Professor Katiraie MA103 Test Three Form A (Chapters 7, 8, 9, and 10.3) Spring 07

Name Solution

Total Possible Points = 200 Points

Note: Show all work. Unless a problem is marked with an asterisk (*), use a calculator only to check.
When asked for the equation of a line, the equation should be given in slope-intercept form.

1. Write an equation of the line that passes through (1, 4) and is parallel to the line passing through the points (3, -6) and (-1, 2). (10 points)

$m = \frac{2-(-6)}{-1-3} = \frac{8}{-4} = -2$

$y = mx+b \Rightarrow 4 = -2(1)+b \Rightarrow \boxed{b=6}$

$\boxed{y = -2x+6}$

2. Find the equation of a line perpendicular to $y = -2x+7$ and passing through $(-4,-2)$ **(Must Show Procedure)** (7 points)

$m_{\perp} = \frac{1}{2}$

$y = mx+b$

$-2 = \frac{1}{2}(-4)+b \Rightarrow b = -2+2 = 0$

$\boxed{y = \frac{1}{2}x}$

3. Solve the following system of equations using Elimination method. (10 points)

$\begin{cases} 2x+3y=6 \\ x-2y=-4 \end{cases} \xrightarrow{-2} \begin{cases} 2x+3y=6 \\ -2x+4y=8 \end{cases}$

$7y = 14 \quad \boxed{y=2}$

$2x+3(2)=6$

$2x=0$

$\boxed{x=0}$

Solution $\boxed{(0,2)}$

4. A student takes out two loans to help pay for college. One loan is at 11% simple interest, and the other is at 7% simple interest. The total amount borrowed is $4000, and the interest after 1 year for both loans is $380. Find the amount of each loan. (10 points)

$$\begin{cases} X+Y=4000 \\ 0.11X+0.07Y=380 \end{cases} \Rightarrow \begin{cases} -7(X+Y=4000) \\ 11X+7Y=38000 \end{cases} \Rightarrow \begin{cases} -7X-7Y=-28000 \\ 11X+7Y=38000 \end{cases}$$

$4X = 10000$

$X = \$2500$

$Y = \$1500$

$2500 at 11%
$1500 at 7%

5. A vending machine will only accept quarters and dimes. When the coins are collected, the machine has 226 coins worth $24.10. How many quarters were there? How many dimes? Show your work! (10 points)

$$\begin{cases} X+Y=226 \\ 0.25X+0.10Y=24.10 \end{cases} \Rightarrow \begin{cases} -10(X+Y=226) \\ 25X+10Y=2410 \end{cases} \Rightarrow \begin{cases} -10X-10Y=-2260 \\ 25X+10Y=2410 \end{cases}$$

$15X = 150$

$X = 10$

$Y = 216$

10 quarters and 216 Dimes

6. PERFORM THE OPERATION & SIMPLIFY: (3 points)

$$\frac{3}{6x^2} - \frac{4}{21x^7} = \frac{21x^5}{42x^7} - \frac{8}{42x^7}$$

$7x^5$

$$= \frac{21x^5 - 8}{42x^7}$$

7. Simplify the following: (12 points)

A. $\sqrt{72x^{18}} = \sqrt{36}\sqrt{2}\,x^9 = 6\sqrt{2}\,x^9$

B. $\left(\frac{x^6}{27}\right)^{\frac{2}{3}} = \frac{x^4}{9}$

C. $\sqrt{(x+7)^{21}} = \sqrt{(x+7)^{20}}\sqrt{x+7} = (x+7)^{10}\sqrt{x+7}$

D. $\sqrt{\sqrt{y}} = \left(y^{\frac{1}{2}}\right)^{\frac{1}{2}} = y^{\frac{1}{4}}$

E. $\sqrt{72x^8y^5} = 6\sqrt{2}\,x^4y^2\sqrt{y} = 6x^4y^2\sqrt{2y}$

F. $\left(x^2y^8\right)^{\frac{1}{2}} = x^1y^4$

8. Assume $1500 is deposited in an account that earns 4% interest compounded annually. (10 points)

a. Find a formula for g(t) where t is time and g(t) is the amount of money in the account after **t years**.

$g(t) = 1500(1+0.04)^t = 1500(1.04)^t$

b. How long will it take for the money to double.

$3000 = 1500(1.04)^t$

$2 = 1.04^t \Rightarrow t = \log_{1.04} 2 = \frac{\log 2}{\log 1.04} = 17.7\text{ YRS}$

9. Find the inverse of the following functions (10 points)

a. $f(x) = 7x + 105$

$y = 7x + 105$

$7x = y + 105 \Rightarrow x = \frac{y+105}{7} \Rightarrow$

$f^{-1}(x) = \frac{x+105}{7}$

b. $g(x) = e^x$

$g^{-1}(x) = \ln x$

10. Assume that the growth of the population of bacteria triples every hour. The colony of bacteria start out with 1000 bacteria. Let f(t) represent the population of bacteria at time t, where **t is in hours.** (10 points)

a. Find the formula for f(t) $= 1000(3)^t$

b. Predict when there will be 100,000 bacteria.

$100000 = 1000(3)^t \Rightarrow 100 = 3^t \Rightarrow t = \log_3 100$

$t = \frac{\log 100}{\log 3} = 4.19$ HRS

11. For problems a through g, algebraically find all solutions, real and non real. Complex solutions should be written in the form $a+bi$ (5 points each)

a. $16x^2 - 81 = 0$

$16x^2 = 81$

$x^2 = \frac{81}{16}$

$x = \pm \frac{9}{4}$

b. $4x^2 + 11x - 3 = 0$

$(4x-1)(x+3) = 0$

$x = \frac{1}{4}$ $\quad x = -3$

c. $x^2 + x + 5 = 0$

$x = \frac{-1 \pm \sqrt{1-4(1)(5)}}{2} = \frac{-1 \pm \sqrt{-19}}{2}$

$= \frac{-1 \pm \sqrt{19}\, i}{2}$

d. $x^2 - 5x = 50$

$x^2 - 5x - 50 = 0$

$(x-10)(x+5) = 0$

$x = 10$ $\quad x = -5$

e. $x(-3x+4) = 2$

$-3x^2 + 4x - 2 = 0$

$x = \frac{-4 \pm \sqrt{16-4(-3)(-2)}}{-6} = \frac{-4 \pm 2\sqrt{2}\, i}{-6}$

$= \frac{+2}{3} \pm \frac{1}{3}\sqrt{2}\, i$

f. $2x^2 = x + 4$

$2x^2 - x - 4 = 0$

$x = \frac{1 \pm \sqrt{1-4(2)(-4)}}{4}$

$= \frac{1 \pm \sqrt{33}}{4}$

12. The height of a thrown math book is given by the formula $h(t) = -16t^2 + 44t + 4$ Where, h(t) is the height measured in feet and t is time measured in seconds. (15 points)

a. When does the book reach its maximum height?

$$t = \frac{-b}{2a} = \frac{-44}{2(-16)} = 1.375 \text{ seconds}$$

b. What is the maximum height of the book?

$$h(1.375) = -16(1.375)^2 + 44(1.375) + 4 = 34.25 \text{ feet}$$

c. How long does it take for the book to hit the ground?

$$-16t^2 + 44t + 4 = 0$$

$$t = 2.84 \text{ seconds}$$

13. Solve the following system by substitution method. (10 points)

$$\begin{cases} y = 2x \\ x^2 + y^2 = 45 \end{cases}$$

$$x^2 + (2x)^2 = 45$$

$$x^2 + 4x^2 = 45$$

$$5x^2 = 45$$

$$x^2 = 9$$

$$x = \pm 3 \Rightarrow y = 2x \quad y = \pm 6$$

$(3, 6)$

$(-3, -6)$

14. The following table represents an exponential function of the form $y = ab^x$. Find the value of a and b, and write the formula for the function in the form $y = ab^x$.

(Please very clearly show of all the mathematical steps) (10 points)

x	y
1	12
2	48
3	192
4	768

$$48 = ab^2$$

$$12 = ab^1 \Rightarrow \frac{48}{12} = \frac{ab^2}{ab^1} \Rightarrow b = 4$$

$$12 = a(4)^1 \Rightarrow a = 3$$

$$y = 3(4)^x$$

15. Let $f(x) = \ln x$ Evaluate f at the indicated values. (15 points)

a. $f(-5) = \ln(-5) =$ undefined

b. $f^{-1}(1) = e$ because $\ln e = 1$

c. Find x when $f(x) = -5$

$\ln x = -5 \Rightarrow x = e^{-5}$

16. Algebraically Solve $6x^3 = 108$ for x. (5 points)

$x^3 = \frac{108}{6} \Rightarrow \sqrt[3]{x^3} = \sqrt[3]{\frac{108}{6}} \Rightarrow x = 2.621$

17. Some values for the function f is shown in the table below. (2.5 points each)

x	0	1	2	3
$f(x)$	3	2	1	0

x	0	1	2	3
$g(x)$	1	2	3	0

a. Find $(f \circ g)(2) = f(3) = 0$

b. Find $(g \circ f)(1) = g(2) = 3$

c. Find $(f \circ g^{-1})(3) = f(2) = 1$

d. Find $(g \circ f^{-1})(2) = g(1) = 2$

18. Evaluate the following. (2.5 points each)

a. $\log_b\left(\sqrt[5]{b}\right)$

$\log_b\left(b^{\frac{1}{5}}\right) = \frac{1}{5}$

b. $\ln\left(e^R\right)$

$= R$

19. Solve for x (algebraically). (4 points Each)

a. $\sqrt[3]{x-1} = 2$

$\left(\sqrt[3]{x-1}\right)^3 = 2^3 \Rightarrow x-1 = 8$

$x = 9$

b. $\sqrt{2x} = x-4$

$2x = (x-4)^2$

$2x = x^2 - 8x + 16$

$-2x \qquad -2x$

$x^2 - 10x + 16 = 0$

$(x-8)(x-2) = 0$

$x = 8 \qquad x = 2$