In fact, there are 4 times as many dimes as nickels, and the value of the dimes is \$3.50 more than the value of the nickels. How many nickels and dimes does Allison have?
  - e) **Finding Rates:** Kyle and Jason live 28 miles apart in Central Massachusetts. They decide to bicycle towards each other and meet somewhere in between. Kyle's rate of speed is 40% of Jason's. They start out at the same time and meet 2 hours later. Find Kyle's rate of speed.
  - f) **Finding Amounts of Solutions:** Amy has 3 liters of a 35% solution of sodium hydroxide in a container. What is the amount and concentration of sodium hydroxide solution she must add to this in order to end up with 7 liters of 27% solution?

#### Teaching Notes:

- Most students struggle with word problems.
- Refer students to the textbook's *Problem-Solving Steps* for guidance.
- Encourage students to draw and label diagrams or construct charts whenever possible.
- Entertain a discussion around which algebraic method, substitution or addition, is appropriate for the word problem.
- Remind students to always check their answer.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 4,3; 1b) 7,3; 1c) \$25, \$18; 1d) 10 nickels, 40 dimes; 1e) 4 mph; 1f) 4 liters of 21% solution

## Mini-Lecture 4.5

### Graphing Linear Inequalities

#### Learning Objectives:

1. Graph a linear inequality in two variables.

#### Examples:

1. Determine whether the ordered pairs given are solutions of the linear inequality in two variables.

a)  $x - y > -2$ ;  $(0, -1)$ ,  $(1, 4)$       b)  $2x + 4y \geq 6$ ;  $(4, -1)$ ,  $(-3, 3)$

c)  $x > -y$ ;  $(0, 0)$ ,  $(3, -2)$       d)  $y > \frac{1}{3}x - 1$ ;  $(0, 0)$ ,  $(-3, -1)$

Graph each inequality.

e)  $x + y \geq 2$       f)  $y < -\frac{1}{5}x$       g)  $x - y > -3$

h)  $2x + y \leq -5$       i)  $-2x - 3y < 6$       j)  $x > y$

k)  $y \geq 2$       l)  $x < 5$       m)  $y \geq 0$

#### Teaching Notes:

- Most students who are good at graphing equalities will find this section easy.
- Although many students do not understand the region they are testing in problems 1a) – 1d), most need practice in testing before they begin graphing inequalities.
- Remind students to always use a test point from their proposed solution region to check their work.
- Remind students that the boundary line is dashed for  $<$  or  $>$  and solid for  $\leq$  or  $\geq$ .
- Refer students to the gray instruction block: *To Graph a Linear Inequality in Two Variables*.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) yes, no; 1b) no, yes; 1c) no, yes; 1d) yes, yes; 1e) – 1m) see mini-lecture graphing answers.

## Mini-Lecture 4.6

### Systems of Linear Inequalities

#### Learning Objectives:

1. Graph a system of linear inequalities.

#### Examples:

1. Graph the solution to the following system.

a) 
$$\begin{cases} 2x \leq y \\ x + y \geq 2 \end{cases}$$

b) 
$$\begin{cases} x - y > 3 \\ y < 2 \end{cases}$$

c) 
$$\begin{cases} x \geq -3 \\ y < 2 \end{cases}$$

d) 
$$\begin{cases} 3x > -6 \\ x + y \leq -2 \end{cases}$$

#### Teaching Notes:

- Students may have difficulty finding the solution region even when both inequalities are graphed correctly. Have students shade each inequality with a different color pencil or shading each at a different angle.
- Each section in the text has three worksheets in the Extra Practice featuring differentiated learning.

#### Answers

1a-d) graph answers at end of mini-lectures

# Mini-Lecture 5.1

## Exponents

### Learning Objectives:

1. Evaluate exponential expressions.
2. Use the product rule for exponents.
3. Use the power rule for exponents.
4. Use the power rule for products and quotients.
5. Use the quotient rule for exponents, and define a number raised to the 0 power.
6. Decide which rule(s) to use to simplify an expression.

### Examples:

1. Evaluate each expression.

a)  $3^3$                       b)  $(-7)^2$                       c)  $-6^2$                       d)  $-4y^2$  when  $y = -5$

2. Use the product rule to simplify each expression. Write the results using exponents.

a)  $x^5 \cdot x^3$                       b)  $(4z^3)(9z^5)$                       c)  $(-3x^3y^2)(-5x^4y^6)$                       d)  $(9ab^2c^4)(-11a^3b)(-2b^2c^5)$

3. Use the power rules to simplify each expression.

a)  $(x^7)^3$                       b)  $(y^3)^{11}$                       c)  $(xy)^5$                       d)  $(5x^3y^2z)^3$

4. Use the power rule for products and quotients.

a)  $(-7a^3b^3)^2$                       b)  $\left(\frac{ab}{c}\right)^7$                       c)  $\left(\frac{-3xy}{z^3}\right)^4$                       d)  $\left(\frac{3x^2y^4}{-2z^3}\right)^2$

5. Use the quotient rule and simplify each expression.

a)  $\frac{x^5}{x^2}$                       b)  $\frac{(-6)^{11}}{(-6)^9}$                       c)  $\frac{x^{12}y^5}{x^8y^4}$                       d)  $\frac{8a^3b^8c^3}{18ab^5c^2}$

Simplify each expression.

e)  $8^0$                       f)  $\left(\frac{3}{7}\right)^0$                       g)  $(5x^2y)^0$                       h)  $x^0 + 9^0$

6. Mixed practice. Decide which rules to use and simplify each expression.

a)  $(8a^3b^2c^0)^2$                       b)  $\left(\frac{-3x^2y^5}{2xz^2}\right)^3$                       c)  $(-4a^2c^3)(-6a^3b^2c^7)$                       d)  $\frac{(12ab)^4}{(6a^2b^2)^2}$

### Teaching Notes:

- Most students need a lot of practice to master these rules.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a) 27; 1b) 49; 1c) -36; 1d) -100; 2a)  $x^8$ ; 2b)  $36z^8$ ; 2c)  $15x^7y^8$ ; 2d)  $198a^4b^5c^9$ ; 3a)  $x^{21}$ ; 3b)  $y^{33}$ ;  
3c)  $x^5y^5$ ; 3d)  $125x^9y^6z^3$ ; 4a)  $49a^6b^6$ ; 4b)  $\frac{a^7b^7}{c^7}$ ; 4c)  $\frac{81x^4y^4}{z^{12}}$ ; 4d)  $\frac{9x^4y^8}{4z^6}$ ; 5a)  $x^3$ ; 5b) 36; 5c)  $x^4y$ ;  
5d)  $\frac{4a^2b^3c}{9}$ ; 5e) 1; 5f) 1; 5g) 1; 5h) 2; 6a)  $64a^6b^4$ ; 6b)  $\frac{-27x^4y^{15}}{4z^6}$ ; 6c)  $24a^5b^2c^{10}$ ; 6d) 576

## Mini-Lecture 5.2

### Adding and Subtracting Polynomials

#### Learning Objectives:

1. Define polynomial, monomial, binomial, trinomial, and degree.
2. Find the value of a polynomial given replacement values for the variables.
3. Simplify a polynomial by combining like terms.
4. Add and subtract polynomials.

#### Examples:

1. Find the degree of each polynomial and determine whether it is a monomial, binomial, trinomial, or none of these.

a)  $x^2 + x - 6$

b)  $3x + 10$

c)  $10x^3y^2z$

d)  $8z^5 + 9$

Identify the degrees of the terms and degree of the polynomial.

e)  $2xy - 5x + 6xy$

f)  $4a^3 - 3a + 6$

g)  $x^3y - x^2y^2 + xy^3$

h)  $s^5t^2 - 3s^4t + 5st$

2. Evaluate each polynomial when (a)  $y = 0$ ; and (b)  $y = -2$

a)  $4y - 7$

b)  $y^3 - 4$

c)  $3y^2 + 8y - 9$

d)  $-13 - 4y - y^2$

3. Simplify each expression by combining like terms.

a)  $3x - 10x$

b)  $3y^3 - 6x^2 + 2x^2 - 5y^3$

c)  $3.7x^3 - 6.3x + 11.6 + 1.8x - x^3 - 8.2$

4. Perform the indicated operation.

a)  $(4x - 3) + (2x - 7)$

b)  $(5x^2 + 3x - 7) - (-5x - 3)$

c) Subtract  $(3x + 2y)$  from  $(5x - 7y)$

#### Teaching Notes:

- Most students find these objectives easy.
- Some students, when identifying the degree of a polynomial, get confused when one term is made of different variables.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 2, trinomial; 1b) 1, binomial; 1c) 3, monomial; 1d) 5, binomial; 1e) 2, 1, 2, 2; 1f) 3, 1, 3; 1g) 4, 4, 4, 4; 1h) 7, 5, 2, 7; 2a) -7, -15; 2b) -4, -12; 2c) -9, -13; 2d) -13, -9; 3a) -7x; 3b)  $-2y^2 - 4x^2$ ; 3c)  $2.7x^3 - 4.5x + 3.4$ ; 4a)  $6x - 10$ ; 4b)  $5x^2 + 8x - 4$ ; 4c)  $2x - 9y$

## Mini-Lecture 5.3

### Multiplying Polynomials

#### Learning Objectives

1. Use the distributive property to multiply polynomials.
2. Multiply polynomials vertically.

#### Examples:

1. Multiply the following monomials.

a)  $4x^3 \cdot 2x^6$       b)  $(-3t^4)(5t^3)$       c)  $(-4.2x^3)(5.1x^5)$       d)  $\left(-\frac{2}{7}a^4\right)\left(\frac{7}{8}a^7\right)$

Multiply the monomial by the polynomial.

e)  $5a(-12a - 6)$       f)  $4x^3(-7x + 1)$   
g)  $-6y^5(8y^4 - 12y^2)$       h)  $3ab^7(3ab^3 - 12b^2 - 4a)$

Multiply the following binomials.

i)  $(x + 3)(x - 5)$       j)  $(3x^2 - 4)(2x^3 + 5)$       k)  $\left(x + \frac{3}{4}\right)^2$       l)  $(1 - 5x)(2 - 3x)$   
m)  $(y - 12)(y^2 + 6y - 3)$       n)  $(x - 8)(4 - 5x - x^2)$       o)  $(8ab - b)^2$

2. Multiply vertically.

a)  $(x - 3y)(4x - 5y)$       b)  $(y - 2)(3y^2 + 4y - 1)$       c)  $(x^2 + x + 7)(x^2 + x + 1)$

#### Teaching Notes:

- Most students find this section relatively easy.
- Remind students to be cautious with signs when distributing.
- In 3c) and 3g), many students will “distribute” the exponent to each term in the base instead of squaring the binomial.
- Some students are very hesitant to work vertically.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a)  $8x^9$ ; 1b)  $-15t^7$ ; 1c)  $-21.42x^8$ ; 1d)  $-1/4a^{11}$ ; 1e)  $-60a^2-30a$ ; 1f)  $-28x^4+4x^3$ ; 1g)  $-48y^9+72y^7$ ;  
1h)  $9a^2b^{10}-36ab^9-12a^2b^7$ ; 1i)  $x^2-2x-15$ ; 1j)  $6x^5-8x^3-15x^2-20$ ; 1k)  $x^2 + \frac{6}{4}x + \frac{9}{16}$ ; 1l)  $15x^2-13x+2$ ;  
1m)  $y^3-6y^2-75y+36$ ; 1n)  $-x^3-13x^2-36x-32$ ; 1o)  $64a^2b^2-16ab^2+b^2$ ; 2a)  $4x^2-17xy+15y^2$ ; 2b)  $3y^3-2y^2-9y+2$ ;  
2c)  $x^4 + 2x^3 + 9x^2 + 8x + 7$



## Mini-Lecture 5.4

### Special Products

#### Learning Objectives:

1. Multiply two binomials using the FOIL Method.
2. Square a binomial.
3. Multiply the sum and difference of two terms.

#### Examples:

1. Multiply using FOIL.

a)  $(x + 7)(x - 12)$       b)  $(3x - 1)(2x + 5)$       c)  $(a - 2b)(a + 12b)$       d)  $\left(x + \frac{3}{7}\right)\left(x - \frac{1}{6}\right)$

2. Multiply. (Square a binomial).

a)  $(x + 4)^2$       b)  $(3x - 5)^2$       c)  $(5x - 3y)^2$       d)  $(7a^3 - 4)^2$

3. Multiply the sum and difference of two terms.

a)  $(y - 3)(y + 3)$       b)  $(5x - 1)(5x + 1)$       c)  $\left(2x - \frac{3}{5}\right)\left(2x + \frac{3}{5}\right)$       d)  $(10x - 7y)(10x + 7y)$

4. Mixed practice. Multiply using special products.

a)  $(n + 13)^2$       b)  $(a - 4y)(a + 11y)$       c)  $(t - 2)(t + 13)$       d)  $(3x + 5)(3x - 5)$

e)  $(-5a^2 + 10b)(-5a^2 - 7b)$       f)  $\left(2x - \frac{4}{7}\right)\left(2x + \frac{4}{7}\right)$       g)  $(4x + 13y)^2$

#### Teaching Notes:

- Many students find FOIL easy.
- In examples 2, some students will incorrectly “distribute” the exponent rather than squaring the binomial. For example:  $(x+4)^2 = x^2 + 4^2$ .
- Encourage students to recognize the special products rather than just “FOIL”-ing them.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $x^2 - 5x - 84$ ; 1b)  $6x^2 + 13x - 5$ ; 1c)  $a^2 + 10ab - 24b^2$ ; 1d)  $x^2 + \frac{11}{42}x - \frac{1}{14}$ ; 2a)  $x^2 + 8x + 16$ ;

2b)  $9x^2 - 30x + 25$ ; 2c)  $25x^2 - 30xy + 9y^2$ ; 2d)  $49a^6 - 56a^3 + 16$ ; 3a)  $y^2 - 9$ ; 3b)  $25x^2 - 1$ ; 3c)  $4x^2 - \frac{9}{25}$ ;

3d)  $100x^2 - 49y^2$ ; 4a)  $n^2 + 26n + 169$ ; 4b)  $a^2 + 7ay - 44y^2$ ; 4c)  $t^2 + 11t - 26$ ; 4d)  $9x^2 - 25$ ; 4e)  $25a^4 - 15a^2b - 70b^2$ ;

4f)  $4x^2 - \frac{16}{49}$ ; 4g)  $16x^2 + 104xy + 169y^2$

## Mini-Lecture 5.5

### Negative Exponents and Scientific Notation

#### Learning Objectives:

1. Simplify expressions containing negative exponents.
2. Use all the rules and definitions for exponents to simplify exponential expressions.
3. Write numbers in scientific notation.
4. Convert numbers from scientific notation to standard form.

#### Examples:

1. Simplify each expression. Write each result using positive exponents only.

a)  $3^{-2}$                       b)  $8a^{-3}$                       c)  $\frac{x^{-4}}{y^{-3}}$                       d)  $4^{-2} + 4^0$

2. Simplify each expression. Write each result using positive exponents only.

a)  $a^{-6} \cdot a^{-3} \cdot a \cdot a^{-2}$                       b)  $\frac{(2x^4)^3}{x^{15}}$                       c)  $(x^{-2}y^8)^{-2}$                       d)  $\frac{-6x^2y^{-3}}{-12x^5y^{-6}}$

e)  $\frac{m^2(m^{-4})^{-2}}{(m^{-2})^5}$                       f)  $\left(\frac{3xy^5}{2x^4y^7}\right)^{-3}$                       g)  $\left(\frac{3a^{-3}b^{-2}}{6a^{-2}b^{-5}}\right)^0$                       h)  $(-2r^{-3}s^{-2}t)(-5t^{-4})$

3. Convert the following numbers in standard form to scientific form.

a) 83,000                      b) 1,250,000                      c) 0.000154                      d) 0.00000689

4. Convert the following numbers in scientific form to standard form.

a)  $1.03 \times 10^6$                       b)  $8.7 \times 10^{-5}$                       c)  $6.003 \times 10^{10}$                       d)  $2.02 \times 10^{-3}$

#### Teaching Notes:

- Many students move the numerical coefficient along with the variable. For example, in 1b) a common incorrect answer is  $8a^{-3} = \frac{1}{8a^3}$
- Overall, students need a lot of practice with these rules to master these objectives.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $1/9$ ; 1b)  $8/a^3$ ; 1c)  $y^3/x^4$ ; 1d)  $17/16$ ; 2a)  $1/a^{10}$ ; 2b)  $8/x^3$ ; 2c)  $x^4/y^{16}$ ; 2d)  $y^3/2x^3$ ; 2e)  $m^{20}$ ;  
 2f)  $\frac{8x^9y^6}{27}$ ; 2g) 1; 2h)  $\frac{10}{r^3s^2t^3}$ ; 3a)  $8.3 \times 10^4$ ; 3b)  $1.25 \times 10^6$ ; 3c)  $1.54 \times 10^{-4}$ ; 3d)  $6.89 \times 10^{-6}$ ; 4a) 1,030,000;  
 4b) 0.000087; 4c) 60,030,000,000; 4d) 0.00202

## Mini-Lecture 5.6

### Dividing Polynomials

#### Learning Objectives:

1. Divide a polynomial by a monomial.
2. Use long division to divide a polynomial by another polynomial.

#### Examples:

1. Perform each division.

a)  $\frac{10x^6 - 40x^3}{5x^2}$

b)  $\frac{6a^7 - 10a^5}{-2a^7}$

c)  $\frac{-14x^7 + 6x^6 - 6x^5}{-2x^5}$

2. Find each quotient using long division.

a)  $\frac{x^2 + 9x + 20}{x + 5}$

b)  $\frac{6m^3 + 26m^2 - 17m + 15}{m + 5}$

c)  $\frac{-20x^3 + 17x^2 + 15x + 13}{-5x - 2}$

d)  $(4m^3 + 14m^2 - 5m + 12) \div (m + 4)$

Find each quotient using long division. Don't forget to write the polynomials in descending order and fill in any missing terms.

e)  $(x^4 + 81) \div (x - 3)$

f)  $\frac{9 - 5x - 25x^3 - 15x^2}{-5x + 2}$

#### Teaching Notes:

- Encourage students to write out each step before simplifying in 1a), 1b), 1c). Many students will “cancel” the monomial and one of the terms instead of dividing.
- Most students will need slow, methodical modeling to understand the concept of dividing by a monomial.
- Many students need to see a numerical long division done in parallel with long division of polynomials.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $2x^4 - 8x$ ; 1b)  $-3 + \frac{5}{a^2}$ ; 1c)  $7x^2 - 3x + 3$ ; 2a)  $x + 4$ ; 2b)  $6m^2 - 4m + 3$ ; 2c)  $4x^2 - 5x - 1 + \frac{11}{-5x - 2}$ ;

2d)  $4m^2 - 2m + 3$ ; 2e)  $x^3 + 3x^2 + 9x + 27 + \frac{162}{x - 3}$ ; 2f)  $5x^2 + 5x + 3 + \frac{3}{-5x + 2}$

## Mini-Lecture 6.1

### The Greatest Common Factor and Factoring by Grouping

#### Learning Objectives:

1. Find the greatest common factor of a list of integers.
2. Find the greatest common factor of a list of terms.
3. Factor out the greatest common factor from a polynomial.
4. Factor a polynomial by grouping.

#### Examples:

1. Find the greatest common factor for each list.

a) 16, 6

b) 18, 24

c) 15, 21

d) 12, 28, 40

2. Find the GCF for each list.

a)  $15m^2, 25m^5$

b)  $40x^2, 20x^7$

c)  $-28x^4, 56x^5$

d)  $21m^2n^5, 35mn^4$

3. Factor out the GCF from each polynomial.

a)  $5a + 15$

b)  $56z + 8$

c)  $y^3 + 2y$

d)  $5x^3 + 10x^4$

e)  $16z^5 + 8z^3 - 12z$

f)  $x(y^2 - 2) + 3(y^2 - 2)$

g)  $6a^8b^9 - 8a^3b^4 + 2a^2b^3 + 4a^5b^3$

4. Factor each four-term polynomial by grouping.

a)  $8y^2 - 12y + 10y - 15$

b)  $15a^6 - 25a^3 - 6a^3 + 10$

c)  $15x^3 - 25x^2y - 6xy^2 + 10y^3$

#### Teaching Notes:

- Many students remove common factors, not the *greatest* common factor.
- Encourage students to factor in a step-by-step manner: first factor out the GCF for the coefficients, then the GCF for each variable.
- Most students have trouble factoring by grouping when it entails factoring a negative from the second group. Encourage students to always write a sign and check by distributing. If the check has the correct terms but wrong sign; switch the sign.
- Remind students that they can check their work by multiplying.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a) 2; 1b) 6; 1c) 3; 1d) 4; 2a)  $5m^2$ ; 2b)  $20x^2$ ; 2c)  $-28x^4$ ; 2d)  $7mn^4$ ; 3a)  $5(a+3)$ ; b)  $8(7z+1)$ ; 3c)  $y(y^2+2)$ ; 3d)  $5x^3(1+2x)$ ; 3e)  $4z(4z^4+2z^2-3)$ ; 3f)  $(y^2-2)(x+3)$ ; 3g)  $2a^2b^3(3a^6b^6-4ab+1+2a^3)$ ; 4a)  $(2y-3)(4y+5)$ ; 4b)  $(3a^3-5)(5a^3-2)$ ; 4c)  $(3x-5y)(5x^2-2y^2)$

## Mini-Lecture 6.2

### Factoring Trinomials of the Form $x^2 + bx + c$

#### Learning Objectives:

1. Factor trinomials of the form  $x^2 + bx + c$ .
2. Factor out the greatest common factor and then factor a trinomial of the form  $x^2 + bx + c$ .

#### Examples:

1. Factor each trinomial completely. If a polynomial can't be factored, write "prime".

a)  $x^2 + 11x + 30$

b)  $y^2 + 7y + 10$

c)  $x^2 + 3x - 4$

d)  $x^2 - 4x - 21$

e)  $x^2 - 13x + 30$

f)  $x^2 - x + 32$

g)  $m^2 + 17m + 16$

h)  $5x - 14 + x^2$

i)  $a^2 + 13ab + 40b^2$

2. Factor each trinomial completely. Some of these trinomials contain a greatest common factor (other than 1). Don't forget to factor out the GCF first.

a)  $2x^2 - 18x + 28$

b)  $3x^2 + 6x - 9$

c)  $x^2 + 10x + 24$

d)  $2x^2 + 20x - 22$

e)  $5x^2 + 20x + 15$

f)  $-x^3 + 3x^2 + 10x$

g)  $4x^4 - 36x^3 + 56x^2$

h)  $x^3y + 10x^2y^2 + 24xy^3$

i)  $\frac{1}{3}y^2 - \frac{8}{3}y - 11$

#### Teaching Notes:

- When factoring trinomials of this form, many students find it helpful to make a table listing all possible factor pairs for  $c$  in the first column and their sums in the second column.
- Some students have trouble factoring a trinomial when the last term is negative.
- Remind students that when the last term (the constant) of a trinomial is positive, the factors have the same sign. When the last term (the constant) of a trinomial is negative, the factors have different signs.
- Refer students to: **To Factor a Trinomial of the Form  $x^2 + bx + c$**  in the textbook.
- Remind students that they can always check their work by multiplication.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $(x+6)(x+5)$ ; 1b)  $(y+5)(y+2)$ ; 1c)  $(x+4)(x-1)$ ; 1d)  $(x-7)(x+3)$ ; 1e)  $(x-10)(x-3)$ ; 1f) prime;  
1g)  $(m+16)(m+1)$ ; 1h)  $(x+7)(x-2)$ ; 1i)  $(a+8b)(a+5b)$ ; 2a)  $2(x-7)(x-2)$ ; 2b)  $3(x+3)(x-1)$ ; 2c)  $(x+6)(x+4)$ ;  
2d)  $2(x+11)(x-1)$ ; 2e)  $5(x+3)(x+1)$ ; 2f)  $-x(x-5)(x+2)$ ; 2g)  $4x^2(x-7)(x-2)$ ; 2h)  $xy(x+6y)(x+4y)$ ;

2i)  $\frac{1}{3}(y-11)(y+3)$

## Mini-Lecture 6.3

### Factoring Trinomials of the Form $ax^2 + bx + c$ and Perfect Square Trinomials

#### Learning Objectives

1. Factor trinomials of the form  $ax^2 + bx + c$ , where  $a \neq 1$ .
2. Factor out the GCF before factoring a trinomial of the form  $ax^2 + bx + c$ .
3. Factor perfect square trinomials.

#### Examples:

1. Complete each factored form.

a)  $3x^2 + 8x + 4 = (3x + 2)(\quad)$                       b)  $2y^2 + 7y - 15 = (2y - 3)(\quad)$

Factor each trinomial completely.

c)  $2x^2 + 7x + 3$                       d)  $5x^2 + 17x + 6$                       e)  $8x^2 + x - 7$

f)  $20r^2 + 31r - 7$                       g)  $6x^2 + 19x - 11$                       h)  $3x^2 - 7x - 20$

2. Factor each trinomial completely. If necessary, factor out the GCF first.

a)  $14x^2 + 4x - 10$                       b)  $9x^2 - 6x - 15$                       c)  $14x^3 + 66x^2 - 20x$

d)  $25x^3 - 15x^2 - 10x$                       e)  $4x^2y^2 - xy^2 - 105y^2$                       f)  $12x^2 - 25xt + 12t^2$

g)  $-7x^2 - 33x + 10$                       h)  $18x^4 - 3x^3 - 21x^2$                       i)  $2x^5 - x^3y^2 - 15xy^4$

3. Factor each Perfect Square Trinomial completely.

a)  $x^2 + 2x + 1$                       b)  $4x^2 - 12x + 9$                       c)  $25x^2 + 60xy + 36y^2$

d)  $16x^3 - 8x^2y + xy^2$                       e)  $5x^3 - 10x^2 + 5x$                       f)  $2a - 24ay + 72ay^2$

#### Teaching Notes:

- Some students remember factoring from high school and are able to use the trial-and-error method to factor.
- Some students may need to see Section 4.4, *Factoring Trinomials by Grouping* before being able to factor successfully.
- Encourage students to use strategies when factoring. For example, identify any prime numbers to reduce the number of combinations.
- Many students will forget to put the GCF in their final answer.
- Remind students that they can check their work by multiplying.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers 1a)  $(x+2)$ ; 1b)  $(y+5)$ ; 1c)  $(2x+1)(x+3)$ ; 1d)  $(5x+2)(x+3)$ ; 1e)  $(8x-7)(x+1)$ ; 1f)  $(5r-1)(4r+7)$ ;  
1g)  $(3x+11)(2x-1)$ ; 1h)  $(3x+5)(x-4)$ ; 2a)  $2(7x-5)(x+1)$ ; 2b)  $3(3x-5)(x+1)$ ; 2c)  $2x(7x-2)(x+5)$ ;  
2d)  $5x(5x+2)(x-1)$ ; 2e)  $y^2(x+5)(4x-21)$ ; 2f)  $(4x-3t)(3x-4t)$ ; 2g)  $(-7x+2)(x+5)$ ; 2h)  $3x^2(6x-7)(x+1)$ ;  
2i)  $x(2x^2+5y^2)(x^2-3y^2)$ , 3a)  $(x+1)^2$ , 3b)  $(2x-3)^2$ , 3c)  $(5x+6y)^2$  3d)  $x(4x-y)^2$ , 3e)  $5x(x-1)^2$ , 3f)  $2a(1-6y)^2$

## Mini-Lecture 6.4

### Factoring Trinomials of the Form $ax^2 + bx + c$ by Grouping

#### Learning Objectives

1. Use the grouping method to factor trinomials of the form  $ax^2 + bx + c$ .

#### Examples:

1. Factor the following trinomial by grouping. Complete the outlined steps.

a)  $12y^2 + 17y + 6$

Find two numbers whose product is 72 ( $12 \cdot 6$ ) and whose sum is 17: \_\_\_\_\_

Write 17y using the factors from previous step: \_\_\_\_\_

Factor by grouping: \_\_\_\_\_

b)  $10x^2 + 9x - 9$

Find two numbers whose product is  $-90[10 \cdot (-9)]$  and whose sum is (-9): \_\_\_\_\_

Write (-9x) using the factors from part (a): \_\_\_\_\_

Factor by grouping: \_\_\_\_\_

Factor by grouping.

c)  $8x^2 + 18x + 9$

d)  $6x^2 + 7x - 3$

e)  $7x^2 - 19x - 6$

f)  $4x^2 - 12x + 9$

g)  $6x^2 - 17x + 5$

h)  $20x^2 - 15x - 50$

i)  $45x^3 + 45x^2 - 50x$

j)  $x - 15 + 6x^2$

k)  $10z^2 - 12z - 1$

#### Teaching Notes:

- Most students appreciate seeing the grouping method. This method gives the student a step-by-step guide to factoring.
- Encourage students to use whatever method works for them (trial-and-error or grouping).
- Remind students to put the trinomial into standard form before attempting to factor.
- Encourage students to check their factoring answers by multiplication.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers: 1a) 9,8; 9y+8y; (4y+3)(3y+2); 1b) -6, 15; -6x+15x; (5x-3)(2x+3); 1c) (4x+3)(2x+3); 1d) (2x+3)(3x-1); 1e) (7x+2)(x-3); 1f) (2x-3)<sup>2</sup>; 1g) (2x-5)(3x-1); 1h) 5(x-2)(4x+5); 1i) 5x(3x-2)(3x+5); 1j) (3x+5)(2x-3); 1k) prime*

## Mini-Lecture 6.5

### Factoring Binomials

#### Learning Objectives:

1. Factor the difference of two squares.
2. Factor the sum or difference of two cubes.

#### Examples:

1. Factor each binomial completely.

a)  $x^2 - 9$

b)  $x^2 - 25$

c)  $y^2 - 64$

d)  $4a^2 - 9$

e)  $49x^2 - 1$

f)  $9a^2 + 16b^2$

g)  $36m^2 - 100n^2$

h)  $\frac{1}{4}x^2 - 1$

i)  $64 - \frac{9}{25}a^2$

2. Factor each sum or difference of two cubes completely.

a)  $8x^3 + 1$

b)  $a^3 - 1$

c)  $64x^3 + 27y^3$

d)  $54y^4 - 2y$

e)  $125b^5 + b^2$

f)  $a^6 - 1$

#### Teaching Notes:

- Some students will have a better understanding of a difference of two squares if they are first shown 3a) and 3b) with a middle term of  $0x$ .
- Encourage students to become proficient with special case factoring as it will be important for future algebra topics such as completing the square.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a)  $(x+3)(x-3)$ ; 1b)  $(x+5)(x-5)$ ; 1c)  $(y+8)(y-8)$ ; 1d)  $(2a+3)(2a-3)$ ; 1e)  $(7x+1)(7x-1)$ ; 1f) cannot be factored; 1g)  $(6m+10n)(6m-10n)$ ; 1h)  $(\frac{1}{2}x+1)(\frac{1}{2}x-1)$ ; 1i)  $(8+\frac{3}{5}a)(8-\frac{3}{5}a)$ ; 2a)  $(2x+1)(4x^2-2x+1)$ ; 2b)  $(a-1)(a^2+a+1)$ ; 2c)  $(4x+3y)(16x^2-12xy+9y^2)$ ; 2d)  $2y(3y-1)(9y^2+3y+1)$ ; 2e)  $b^2(5b+1)(25b^2-5b+1)$ ; 2f)  $(a^2-1)(a^4+a^2+1)$



## Mini-Lecture 6.6

### Solving Quadratic Equations by Factoring

#### Learning Objectives:

1. Solve quadratic equations by factoring.
2. Solve equations with degree greater than 2 by factoring.
3. Find the  $x$ -intercepts of the graph of a quadratic equation in two variables.

#### Examples:

1. Solve each equation.

a)  $(x-1)(x+4)=0$

b)  $(x+5)(x+9)=0$

c)  $(x-10)(x+8)=0$

d)  $5x(x-15)=0$

e)  $(2x-5)(x+3)=0$

f)  $\left(x-\frac{2}{7}\right)\left(x-\frac{1}{3}\right)=0$

g)  $x^2-x-30=0$

h)  $x^2-9x+20=0$

i)  $y^2-y-42=0$

j)  $x^2-7x=0$

k)  $x^2=25$

l)  $x^2+2x=15$

m)  $5x^2-30x+40=0$

n)  $x(x-4)=21$

o)  $x(x-6)=16$

2. Solve the following equations with degree greater than 2 by factoring.

a)  $y^3+14y^2+49y=0$

b)  $24x^3-4x^2-20x=0$

c)  $(x-4)(x^2-3x+2)=0$

d)  $16a^3-9a=0$

e)  $49t^3-4t=0$

f)  $(9x+5)(10x^2-3x-4)=0$

3. Find the  $x$ -intercepts of the graph.

a)  $y=(x-4)(x+5)$

b)  $y=(x-1)(x+1)$

c)  $y=x^3-x^2-4x-4$

#### Teaching Notes:

- Remind students to always put the equation into standard form.
- Some students try to use the zero factor property before the equation is in standard form. For example 1n)  $x(x-4)=21 \rightarrow x=21, x-4=21, etc.$
- Students will find this section challenging.
- Remind students to always check their answers.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 1, -4; 1b) -5, -9; 1c) 10, -8; 1d) 0, 15; 1e) 5/2, -3; 1f) 2/7, 1/3; 1g) 6, -5; 1h) 4, 5; 1i) 7, -6; 1j) 0, 7; 1k) 5, -5; 1l) -5, 3; 1m) 2, 4; 1n) 7, -3; 1o) -2, 8; 2a) 0, -7; 2b) 0, -5/6, 1; 2c) 4, 2, 1; 2d) 0, 3/4, -3/4; 2e) 0, 2/7, -2/7; 2f) -5/9, -1/2, 4/5, 3a) (4,0), (-5,0), 3b) (1,0), (-1,0), 3c) (2,0), (-2,0), (1,0)

## Mini-Lecture 6.7

### Quadratic Equations and Problem Solving

#### Learning Objectives:

1. Solve problems that can be modeled by quadratic equations.

#### Examples:

1. Represent each given condition using a single variable,  $x$ .
  - a) The length and width of a rectangle whose width is half the measurement of the length.
  - b) Two consecutive integers.
  - c) The base and height of a triangle whose height is 3 less than 5 times its base.
2. Use the information given to solve the following problems.
  - a) The area of a square is  $144 \text{ ft}^2$ . Find the length of its side.
  - b) The area of the circle is  $81\pi$  square inches. Find the radius.
  - c) The sum of two numbers is 16. The sum of their squares is 130. Find the numbers.
  - d) A roofer drops a hammer from the top of a 64-foot roof. The height of the hammer after  $t$  seconds is given by  $h = -16t^2 + 64$ . When will the hammer hit the ground?
  - e) The hypotenuse of a right triangle is 6 inches more than the shorter leg. The longer leg is 3 inches more than the shorter leg. Find the lengths of all three sides.

#### Teaching Notes:

- Many students struggle with word problems.
- Most students have trouble with converting the words to mathematical expressions.
- Refer students to Chapter 2, Section 2.4, **General Strategy for Problem Solving**.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $x = \text{length}$ ,  $1/2x = \text{width}$ ; 1b)  $x, x+1$ ; 1c)  $b=x, h=5x-3$ ; 2a) 12 ft.; 2b) 9 in.; 2c) 7, 9; 2d) after 2 seconds; 2e) 9, 12, 15

## Mini-Lecture 7.1

### Simplifying Rational Expressions

#### Learning Objectives:

1. Find the value of a rational expression given a replacement number.
2. Identify values for which a rational expression is undefined.
3. Simplify or write rational expressions in lowest terms.
4. Write equivalent rational expressions of the form  $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$ .

#### Examples:

1. Find the value of the expression for the given value (s).

a)  $\frac{x+4}{x-2}; x=4$                       b)  $\frac{y^2}{y-3}; y=5$                       c)  $\frac{x^2+5x-2}{x^2-x-2}; x=-3$

2. Find any numbers for which each rational expression is undefined.

a)  $\frac{7}{4a}$                       b)  $\frac{3y^3}{y^2-5y}$                       c)  $\frac{x}{2x^2-7x-4}$

3. Simplify each expression.

a)  $\frac{(y+3)(y-1)}{(y-1)(y+5)}$                       b)  $\frac{6-a}{a-6}$                       c)  $\frac{-2x-2y}{x+y}$

d)  $\frac{6x-12}{3x^2-12}$                       e)  $\frac{y^2+8y+15}{y^2+9y+18}$                       f)  $\frac{x^2-xy+5x-5y}{x^2+5x}$

4. List four equivalent forms for each rational expression.

a)  $-\frac{x-2}{x+5}$                       b)  $-\frac{y+3}{y-11}$                       c)  $-\frac{5y+2}{3y-1}$

#### Teaching Notes:

- Some students need a review of simplifying numerical fractions before applying to rational expressions.
- Many students need to be reminded to factor completely before simplifying.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a) 4; 1b) 25/2; 1c) -4/5; 2a) 0; 2b) 0, 5; 2c) -1/2, 4; 3a)  $\frac{y+3}{y+5}$ ; 3b) -1; 3c) -2; 3d)  $\frac{2}{x+2}$ ;  
 3e)  $\frac{y+5}{y+6}$ ; 3f)  $\frac{x-y}{x}$ ; 4a)  $\frac{-(x-2)}{x+5}, \frac{2-x}{x+5}, \frac{x-2}{-(x+5)}, \frac{x-2}{-x-5}$ ; 4b)  $\frac{-(y+3)}{y-11}, \frac{-y-3}{y-11}, \frac{y+3}{-(y-11)}, \frac{y+3}{11-y}$ ;  
 4c)  $\frac{-(5y+2)}{3y-1}, \frac{-5y-2}{3y-1}, \frac{5y+2}{-(3y-1)}, \frac{5y+2}{1-3y}$

## Mini-Lecture 7.2

### Multiplying and Dividing Rational Expressions

#### Learning Objectives:

1. Multiply rational expressions.
2. Divide rational expressions.
3. Multiply or divided rational expressions.

#### Examples:

1. Find each product and simplify if possible.

$$\text{a) } \frac{8p-8}{p} \cdot \frac{7p^2}{9p-9} \qquad \text{b) } \frac{x^2+5x+6}{x^2+11x+24} \cdot \frac{x^2+8x}{x^2-4x+3} \qquad \text{c) } \frac{x^3-x^2+x}{x^3+1} \cdot \frac{-45x-45}{5x}$$

2. Find each quotient and simplify.

$$\text{a) } \frac{-4k^2}{4k^5} \div \frac{16k^4}{12k^6} \qquad \text{b) } \frac{2-2x}{x} \div \frac{5x-5}{3x^2} \qquad \text{c) } \frac{y^2-11y+30}{y^2-36} \div \frac{y^2-9}{y^2-3y-18}$$

3. Multiply or divide. Simplify if possible.

$$\text{a) } \frac{x^2-11x+10}{x^2-11x+28} \cdot \frac{x^2-9x+20}{x^2-16x+60} \qquad \text{b) } \frac{2k^2+8k+6}{k^2-9} \div \frac{4k^2+18k+14}{2k-6}$$

#### Teaching Notes:

- Many students need to review multiplying and dividing numerical fractions.
- When dividing, remind students to change division to multiplication by the reciprocal first, then begin factoring. Many students will begin factoring and forget that they are dividing.
- Some students will need additional practice using unit fractions.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $\frac{56p}{9}$ ; 1b)  $\frac{x(x+2)}{(x-3)(x-1)}$ ; 1c) -9; 2a)  $\frac{-3}{4k}$ ; 2b)  $\frac{-6x}{5}$ ; 2c)  $\frac{(y-5)(y-6)}{(y+6)(y-3)}$ ;  
3a)  $\frac{(x-5)(x-1)}{(x-6)(x-7)}$ ; 3b)  $\frac{2}{2k+7}$ ;

## Mini-Lecture 7.3

### Adding and Subtracting Rational Expressions with Common Denominators and Least Common Denominator

#### Learning Objectives:

1. Add and subtract rational expressions with the same denominator.
2. Find the least common denominator of a list of rational expressions.
3. Write a rational expression as an equivalent expression whose denominator is given.
4. Key Vocabulary: *least common denominator (LCD), equivalent expressions.*

#### Examples:

1. Add or subtract as indicated. Simplify the result if possible.

a)  $\frac{3}{12x} + \frac{4}{12x}$

b)  $\frac{8a+2b}{2} - \frac{8a-2b}{2}$

c)  $\frac{7y^2}{y-1} + \frac{-7y}{y-1}$

d)  $\frac{m^2-7m}{m-3} + \frac{12}{m-3}$

e)  $\frac{3x+2}{x^2+4x-21} - \frac{2x-5}{x^2+4x-21}$

2. Find the LCD for each list of rational expressions.

a)  $\frac{1}{20x^5}, \frac{1}{36x^4}$

b)  $\frac{2}{x+3}, \frac{3}{x-5}$

c)  $\frac{4}{3a+27}, \frac{6}{a^2+9a}$

3. Rewrite each rational expression as an equivalent rational expression with the given denominator.

a)  $\frac{3}{8m} = \frac{?}{72m}$

b)  $\frac{x}{x+4} = \frac{?}{5x+20}$

c)  $\frac{a}{a+3b} = \frac{?}{a^2-9b^2}$

d)  $\frac{-25}{x^3+2x^2-3x} = \frac{?}{x(x-1)(x+3)(x+5)}$

#### Teaching Notes:

- Most students need a review of finding the LCD for numerical fractions.
- Many students have a difficult time finding the LCD. Refer them back to the process they use when finding the LCD for numerical fractions.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $\frac{7}{12x}$ ; 1b)  $2b$ ; 1c)  $7y$ ; 1d)  $m-4$ ; 1e)  $\frac{1}{x-3}$ ; 2a)  $180x^5$ ; 2b)  $x^2 - 2x - 15$ ; 2c)  $3a^2 + 27a$ ;  
3a)  $27$ ; 3b)  $5x$ ; 3c)  $a^2 - 3ab$ ; 3d)  $-25x - 125$

## Mini-Lecture 7.4

### Adding and Subtracting Rational Expressions With Unlike Denominators

#### Learning Objectives:

1. Add and subtract rational expressions with unlike denominators.

#### Examples:

1. Perform each indicated operation. Simplify if possible.

a)  $\frac{x}{9} + \frac{8}{7}$

b)  $\frac{x}{5} - \frac{3}{11}$

c)  $\frac{9}{40} - \frac{2}{5x}$

d)  $\frac{6}{z^2} - \frac{4}{z}$

e)  $\frac{5}{r} + \frac{9}{r-3}$

f)  $\frac{9-5y}{63} - \frac{8-7y}{18}$

g)  $\frac{-8x+3}{x} + \frac{-8x+2}{5x}$

h)  $\frac{6x}{x+6} + \frac{3}{x-6}$

i)  $\frac{7}{6-m} + \frac{2}{m-6}$

j)  $\frac{-5}{y-4} - \frac{7}{4-y}$

k)  $\frac{m-5}{m^2+9m+20} + \frac{4m-1}{m^2+7m+10}$

l)  $\frac{x}{x^2-16} - \frac{6}{x^2+5x+4}$

m)  $\frac{3}{y^2-3y+2} + \frac{5}{y^2-1}$

#### Teaching Notes:

- Most students have difficulty with adding and subtracting rational expressions, mainly with finding the LCD.
- Remind students that with subtraction, they are subtracting the entire numerator (i.e. Distributive Property).
- Some students may find working in a vertical format easier to build equivalent expressions.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a)  $\frac{7x+72}{63}$ ; 1b)  $\frac{11x-15}{55}$ ; 1c)  $\frac{-9x-16}{40x}$ ; 1d)  $\frac{6-4z}{z^2}$ ; 1e)  $\frac{14r-15}{r^2-3r}$ ; 1f)  $\frac{39y-38}{126}$ ;

1g)  $\frac{-48x+17}{5x}$ ; 1h)  $\frac{6x^2-33x+18}{x^2-36}$ ; 1i)  $\frac{6}{6-m}$ ; 1j)  $\frac{2}{y-4}$ ; 1k)  $\frac{5m^2+12m-14}{m^3+11m^2+38m+40}$ ;

1l)  $\frac{x^2-5x+24}{x^3+x^2-16x-16}$ ; 1m)  $\frac{8y-7}{y^3-2y^2-y+2}$

## Mini-Lecture 7.5

### Solving Equations Containing Rational Expressions

#### Learning Objectives:

1. Solve equations containing rational expressions.
2. Solve equations containing rational expressions for a specified variable.
3. Key Vocabulary: *rational expressions*.

#### Examples:

1. Solve each equation and check each solution.

a)  $\frac{x}{3} - \frac{x}{9} = 8$

b)  $\frac{4x}{5} - 8 = x$

c)  $\frac{x-3}{7} = \frac{x+4}{3}$

d)  $\frac{x+10}{x-5} = \frac{9}{5-x}$

e)  $\frac{4}{x-4} + \frac{4}{2x-8} = 6$

f)  $\frac{x}{x-7} + 6 = \frac{7}{x-7}$

g)  $\frac{-5x}{3x+3} = \frac{2x}{6x+6} + \frac{6x-4}{x+1}$

h)  $\frac{1}{x} + \frac{1}{x-8} = \frac{x-7}{x-8}$

i)  $\frac{-2}{m+5} - \frac{3}{m-5} = \frac{15}{m^2-25}$

j)  $\frac{x+4}{x^2+2x-15} - \frac{4}{x^2+10x+25} = \frac{x-4}{x^2+2x-15}$

2. Solve each equation for the indicated variable.

a)  $\frac{1}{a} + \frac{1}{b} = c$  for  $b$

b)  $A = \frac{1}{2}h(B+b)$  for  $B$

c)  $F = \frac{-GMm}{r^2}$  for  $m$

#### Teaching Notes:

- Remind students to always determine the values for  $x$  that will make the denominators of the equation equal to zero.
- Some students prefer to create equivalent expressions using the LCD then set numerators equal.
- Most students have difficulty solving a formula for a specific variable.
- To help students focus on solving a formula for a specific variable, encourage them to write the variable with a different color pencil.

Answers: 1a) 36; 1b) -40; 1c) -37/4; 1d) -19; 1e) 5; 1f) no solution; 1g) 1/2; 1h) 1; 1i) -4; 1j) -13;

2a)  $b = \frac{a}{ac-1}$ ; 2b)  $B = \frac{2A}{h} - b$  or  $\frac{2A-bh}{h}$ ; 2c)  $m = -\frac{Fr^2}{GM}$

## Mini-Lecture 7.6

### Proportion and Problem Solving with Rational Equations

#### Learning Objectives:

1. Solve proportions.
2. Use proportions to solve problems.
3. Solve problems about numbers.
4. Solve problems about work.
5. Solve problems about distance.

#### Examples:

1. Solve each proportion.

a)  $\frac{x}{39} = \frac{6}{13}$

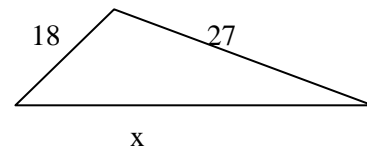
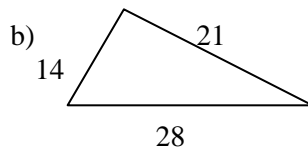
b)  $\frac{2}{7} = \frac{5}{x}$

c)  $\frac{8+x}{5} = \frac{5+x}{3}$

d)  $\frac{7}{2} = \frac{z-5}{z-3}$

2. Solve. Find the unknown length ( $x$ ) in the pair of similar triangles.

- a) There are 170 calories in 3 peanut butter cookies. How many calories are in 7 cookies?



3. Solve the following: problems about numbers.

a) Five divided by the difference of a number and one equals the quotient of ten and the sum of the number and twelve. Find the number.

b) If three times a number added to five is divided by the number plus nine, the result is four thirds. Find the number.

4. Solve the following: problems about work.

a) A painter can paint a sign in 4 hours. His assistant can paint the same sign in 6 hours. How long will it take them to paint the sign if they paint it together.

b) A conveyer belt can move 1000 cans of soup to the loading area in 7 minutes. A smaller conveyer belt can move the same number of cans in 11 minutes. How long will it take to move 1000 cans of soup if both conveyer belts are used.

5. Solve the following: problems about distance.

a) A cyclist bikes at a constant speed for 18 miles. His return trip is a different route of 23 miles long and will take 1 hour longer. Find the speed.

#### Teaching Notes:

- Most students will struggle with applications.
- Refer students to Chapter 2, Section 2.4, **General Strategy for Problem Solving**.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a) 18; 1b)  $35\frac{1}{2}$ ; 1c)  $-\frac{1}{2}$ ; 1d)  $11\frac{1}{5}$ ; 2a)  $396\frac{2}{3}$ ; 2b) 36; 3a) 14; 3b)  $21\frac{1}{5}$ ; 4a) 2.4 hours; 4b)  $4\frac{5}{18}$  min.; 5a) 5 mph.



## Mini-Lecture 7.7

### Variation and Problem Solving

#### Learning Objectives:

1. Solve problems involving direct variation.
2. Solve problems involving inverse variation.
3. Other types of direct and inverse variation.
4. Variation and problem solving.

#### Examples:

1. Write a direct variation equation,  $y=kx$ , that satisfies the ordered pairs in each table.

a) 

<b>x</b>	0	2	5	7
<b>y</b>	0	10	25	35

b) 

<b>x</b>	0	5	-1	$-\frac{5}{8}$
<b>y</b>	0	2	$-\frac{2}{5}$	$-\frac{1}{4}$

- c)  $y$  varies directly as  $x$ . If  $y = 24$  when  $x = 3$ , find  $y$  when  $x$  is 6.

2. Write an inverse variation equation,  $y = \frac{k}{x}$ , that satisfies the ordered pairs in each table.

a) 

<b>x</b>	1	2	4	8
<b>y</b>	4	2	1	0.5

b) 

<b>x</b>	-2	3	10	0.05
<b>y</b>	-0.15	0.1	0.03	6

- c)  $y$  varies inversely as  $x$ . If  $y = 10$  when  $x = 6$ , find  $y$  when  $x$  is 12.

3. Solve.

- a)  $y$  varies directly as  $x^2$ . If  $y = 72$  when  $x = 3$ , find  $y$  when  $x$  is  $\frac{1}{4}$ .
- b)  $z$  varies inversely as  $a^2$ . If  $z = 25$  when  $a = 2$ , find  $z$  when  $a$  is 5.

4. Solve the following.

a) Your paycheck (before deductions) varies directly as the number of hours you work. If your paycheck is \$147.25 for 15.5 hours, find your pay for 24.5 hours.

b) For a constant distance, the rate of travel varies inversely as the time traveled. If a truck driver travels 45 mph and arrives at a destination in 8 hours. How long will the return trip take traveling at 60 mph?

#### Teaching Notes:

- Most students understand direct variation but struggle with inverse variation.
- Most students struggle with the applications that involve both direct and inverse variation.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $y=5x$ ; 1b)  $y = \frac{2}{5}x$ ; 1c) 48; 2a)  $y = \frac{4}{x}$ ; 2b)  $y = \frac{0.3}{x}$ ; 2c) 5; 3a)  $\frac{1}{2}$ ; 3b) 4; 4a) \$232.75;

4b) 6

## Mini-Lecture 7.8

### Simplifying Complex Fractions

#### Learning Objectives

1. Simplify complex fractions using method 1.
2. Simplify complex fractions using method 2.

#### Examples:

1. Simplify using Method 1.

$$\text{a) } \frac{\frac{1}{11}}{\frac{5}{6}}$$

$$\text{b) } \frac{\frac{7m^6n^3}{5m}}{\frac{9m^2n^8}{4n^2}}$$

$$\text{c) } \frac{\frac{8}{a} + 8}{\frac{8}{a} - 8}$$

2. Simplify using Method 2.

$$\text{a) } \frac{\frac{x+7}{8}}{\frac{x+1}{x}}$$

$$\text{b) } \frac{\frac{4}{y}}{\frac{7}{y+10}}$$

$$\text{c) } \frac{\frac{9s^2 - 25t^2}{st}}{\frac{\frac{3}{t} - \frac{5}{s}}{s}}$$

3. Simplify using either method.

$$\text{a) } \frac{\frac{1}{7} - \frac{1}{5}}{\frac{1}{5} + \frac{1}{2}}$$

$$\text{b) } \frac{9 + \frac{3}{x}}{\frac{x}{4} + \frac{1}{12}}$$

$$\text{c) } \frac{\frac{3}{5x-1} - 3}{\frac{3}{5x-1} + 3}$$

#### Teaching Notes:

- A brief review of division with fractions may be needed.
- Students with more mathematical confidence tend to use Method 2 – LCD.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $\frac{6}{55}$ ; 1b)  $\frac{28m^3}{45n^3}$ ; 1c)  $\frac{1+a}{1-a}$ ; 2a)  $\frac{x^2+7x}{8x+8}$ ; 2b)  $\frac{4y+40}{7y}$ ; 2c)  $3s+5t$ ; 3a)  $-4/49$ ;

3b)  $\frac{36}{x}$ ; 3c)  $\frac{2-5x}{5x}$

# Mini-Lecture 8.1

## Introduction to Radicals

### Learning Objectives:

1. Find square roots.
2. Find cube roots.
3. Find  $n$ th roots.
4. Approximate square roots.
5. Simplify radicals containing variables.

### Examples:

1. Find each square root.

a)  $\sqrt{49}$

b)  $\sqrt{\frac{1}{36}}$

c)  $-\sqrt{9}$

d)  $\sqrt{-100}$

e)  $\sqrt{\frac{25}{121}}$

f)  $\sqrt{0.64}$

2. Find each cube root.

a)  $\sqrt[3]{8}$

b)  $\sqrt[3]{-216}$

c)  $\sqrt[3]{-\frac{8}{27}}$

3. Find each root.

a)  $\sqrt[4]{16}$

b)  $\sqrt[3]{-27}$

c)  $-\sqrt[4]{\frac{81}{625}}$

4. Approximate each square root to three decimal places.

a)  $\sqrt{12}$

b)  $\sqrt{22}$

c)  $-\sqrt{120}$

5. Find each root. Assume that all variables represent positive numbers.

a)  $\sqrt{x^2}$

b)  $\sqrt{a^4}$

c)  $\sqrt{m^8}$

d)  $\sqrt{81x^4}$

e)  $\sqrt{x^{10}y^8z^2}$

f)  $\sqrt[3]{27a^6b^9c^3}$

### Teaching Notes:

- Many students confuse 1c), 1d), and 2b).
- Students have a hard time understanding  $\sqrt{x^2} = |x|$  even though we assume that all variable represent positive numbers.
- It is very important to stress that using a calculator gives an *approximation* and leaving an answer in radical form is an *exact* value.

Answers: 1a) 7; 1b) 1/6; 1c) -3; 1d) not a real number; 1e) 5/11; 1f) 0.8; 2a) 2; 2b) -6; 2c) -2/3;  
3a) 2; 3b) -3; 3c) -3/5; 4a) 3.464; 4b) 4.69; 4c) -10.954; 5a)  $x$ ; 5b)  $a^2$ ; 5c)  $m^4$ ; 5d)  $9x^2$ ;  
5e)  $x^5y^4z$ ; 5f)  $3a^2b^3c$

## Mini-Lecture 8.2

### Simplifying Radicals

#### Learning Objectives:

1. Use the product rule to simplify square roots.
2. Use the quotient rule to simplify square roots.
3. Simplify radicals containing variables.
4. Simplify higher roots.
5. Key Vocabulary: *perfect squares*.

#### Examples:

1. Use the product rule to simplify each radical.

a)  $\sqrt{18}$

b)  $\sqrt{12}$

c)  $\sqrt{33}$

d)  $\sqrt{160}$

e)  $5\sqrt{16}$

f)  $-3\sqrt{50}$

2. Use the quotient rule and the product rule to simplify each radical.

a)  $\sqrt{\frac{25}{16}}$

b)  $\sqrt{\frac{99}{4}}$

c)  $\sqrt{\frac{125}{144}}$

3. Simplify each radical. Assume that all variables represent positive numbers.

a)  $\sqrt{x^5}$

b)  $\sqrt{y^9}$

c)  $\sqrt{a^{13}}$

d)  $\sqrt{\frac{18}{x^2}}$

e)  $\sqrt{36y^3}$

f)  $\sqrt{80y^{12}}$

g)  $\sqrt{\frac{98}{p^6}}$

h)  $\sqrt{\frac{300}{x^{20}}}$

i)  $\sqrt{\frac{16x}{z^{10}}}$

4. Simplify each radical.

a)  $\sqrt[3]{40}$

b)  $\sqrt[3]{300}$

c)  $\sqrt[3]{\frac{625}{216}}$

#### Teaching Notes:

- Many students have trouble with radicals.
- When simplifying, students get confused where to write the numbers – outside the radical symbol or in the radicand.
- A common error is to evaluate “ $\sqrt{16} = \sqrt{4} = 2$ ”. Many students do not know when to stop!
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $3\sqrt{2}$ ; 1b)  $2\sqrt{3}$ ; 1c)  $\sqrt{33}$ ; 1d)  $4\sqrt{10}$ ; 1e) 20; 1f)  $-15\sqrt{2}$ ; 2a)  $5/4$ ; 2b)  $\frac{3\sqrt{11}}{2}$ ;

2c)  $\frac{5\sqrt{5}}{12}$ ; 3a)  $x^2\sqrt{x}$ ; 3b)  $y^4\sqrt{y}$ ; 3c)  $a^6\sqrt{a}$ ; 3d)  $\frac{3\sqrt{2}}{x}$ ; 3e)  $6y\sqrt{y}$ ; 3f)  $4y^6\sqrt{5}$ ; 3g)  $\frac{7\sqrt{2}}{p^3}$ ;

3h)  $\frac{10\sqrt{3}}{x^{10}}$ ; 3i)  $\frac{4\sqrt{x}}{z^5}$ ; 4a)  $2\sqrt[3]{5}$ ; 4b)  $\sqrt[3]{300}$ ; 4c)  $\frac{5\sqrt[3]{5}}{6}$

## Mini-Lecture 8.3

### Adding and Subtracting Radicals

#### Learning Objectives:

1. Add or subtract like radicals.
2. Simplify radical expressions, and then add or subtract any like radicals.

#### Examples:

1. Add or subtract as indicated.

a)  $20\sqrt{5} + 3\sqrt{5}$

b)  $11\sqrt{7} - 3\sqrt{7}$

c)  $-7\sqrt{11} - 5\sqrt{11}$

d)  $11\sqrt{3} - 12\sqrt{3} + 35 + 3\sqrt{3}$

e)  $3\sqrt{7} + 5\sqrt{21} - 8\sqrt{21} - 10\sqrt{7}$

2. Add or subtract by first simplifying each radical and then combining any like radicals. Assume that all variables represent positive numbers.

a)  $8\sqrt{5} + 9\sqrt{20}$

b)  $-7\sqrt{2} + 9\sqrt{50}$

c)  $-8\sqrt{3} - 3\sqrt{75}$

d)  $-10\sqrt{48} - 3\sqrt{75}$

e)  $-5\sqrt{8x} - 6\sqrt{18x}$

f)  $-5\sqrt{x^2} + 3x + 8\sqrt{x^2}$

3. Simplify each radical expression.

a)  $5\sqrt[3]{7} + 8\sqrt[3]{7}$

b)  $-3\sqrt[3]{12} + 8\sqrt[3]{12} - 10$

c)  $2\sqrt[3]{25} - 7\sqrt[3]{5} + 6\sqrt[3]{25}$

d)  $\sqrt[3]{40} + 6\sqrt[3]{135}$

e)  $\sqrt[3]{128} - 5\sqrt[3]{250}$

f)  $7\sqrt[3]{x} + \sqrt[3]{64x}$

#### Teaching Notes:

- Many students need extra practice in identifying like radicals.
- Some students combine the coefficients and multiply the like radicals.
- Many students confuse  $\sqrt{\quad}$  and  $\sqrt[3]{\quad}$ . In fact, a common error is to evaluate  $\sqrt[3]{4} = 2$  or  $\sqrt[3]{36} = 6$ . Encourage students to be cautious determining the index.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $23\sqrt{5}$ ; 1b)  $8\sqrt{7}$ ; 1c)  $-12\sqrt{11}$ ; 1d)  $2\sqrt{3} + 35$ ; 1e)  $-3\sqrt{21} - 7\sqrt{7}$ ; 2a)  $26\sqrt{5}$ ; 2b)  $38\sqrt{2}$ ; 2c)  $-23\sqrt{3}$ ; 2d)  $-55\sqrt{3}$ ; 2e)  $-28\sqrt{2x}$ ; 2f)  $6x$ ; 3a)  $13\sqrt[3]{7}$ ; 3b)  $5\sqrt[3]{12} - 10$ ; 3c)  $8\sqrt[3]{25} - 7\sqrt[3]{5}$ ; 3d)  $20\sqrt[3]{5}$ ; 3e)  $-21\sqrt[3]{2}$ ; 3f)  $11\sqrt[3]{x}$

## Mini-Lecture 8.4

### Multiplying and Dividing Radicals

#### Learning Objectives:

1. Multiply radicals.
2. Divide radicals.
3. Rationalize denominators.
4. Rationalize using conjugates.
5. Key Vocabulary: *product rule for radicals, quotient rule for radicals, rationalizing, conjugates.*

#### Examples:

1. Multiply and simplify. Assume that all variables represent positive real numbers.

a)  $\sqrt{3} \cdot \sqrt{5}$

b)  $\sqrt{5x} \cdot \sqrt{5x}$

c)  $\sqrt{2} \cdot \sqrt{6}$

d)  $(3\sqrt{x})^2$

e)  $\sqrt{5x^3} \cdot \sqrt{15x}$

f)  $\sqrt{6}(\sqrt{3} + \sqrt{2})$

g)  $(\sqrt{7} + 3)(\sqrt{7} - 3)$

h)  $(8\sqrt{5} + 9)(9\sqrt{5} + 3)$

i)  $(4\sqrt{3} - 8)^2$

2. Divide and simplify. Assume that all variables represent positive real numbers.

a)  $\frac{\sqrt{12}}{\sqrt{3}}$

b)  $\frac{\sqrt{50}}{\sqrt{2}}$

c)  $\frac{\sqrt{50y^3}}{\sqrt{2y}}$

3. Rationalize each denominator and simplify. Assume that all variables represent positive real numbers.

a)  $\frac{\sqrt{7}}{\sqrt{5}}$

b)  $\sqrt{\frac{5}{12}}$

c)  $\frac{3x}{\sqrt{2}}$

4. Rationalize each denominator and simplify. Assume that all variables represent positive real numbers.

a)  $\frac{2}{6 - \sqrt{3}}$

b)  $\frac{7}{\sqrt{5} + 2}$

c)  $\frac{15}{3 + \sqrt{x}}$

#### Teaching Notes:

- Many students have trouble with problem 1i. They tend to square each term in the binomial rather than squaring the binomial.
- Most students are able to rationalize a denominator with one term.
- Many students have difficulty rationalizing a denominator with 2 terms.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $\sqrt{15}$ ; 1b)  $5x$ ; 1c)  $2\sqrt{3}$ ; 1d)  $9x$ ; 1e)  $5x^2\sqrt{3}$ ; 1f)  $3\sqrt{2} + 2\sqrt{3}$ ; 1g)  $-2$ ;

1h)  $387 + 105\sqrt{5}$ ; 1i)  $112 - 64\sqrt{3}$ ; 2a)  $2$ ; 2b)  $5$ ; 2c)  $5y$ ; 3a)  $\frac{\sqrt{35}}{5}$ ; 3b)  $\frac{\sqrt{15}}{6}$ ; 3c)  $\frac{3x\sqrt{2}}{2}$ ;

4a)  $\frac{12 + 2\sqrt{3}}{33}$ ; 4b)  $7\sqrt{5} - 14$ ; 4c)  $\frac{45 - 15\sqrt{x}}{9 - x}$

## Mini-Lecture 8.5

### Solving Equations Containing Radicals

#### Learning Objectives:

1. Solve radical equations by using the squaring property of equality once.
2. Solve radical equations by using the squaring property of equality twice.

#### Examples:

1. Solve each equation.

a)  $\sqrt{x} = 5$

b)  $\sqrt{x} - 3 = 13$

c)  $3\sqrt{x} - 15 = 60$

d)  $2\sqrt{x} + 11 = 9$

e)  $1 + \sqrt{y + 4} = 11$

f)  $\sqrt{y + 9} = y + 3$

g)  $\sqrt{5x - 2} = \sqrt{2x + 1}$

h)  $\sqrt{x + 8} - x = 2$

i)  $\sqrt{9x^2 + 5x - 20} = 3x$

2. Solve each equation.

a)  $\sqrt{x} + 3 = \sqrt{x + 21}$

b)  $\sqrt{x - 27} = \sqrt{x} - 3$

c)  $\sqrt{x} - 1 = \sqrt{x - 9}$

Mixed Practice. Solve each equation.

d)  $\sqrt{5x - 1} = 3$

e)  $x + 3 = \sqrt{2x} + 7$

f)  $\sqrt{x + 11} = \sqrt{6x - 9}$

#### Teaching Notes:

- Many students have to be reminded to isolate the radical before squaring both sides.
- Refer students to the textbook for *To Solve a Radical Equation Containing Square Roots*.
- Many students find the concept of extraneous solutions confusing.
- Show students a simple example of an extraneous solution, such as:  
 $x = 5 \rightarrow$  square both sides  $\rightarrow x^2 = 25 \rightarrow x = \pm 5 \rightarrow x = -5$  is extraneous.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 25; 1b) 256; 1c) 625; 1d) not real; 1e) 96; 1f) 0; 1g) 1; 1h) 1; 1i) 4; 2a) 4; 2b) 36; 2c) 25; 2d) 2; 2e) 8; 2f) 4

## Mini-Lecture 8.6

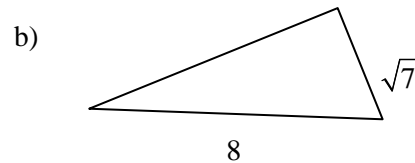
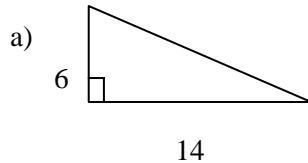
### Radical Equations and Problem Solving

#### Learning Objectives:

1. Use the Pythagorean formula to solve problems.
2. Use the distance formula.
3. Solve problems using formulas containing radical.

#### Examples:

1. Use the Pythagorean Theorem to find the length of the unknown side of each right triangle. Give an exact answer and a two-decimal-place approximation.



Find the length of the unknown side of each right triangle with sides  $a$ ,  $b$ , and  $c$ , where  $c$  is the hypotenuse. Give an exact answer and a two-decimal-place approximation.

- c)  $a = 15, b = 20$                       d)  $b = 18, c = 82$                       e)  $a = 20, c = 28$
2. Use the distance formula to find the distance between the points given.  
a)  $(4,2), (5,8)$                       b)  $(-4,-6), (-1,5)$                       c)  $(-8,6), (8,-6)$
  3. Solve. Give a two decimal place approximation.  
a) A baseball diamond is a square measuring 90 feet on each side. What is the distance from first base to third base?  
b) A firefighter places a 37-foot ladder 9 feet from the base of the wall of a building. Will the top reach a window ledge that is 35 feet above the ground?  
c) Denise set up a volleyball net in her backyard. One of the poles, which forms a right angle with the ground, is 7 feet high. To secure the poles, he attached a rope from the top of the pole to a stake 3 feet from the bottom of the pole. Find the length of the rope.  
d) Cindy and Dan leave home at the same time. Dan drives eastward at 30 miles per hour, while Cindy drives south at 50 miles per hour. Find how far apart they are to the nearest mile after 3 hours.

#### Teaching Notes:

- Most students can use Pythagorean Theorem when the triangle is in the form of 1a).
- Many students get confused when the triangle is rotated as in 1b). Remind students to locate the right angle; the side directly across from the right angle is the hypotenuse.
- Remind students to draw a picture, label a diagram, etc. when solving applications. Again, always locate the right angle, which will lead to identifying the hypotenuse.

Answers: 1a)  $2\sqrt{58}, 15.23$ ; 1b)  $\sqrt{57}, 7.55$ ; 1c) 25; 1d) 80; 1e)  $8\sqrt{6}, 19.60$ ; 2a)  $\sqrt{37}, 6.08$ ; 2b)  $\sqrt{130}, 11.40$ ; 2c) 2; 3a) 127.28 ft. 3b) yes; 3c) 7.62 ft.; 3d) 175 mi.



## Mini-Lecture 8.7

### Rational Exponents

#### Learning Objectives:

1. Evaluate exponential expressions of the form  $a^{1/n}$ .
2. Evaluate exponential expressions of the form  $a^{m/n}$ .
3. Evaluate exponential expressions of the form  $a^{-m/n}$ .
4. Use rules for exponents to simplify expressions containing fractional exponents.

#### Examples:

1. Evaluate exponential expressions of the form  $a^{1/n}$ . Write in radical form and then simplify.

a)  $36^{\frac{1}{2}}$

b)  $1^{\frac{1}{3}}$

c)  $\left(\frac{1}{1000}\right)^{\frac{1}{3}}$

2. Evaluate exponential expressions of the form  $a^{m/n}$ . Write first in radical form, then simplify.

a)  $9^{\frac{3}{2}}$

b)  $64^{\frac{2}{3}}$

c)  $-81^{\frac{3}{4}}$

3. Evaluate exponential expressions of the form  $a^{-m/n}$ . Write first in radical form, then simplify.

a)  $4^{\frac{1}{2}}$

b)  $16^{-\frac{3}{4}}$

c)  $125^{-\frac{2}{3}}$

4. Use rules for exponents to simplify expression containing fractional exponents.

a)  $27^{\frac{4}{3}} \cdot 27^{\frac{1}{3}}$

b)  $\frac{64^{\frac{4}{3}}}{64^{\frac{5}{3}}}$

$-25^{\frac{3}{2}}$

#### Teaching Notes:

- Identify the power and the index of the exponent before students work the problem.
- Remind students a negative outside the parentheses is not affected by the exponent
- Review exponent rules before simplifying expressions.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $\sqrt{36} = 6$ ; 1b)  $\sqrt[3]{1} = 1$ ; 1c)  $\sqrt[3]{\frac{1}{1000}} = \frac{1}{10}$ ; 2a)  $(\sqrt{9})^3$ , 27; 2b)  $(\sqrt[3]{64})^2$ , 16; 2c)  $-(\sqrt[4]{81})^3$ , -27;

3a)  $\frac{1}{2}$ , 3b)  $\frac{1}{8}$ ; 3c)  $\frac{1}{25}$ ; 4a)  $(\sqrt[3]{27})^5$ ,  $3^5$ ; 4b)  $\frac{1}{\sqrt[3]{64}}$ ,  $\frac{1}{4}$ ; 4c)  $(-\sqrt{25})^3$ ,  $-5^3$

## Mini-Lecture 9.1

### Solving Quadratic Equations by the Square Root Property

#### Learning Objectives:

1. Use the square root property to solve quadratic equations.
2. Solve problems modeled by quadratic by quadratic equations.

#### Examples:

1. Solve each equation by factoring.

a)  $x^2 - 36 = 0$       b)  $5x^2 - 125 = 0$       c)  $x^2 - 4x = 21$       d)  $2x^2 = x + 10$

Use the square root property to solve each quadratic equation.

e)  $x^2 = 49$       f)  $x^2 - 33 = 0$       g)  $x^2 + 100 = 0$       h)  $25x^2 = 8$

i)  $(x-4)^2 = 25$       j)  $\left(x - \frac{1}{5}\right)^2 = \frac{1}{25}$       k)  $(2x-1)^2 = 49$       l)  $(a+2)^2 = 15$

2. Solve the following applications. Round each answer to the nearest tenth of a second.

a) Use the formula  $h = 16t^2$  to solve the following: determine the time of a stuntman's fall if he jumped from a height of 450 feet.

b) Use the formula for the area of a square  $A = s^2$  where  $s$  is the length of a side. If the area of a square is 40 square inches, find the length of the side.

c) Use the formula for the area of a square  $A = s^2$  where  $s$  is the length of a side. If the area of the square base of a monument is 2200 square feet, find the length of the side.

#### Teaching Notes:

- Many students will need a review of factoring, especially factoring difference of two squares.
- Many students need extra practice working with the “ $\pm$ ” symbol.
- Some students need to always write their answers by separating the “ $\pm$ ” symbol, and writing the positive solution and negative solution.
- Remind students with applications that the negative solution will be an *extraneous solution*.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $\pm 6$ ; 1b)  $\pm 5$ ; 1c)  $-3, 7$ ; 1d)  $-2, \frac{5}{2}$ ; 1e)  $\pm 7$ ; 1f)  $\pm\sqrt{33}$ ; 1g) *not real*; 1h)  $\pm\frac{2\sqrt{2}}{5}$ ;

1i)  $-1, 9$ ; 1j)  $0, \frac{2}{5}$ ; 1k)  $-3, 4$ ; 1l)  $-2 \pm \sqrt{15}$ ; 2a)  $5.3$  s; 2b)  $6.3$  in.; 2c)  $46.9$  ft.

## Mini-Lecture 9.2

### Solving Quadratic Equations by Completing the Square

#### Learning Objectives:

1. Write perfect square trinomials.
2. Solve quadratic equations of the form  $x^2 + bx + c = 0$  by completing the square.
3. Solve quadratic equations of the form  $ax^2 + bx + c = 0$  by completing the square.

#### Examples:

1. Solve each quadratic equation by completing the square.

a)  $x^2 + 12x = -35$

b)  $a^2 - 10a + 21 = 0$

c)  $z^2 + 10z + 3 = 0$

d)  $x^2 = 7 - 4x$

e)  $x^2 = 5 - 5x$

f)  $a(3a - 2) + 1 = 5$

2. Solve each quadratic equation by completing the square.

a)  $16x^2 + 24x + 5 = 0$

b)  $9y^2 + 18y + 8 = 0$

c)  $9x^2 - 35 = 6x$

d)  $2x^2 + 4x + 11 = 0$

e)  $2n^2 + 6n + 3 = 0$

f)  $2x(2x + 5) = -3$

Mixed Practice. Solve each quadratic equation by completing the square.

g)  $k^2 + 8k + 7 = 0$

h)  $x(6x + 1) - 10 = -8$

i)  $2x^2 + 5x - 2 = 0$

#### Teaching Notes:

- Refer students to the textbook for steps used to solve by completing the square: ***To Solve a Quadratic Equation in  $x$  by Completing the Square.***
- Many students will need a review of perfect square trinomials to understand the goal of completing the square.
- Most students need to see many examples done out for them to understand the process.
- Some students need to write out both solutions instead of using the “ $\pm$ ” symbol.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) -7, -5; 1b) 3, 7; 1c)  $-5 \pm \sqrt{22}$ ; 1d)  $-2 \pm \sqrt{11}$ ; 1e)  $\frac{-5 \pm 3\sqrt{5}}{2}$ ; 1f)  $\frac{1 \pm \sqrt{13}}{3}$  ;

2a) -1/4, -5/4; 2b) -2/3, -4/3; 2c) 7/3, -5/3; 2d) not real; 2e)  $\frac{-3 \pm \sqrt{3}}{2}$  2f)  $\frac{-5 \pm \sqrt{13}}{4}$ ; 2g) -7, -1;

2h) 1/2, -2/3; 2i)  $\frac{-5 \pm \sqrt{41}}{4}$

## Mini-Lecture 9.3

### Solving Quadratic Equations by the Quadratic Formula

#### Learning Objectives:

1. Use the quadratic formula to solve quadratic equations.
2. Approximate solutions to quadratic equations.
3. Determine the number of solutions of a quadratic equation by using the discriminant.

#### Examples:

1. Identify the value of a, b, and c in each quadratic equation.

a)  $2x^2 - 7x - 9 = 0$

b)  $4x^2 - 3x - 7 = 0$

c)  $x^2 - 15 = 0$

Use the quadratic formula to solve each quadratic equation.

d)  $x^2 + 7x + 10 = 0$

e)  $3p^2 - 23p + 14 = 0$

f)  $2x^2 - 7x - 4 = 0$

g)  $x^2 + 5x - 8 = 0$

h)  $x^2 - 7 = 0$

i)  $x^2 - 3x + 4 = -5$

2. Use the quadratic formula to solve each quadratic equation. Find the exact solutions; then approximate these solutions to the nearest tenth.

a)  $4y^2 + 2y - 3 = 0$

b)  $2x^2 + 4x = -1$

c)  $\frac{1}{8}x^2 - \frac{1}{4}x = \frac{1}{2}$

3. Use the discriminant to determine the number of solutions of each quadratic equations.

a)  $x^2 - 5x + 4 = 0$

b)  $2x^2 - 3x + 5 = 0$

c)  $x^2 - 12x + 36 = 0$

#### Teaching Notes:

- Most students have difficulty learning the quadratic formula. Suggest that they put the formula to the tune of “Pop Goes The Weasel”. “x equals negative b plus or minus square root, b squared minus 4ac all over 2a”.
- Most students make careless errors upon evaluation.
- Encourage students to clearly write the values for a, b, and c before substitution.
- Remind students that the fraction bar is under the entire numerator  $-b \pm \sqrt{b^2 - 4ac}$ .
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) a=2, b=-7, c=-9; 1b) a=4, b=-3, c=-7; 1c) a=1, b=0, c=-15; 1d) -2, -5; 1e) 7, 2/3; 1f) 4, -1/2

1g)  $\frac{-5 \pm \sqrt{57}}{2}$ ; 1h)  $\pm\sqrt{7}$ ; 1i) not a real number; 2a)  $\frac{-1 \pm \sqrt{13}}{4}$ , 0.7, -1.2; 2b)  $\frac{-2 \pm \sqrt{2}}{2}$ , -0.3, -1.7;

2c)  $1 \pm \sqrt{5}$ , 3.2, -1.2; 3a) two distinct real solutions; 3b) no real solutions; 3c) one real solution

## Mini-Lecture 9.4

### Complex Solutions of Quadratic Equations

#### Learning Objectives:

1. Write complex numbers using  $i$  notation.
2. Add and subtract complex numbers.
3. Multiply complex numbers.
4. Divide complex numbers.
5. Solve quadratic equations that have complex solutions.

#### Examples:

1. Simplify each of the following using  $\sqrt{-1} = i$ .  
a)  $\sqrt{-49}$                       b)  $\sqrt{-12}$                       c)  $\sqrt{-80}$
2. Add or subtract as indicated.  
a)  $(3-i) + (5+4i)$                       b)  $(2+5i) - (3+2i)$                       c) Subtract  $(8-i)$  from  $(3+4i)$
3. Multiply complex numbers.  
a)  $2i(4-8i)$                       b)  $(5-2i)(3+i)$                       c)  $(1+2i)(1-2i)$
4. Divide complex numbers.  
a)  $\frac{9-12i}{3}$                       b)  $\frac{20+5i}{5i}$                       c)  $\frac{6-i}{3-2i}$
5. Solve the following quadratic equations for complex solutions.  
a)  $(x+2)^2 = -16$                       b)  $(3x-5)^2 = -12$                       c)  $x^2 + 8x + 20 = 0$

#### Teaching Notes:

- Remind students  $i^2 = -1$  and  $i = \sqrt{-1}$ .
- Complex numbers can be written in the form  $a + bi$ .
- Complex numbers written in the form  $0 + bi$   $b \neq 0$  is also called a pure imaginary number.
- Complex number written in the form  $a + bi$  is in standard form.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a)  $7i$ ; 1b)  $2i\sqrt{3}$ ; 1c)  $4i\sqrt{5}$ ; 2a)  $8 + 3i$ ; 2b)  $-9 + 3i$ ; 2c)  $-5 + 5i$ ; 3a)  $16 + 8i$ ; 3b)  $17 - i$ ; 3c)  $5$ ;  
4a)  $3 - 4i$ ; 4b)  $\frac{20+9i}{14}$ ; 4c)  $1 - 4i$ ; 5a)  $2 \pm 4i$ ; 5b)  $\frac{5 \pm 2\sqrt{3}}{3}$ ; 5c)  $-4 \pm 2i$

## Mini-Lecture 9.5

### Graphing Quadratic Equations

#### Learning Objectives:

1. Graph quadratic equations of the form  $y = ax^2$ .
2. Graph quadratic equations of the form  $y = ax^2 + bx + c$ .
3. Use the vertex formula to determine the vertex of a parabola.

#### Examples:

1. Graph each quadratic equation by finding and plotting ordered pair solutions.

a)  $y = x^2$

b)  $y = 4x^2$

c)  $y = -2x^2$

2. Sketch the graph of each equation. Label the vertex and the intercepts.

a)  $y = x^2 - 4$

b)  $y = x^2 + 16$

c)  $y = -x^2 - 2$

d)  $y = x^2 + 7x + 10$

e)  $y = -x^2 + x + 6$

f)  $y = -x^2 + 6x - 9$

g)  $y = \frac{1}{5}x^2$

h)  $y = x^2 + 5x$

i)  $y = -2x^2 + 5x - 3$

#### Teaching Notes:

- Remind students that a parabola is a smooth curve; not a point.
- Encourage students to always find the x- and y-intercepts, if they exist.
- Most students prefer graphing by vertex and intercepts rather than by finding points.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

*Answers:* 1a) – 2i) see mini-lecture graphing answers; 2a)  $V=(0,-4), (2, 0), (-2, 0)$ ; 2b)  $V=(0, 16)$ ;

2c)  $V = (0,-2)$ ; 2d)  $V=\left(-\frac{7}{2}, -\frac{9}{4}\right), (-2, 0), (-5, 0), (0, 10)$ ; 2e)  $V=\left(\frac{1}{2}, \frac{25}{4}\right), (0, 6), (-2,0), (3,0)$ ;

2f)  $V = (3, 0), (0, -9)$ ; 2g)  $V=(0, 0)$ ; 2h)  $V=\left(-\frac{5}{2}, -\frac{25}{4}\right), (0, 0), (-5, 0)$ ; 2i)  $V=\left(\frac{5}{4}, \frac{1}{8}\right), (1, 0), (3/2, 0), (0, -3)$