2.2 Linear Functions: Slope, Graphs, and Models

- Slope-Intercept Form of an Equation
- Applications

PEARSON Addison Wesley

A function given by an equation of the form $f(x) = mx$ is a *linear function*. Its graph is a straight line passing through the origin.

The graph of $y = mx + b$, $b \neq 0$, is a line parallel to $y = mx$, passing through the point $(0, b)$.

The point $(0, b)$ is called the y-intercept.

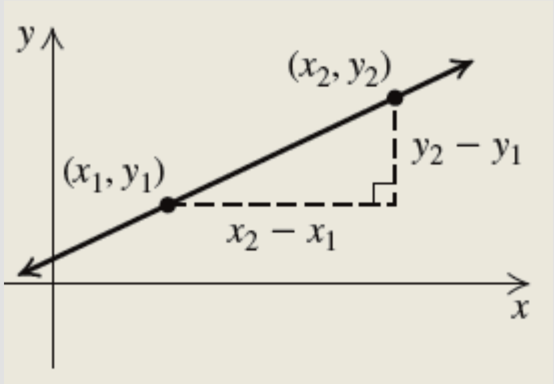
Slope

The *slope* of the line containing points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$= \frac{\text{the difference in } y}{\text{the difference in } x}$$

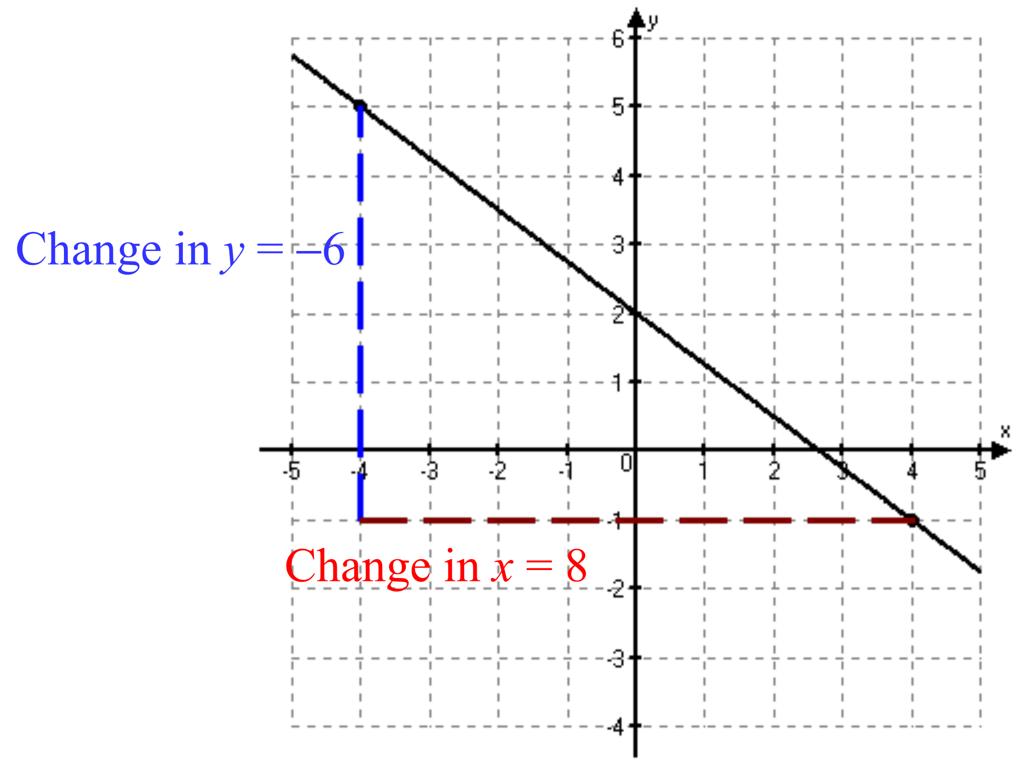
$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}.$$



Example Graph the line containing the points (–4, 5) and (4, –1) and find the slope.

Solution

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{-1-5}{4-(-4)} = \frac{-6}{8} = -\frac{6}{8}, \text{ or } -\frac{3}{4}$$



Example Determine the slope of the line given by $y = \frac{2}{3}x - 1$ and graph the line.

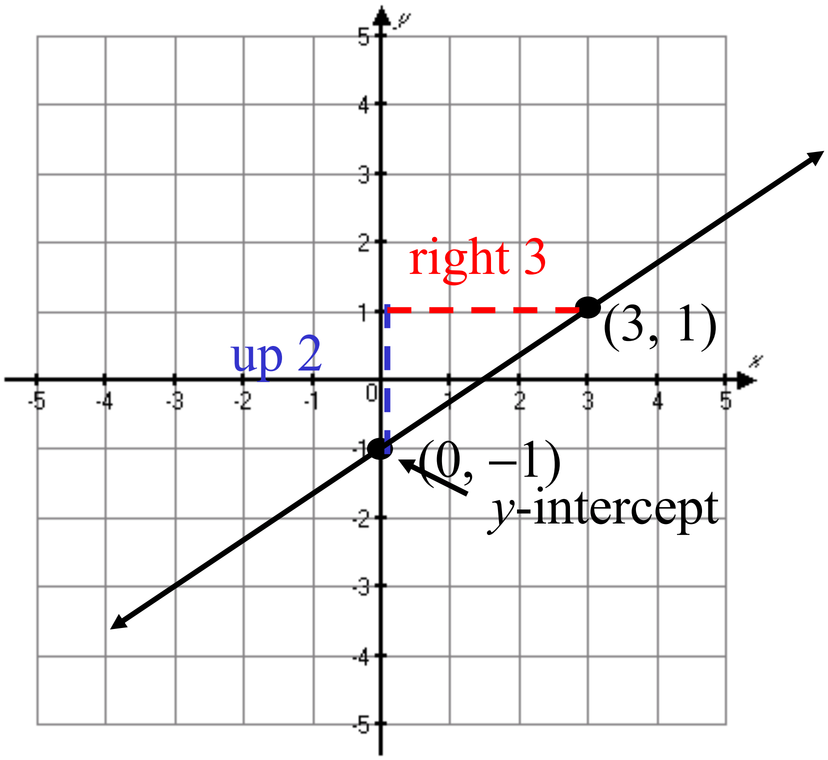
Solution

$m = 2/3$, so the slope is 2/3.

We plot (0, –1) and from there move *up* 2 units.

Then move to the *right* 3 units. This locates the point (3, 1).

Draw a line passing through the two points.



Example Find the slope and the y-intercept of each line whose equation is given.

a) $y = \frac{3}{8}x - 2$ b) $3x + y = 7$ c) $4x - 5y = 10$

Solution

a) $y = \frac{3}{8}x - 2$

The slope is $\frac{3}{8}$. The y-intercept is $(0, -2)$.

b) We first solve for y to find an equivalent form of $y = mx + b$.

$$3x + y = 7$$

$$y = -3x + 7$$

The slope is -3. The y-intercept is $(0, 7)$.

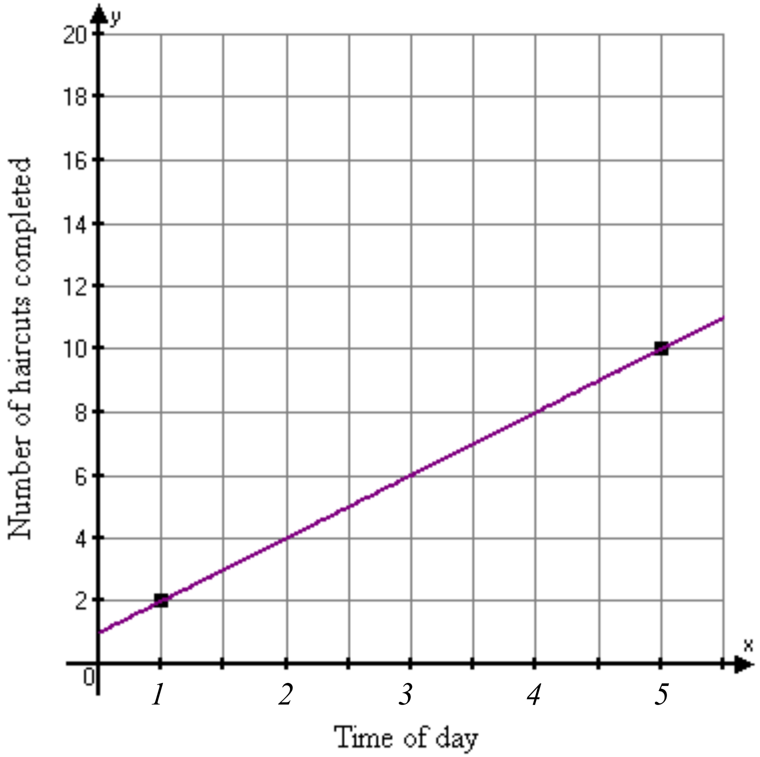
Applications

Graphs allow us to visualize a rate of change. As a rule, the quantity listed in the numerator appears on the vertical axis and the quantity listed in the denominator appears on the horizontal axis.

Example Wanda's Hair Salon has a graph displaying data from a recent day of work.

a) What rate can be determined from the graph?

b) What is that rate?



Solution

a) We can find the rate *Number of haircuts per hour.*

b) $$\frac{10 \text{ haircuts} - 2 \text{ haircuts}}{5:00 - 1:00} = \frac{8 \text{ haircuts}}{4 \text{ hours}} = 2 \text{ haircuts per hour.}$$

Example

Swanton Electronics uses the function $S(t) = -2500t + 12{,}500$ to determine the *salvage value* $S(t)$, in dollars, of a delivery van t years after its purchase.

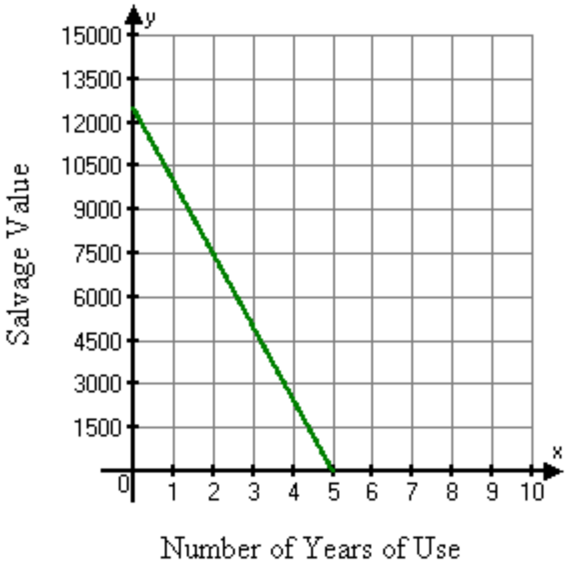
a) What do the numbers –2500 and 12,500 signify?

b) How long will it take the van to *depreciate* completely?

c) What is the domain of S?

Solution $S(t) = -2500t + 12{,}500$

Drawing or at least visualizing the graph can be useful.



a) At time $t = 0$, we have:

$$S(0) = -2500(0) + 12{,}500$$
$$= 12{,}500$$

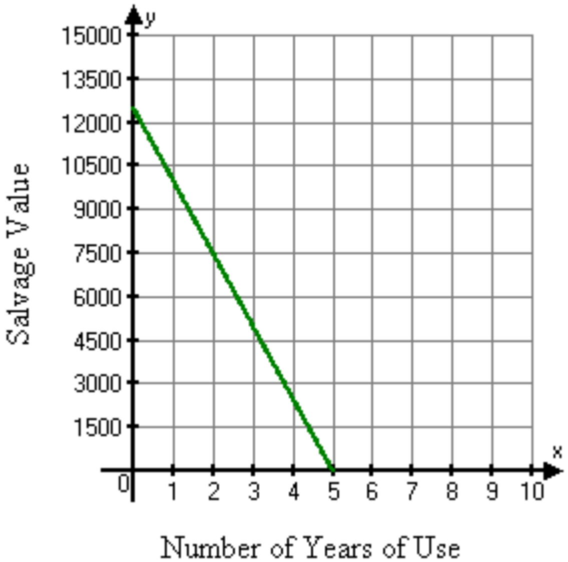
The –2500 represents how much the van decreases in value from year to year.

The $12,500, represents the initial value of the van.

Solution $S(t) = -2500t + 12,500$

b) The van will completely depreciate in value when $S(t) = 0$.

$$S(t) = -2500t + 12,500$$
$$0 = -2500t + 12,500$$
$$-12,500 = -2500t$$
$$5 = t$$



In 5 years the van will have depreciated completely.

c) The domain cannot be negative and after 5 years the van will be depreciated completely.

Domain of S is $\{t \mid 0 \le t \le 5\}$