# 2.3 Another Look at Linear Graphs

- Graphing Horizontal Lines and Vertical Lines
- Graphing Using Intercepts
- Parallel and Perpendicular Lines
- Recognizing Linear Equations

# Graphing Horizontal Lines and Vertical Ines

### *Slope of a Horizontal Line*

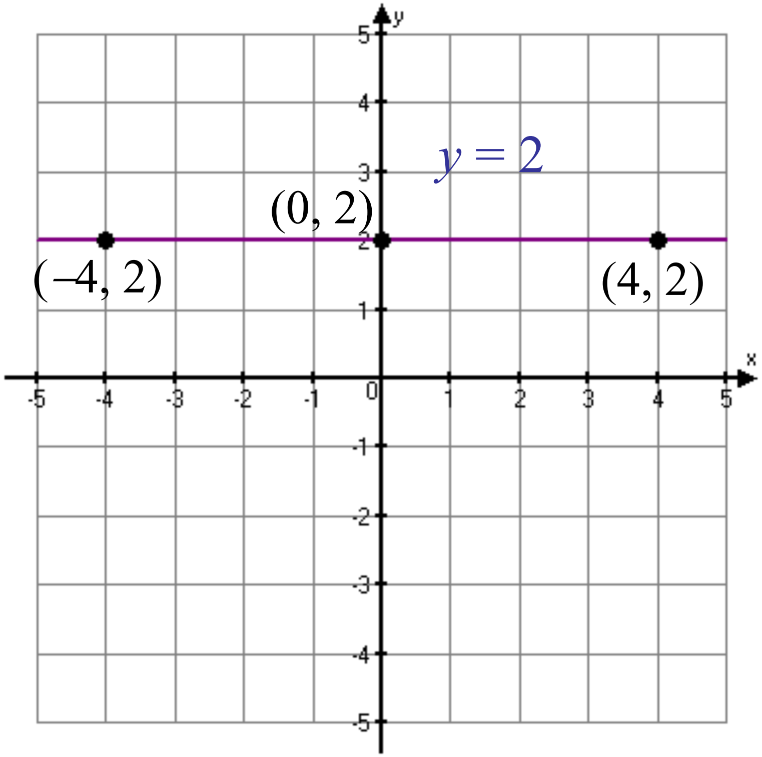
The slope of a horizontal line is 0.

The graph of any function of the form $f(x) = b$ or $y = b$ is a horizontal line that crosses the $y$-axis as $(0, b)$.

# Example continued $y = 2$

*Solution*

When we plot the ordered pairs (0, 2), (4, 2) and (–4, 2) and connect the points, we obtain a horizontal line.




Any ordered pair of the form ($x$, 2) is a solution, so the line is parallel to the $x$-axis with $y$-intercept (0, 2).

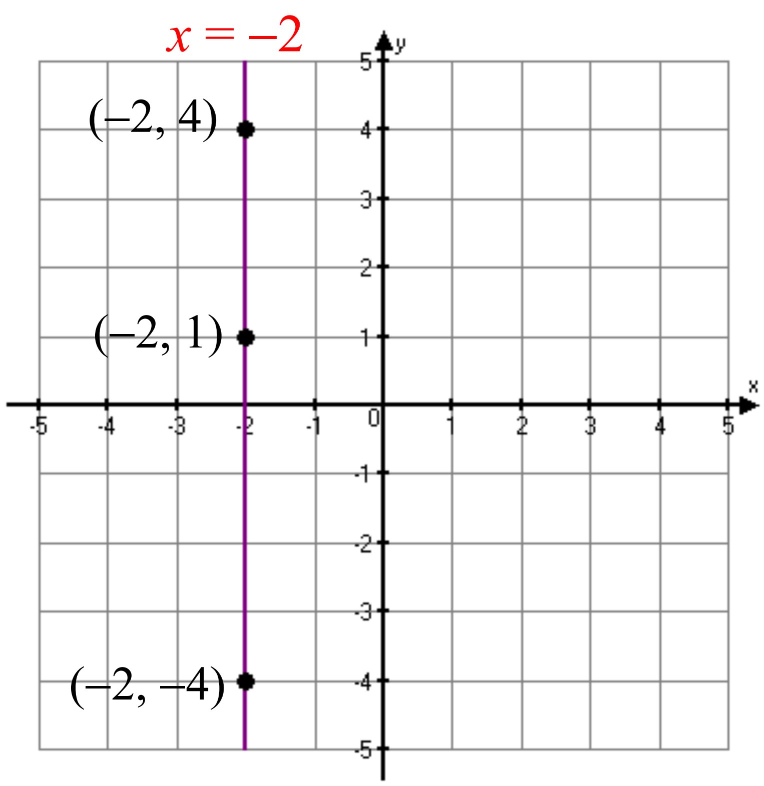
## *Slope of a Vertical Line*

The slope of a vertical line is undefined.

# Example continued $x = -2$

*Solution*

When we plot the ordered pairs (–2, 4), (–2, 1), and (–2, –4) and connect them, we obtain a vertical line.




Any ordered pair of the form (–2, $y$) is a solution. The line is parallel to the $y$-axis with $x$-intercept (–2, 0).

## *Slope of a Vertical Line*

The slope of a vertical line is undefined.

The graph of any equation of the form $(x) = a$ is a vertical line that crosses the $x$-axis as $(a, 0)$.

# Graphing Using Intercepts

The point at which the graph crosses the $y$-axis is called the ***y*-intercept.** The $x$-coordinate of a $y$-intercept is always 0.

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## *To Determine Intercepts*

The $x$-intercept is $(a, 0)$. To find $a$, let $y = 0$ and solve the original equation for $x$.

The $y$-intercept is $(0, b)$. To find $b$, let $x = 0$ and solve the original equation for $y$.

# Example

Graph $5x + 2y = 10$ using intercepts.

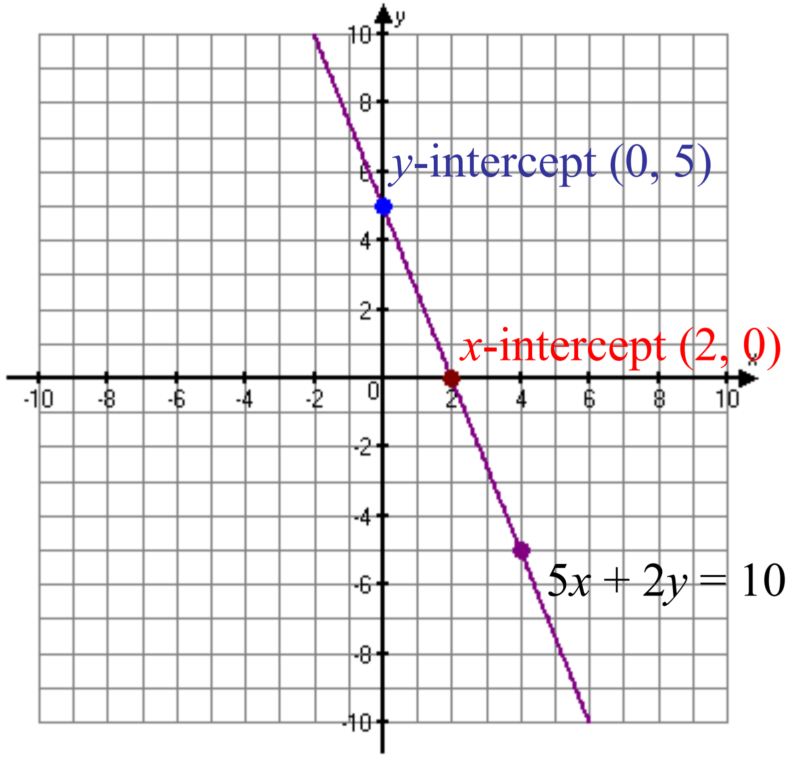
*Solution*

To find the $y$-intercept we let $x = 0$ and solve for $y$.

$$5(0) + 2y = 10$$
$$2y = 10$$
$$y = 5 \quad (0, 5)$$

To find the $x$-intercept we let $y = 0$ and solve for $x$.

$$5x + 2(0) = 10$$
$$5x = 10$$
$$x = 2 \quad (2, 0)$$



**Example** Determine whether the graphs of $y = \frac{3}{2}x + 3$ and $3x - 2y = -5$ are parallel.

***Solution*** When two lines have the same slope but different $y$-intercepts they are parallel.

The line $y = \frac{3}{2}x + 3$ has slope 3/2 and $y$-intercept (0, 3).

Rewrite $3x - 2y = -5$ in slope-intercept form:

$$3x - 2y = -5$$
$$-2y = -3x - 5$$
$$y = \frac{3}{2}x + \frac{5}{2}$$

The slope is 3/2 and the $y$-intercept is 5/2.

Both lines have slope 3/2 and different $y$-intercepts, the graphs are parallel.

**Slope and Perpendicular Lines**

Two lines are perpendicular if the product of their slopes is $-1$ or if one line is vertical and the other is horizontal.

**Example** Determine whether the graphs of $3x - 2y = 1$ and $y = -\frac{2}{3}x + \frac{1}{3}$ are perpendicular.

*Solution*

First, we find the slope of each line.

$$y = -\frac{2}{3}x + \frac{1}{3}$$

The slope is – 2/3.

Rewrite the other line in slope-intercept form.

$$3x - 2y = 1$$
$$-2y = 1 - 3x$$
$$y = \frac{1}{-2} - \frac{3x}{-2}$$
$$y = \frac{3}{2}x - \frac{1}{2}$$

The slope of the line is 3/2. The lines are perpendicular if the product of their slopes is –1.

$$\frac{-2}{3} \cdot \left(\frac{3}{2}\right) = -1$$

*The lines are perpendicular.*

# Recognizing Linear Equations

A linear equation may appear in different forms, but all linear equations can be written in *standard form* $Ax + By = C$.

## *The Standard Form of a Linear Equation*

Any equation $Ax + By = C$, where $A$, $B$, and $C$ are real numbers and $A$ and $B$ are not both 0, is a linear equation in *standard form* and has a graph that is a straight line.

# Example

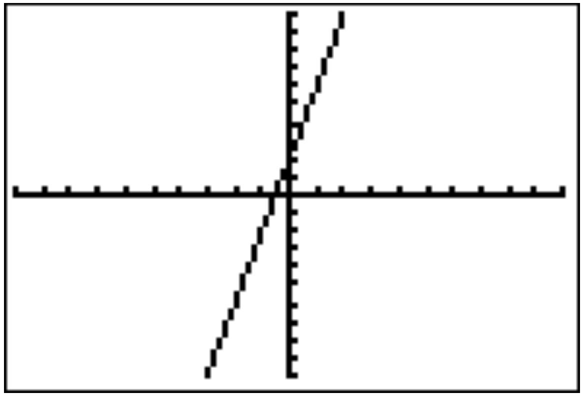
Determine whether each of the following equations is linear:

a) $y = 4x + 2$ b) $y = x^2 + 3$ c) $2y = 7$

*Solution* Attempt to write each equation in standard form.

a) $y = 4x + 2$

$-4x + y = 2$ Adding $-4x$ to both sides

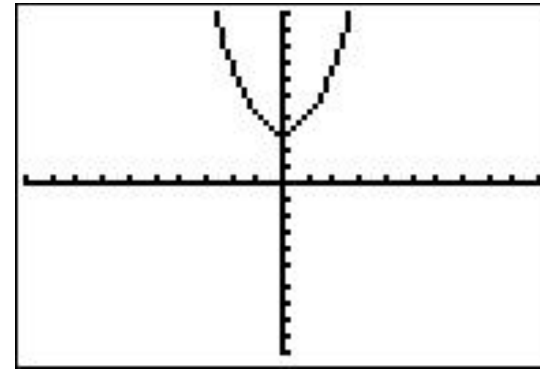


*continued* b) $y = x^2 + 3$ c) $2y = 7$

b) $y = x^2 + 3$

$-x^2 + y = 2$ Adding $-x^2$ to both sides

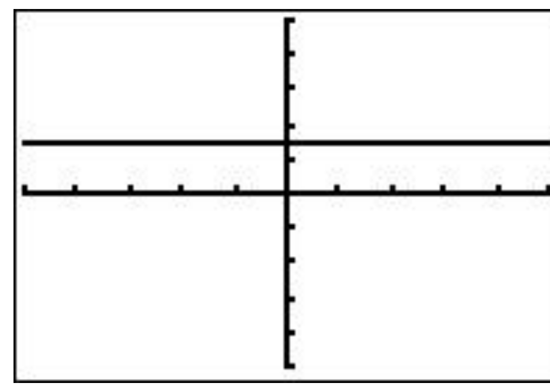
The equation is not linear since it has an $x^2$ term.



c) $2y = 7$

$0 \cdot x + 2y = 7$

The equation is written in standard form, with $A = 0$, $B = 2$ and $C = 7$.



# 2.4 Introduction to Curve Fitting: Point-Slope Form

- **Point-Slope Form**
- **Interpolation and Extrapolation**
- **Curve Fitting**
- **Linear Regression**

## *Point-Slope Form*

Any equation $y - y_1 = m(x - x_1)$ is said to be written in *point-slope* form and has a graph that is a straight line.

The slope of the line is *m*.

The line passes through $(x_1, y_1)$.

# Example

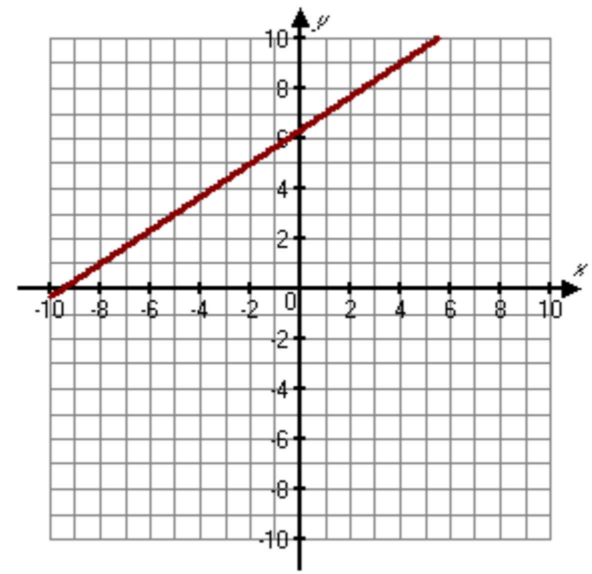
Find and graph an equation of the line passing through (4, 9) with slope 2/3.

## *Solution*

We substitute 2/3 for $m$, and 4 for $x_1$, and 9 for $y_1$:

$$y - y_1 = m(x - x_1)$$

$$y - 9 = \frac{2}{3}(x - 4)$$



# Example

Write the slope-intercept equation for the line with slope 3 and point (4, 3).

## *Solution*

There are two parts to this solution. First, we write an equation in point-slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 3(x - 4)$$

Next, we find an equivalent equation of the form $y = mx + b$:

$$y - 3 = 3(x - 4)$$

$$y - 3 = 3x - 12$$ Using the distributive law

$$y = 3x - 9$$ Adding 3 to both sides to get the slope-intercept form

# Interpolation and Extrapolation

It is possible to use line graphs to estimate real-life quantities that are not already known. To do so, we calculate the coordinates of an unknown point by using two points with known coordinates. When the unknown point is located *between* the two points, this process is called **interpolation**. Sometimes a graph passing through the known points is *extended* to predict future values. Making predictions in this manner is called **extrapolation**.

# Example

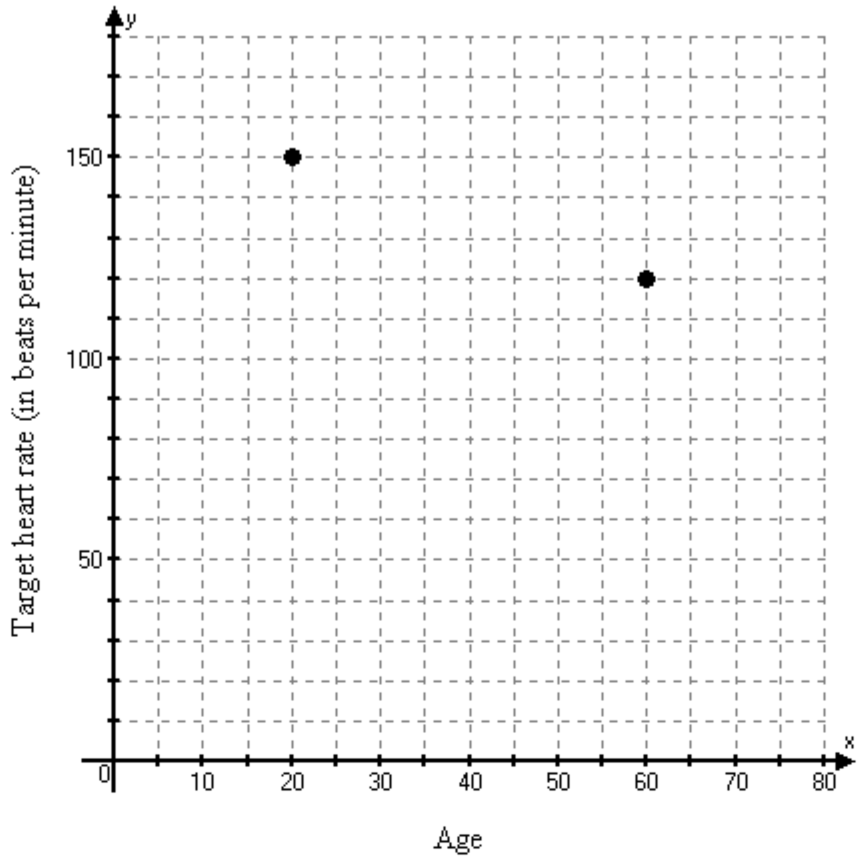
Aerobic exercise. A person's target heart rate is the number of beats per minute that bring the most aerobic benefit to his or her heart. The target heart rate for a 20-year-old is 150 beats per minute and for a 60-year-old, 120 beats per minute.

a) Graph the given data and calculate the target heart rate for a 46-year-old.

b) Calculate the target heart rate for a 70-year-old.

# *Solution*

a) We draw the axes and label, using a scale that will permit us to view both the given and the desired data. The given information allows us to then plot (20, 150) and (60, 120).



# *Solution continued*

We determine the slope of the line.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{150 - 120 \text{ beats per minute}}{20 - 60 \text{ years}}$$

$$= \frac{30 \text{ beats per minute}}{-40 \text{ years}} = -\frac{3}{4} \text{ beats per minute per year}$$

Use one point and write the equation of the line.

$$y - 150 = -\frac{3}{4}(x - 20)$$

$$y - 150 = -\frac{3}{4}x + 15$$

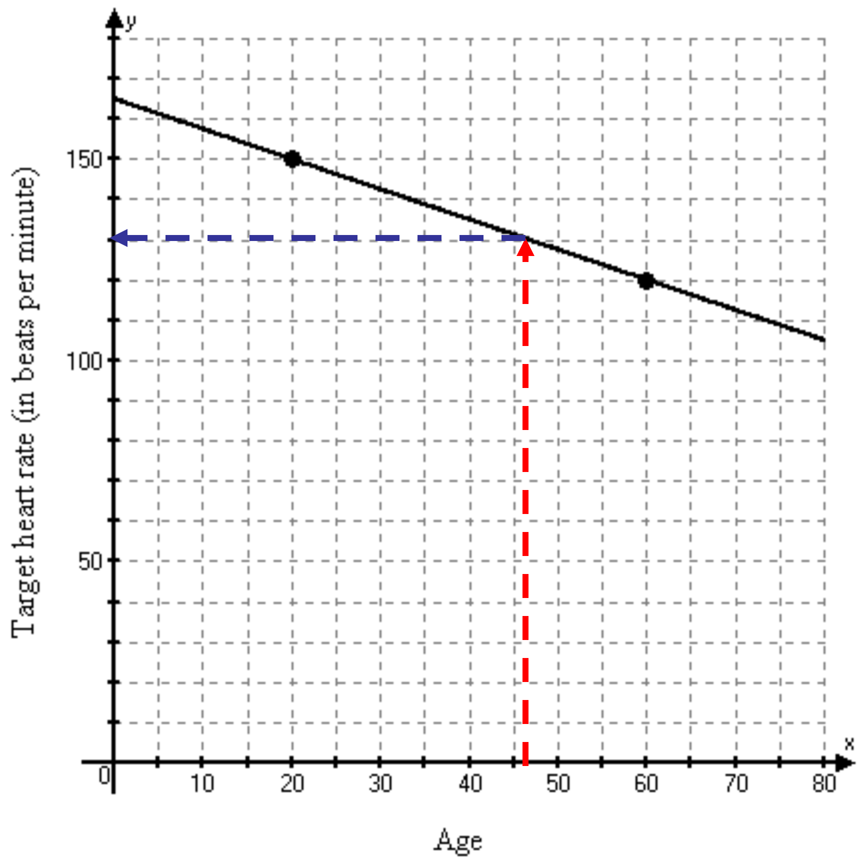
$$y = -\frac{3}{4}x + 165$$

# *Solution continued*

a) To calculate the target heart rate for a 46-year-old, we substitute 46 for *x* in the slope-intercept equation:

$$y = -\frac{3}{4}(46) + 165$$
$$= -34.5 + 165 = 130.5$$

The graph confirms the target heart rate.

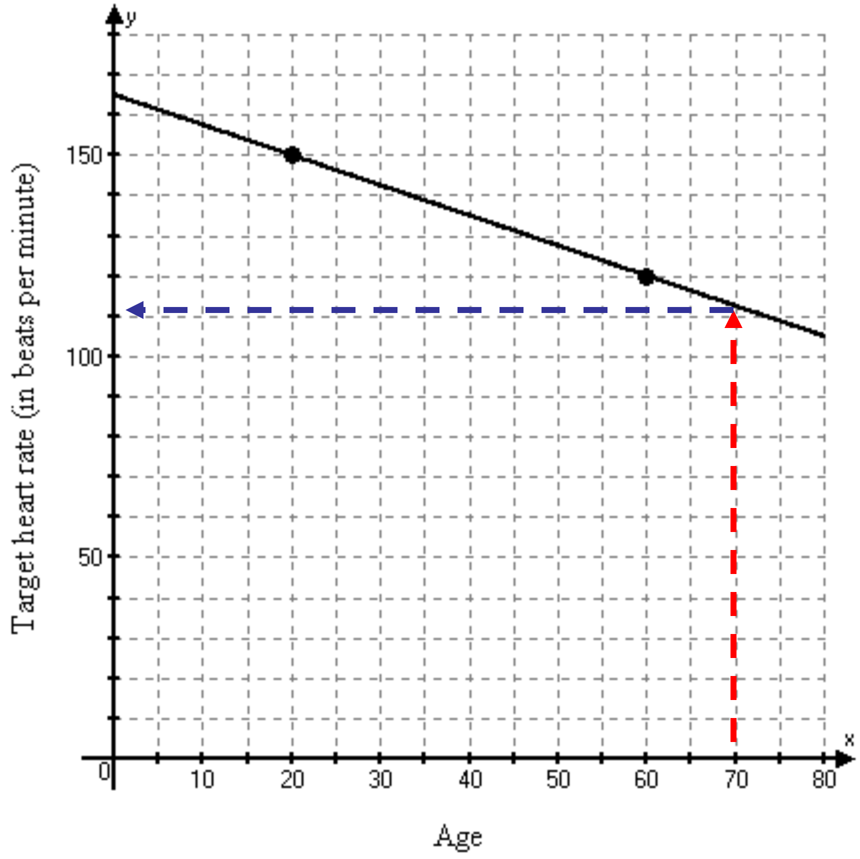


# *Solution continued*

b) To calculate the target heart rate for a 70-year-old, we substitute 70 for *x* in the slope-intercept equation:

$$y = -\frac{3}{4}(70) + 165$$
$$= -52.5 + 165 = 112.5$$

The graph confirms the target heart rate.



# Curve Fitting

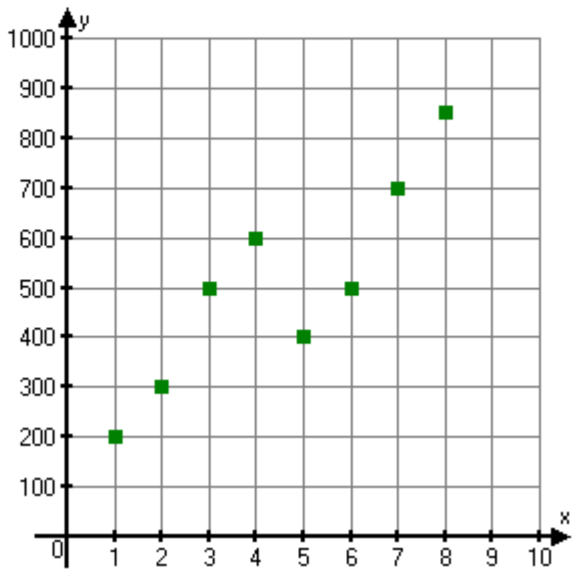
The process of understanding and interpreting *data*, or lists of information, is called *data analysis*.

One helpful tool in data analysis is **curve fitting**, or finding an algebraic equation that describes the data.

# Example

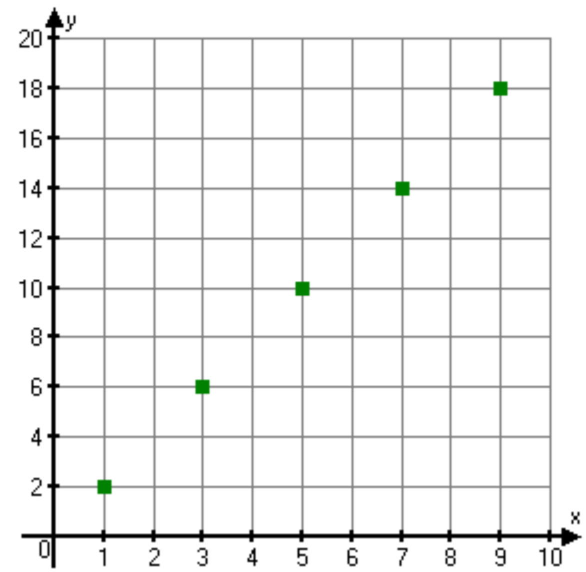
Which graph of sets of data appears to be linear?

a.




Not linear; the points do not lie in a straight line.

b.




Linear; the points appear to lie in a straight line.