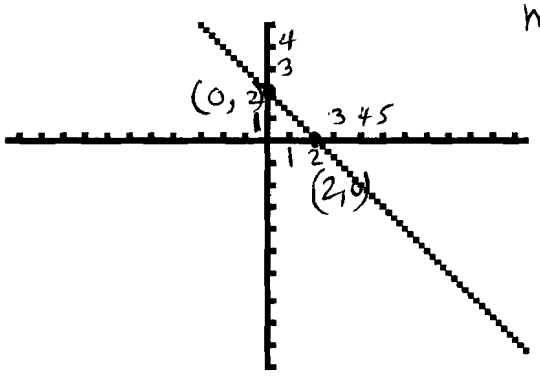
Professor Katiraie MA103 Test Three Fall 07

Name Solutions

Total Possible Points = 200 Points
Plus 10 Points of Extra Credits ☺

Note: Show all work.

1. Find the equation of the graph of the line shown below. (8 points)
(Must Show Procedure)



$m = \frac{0-2}{2-0} = \frac{-2}{2} = -1$

y-intercept $= (0, 2)$

$y = -x + 2$

2. Find the equation of a line perpendicular to $y = -2x + 7$
and passing through $(-4, -2)$ **(Must Show Procedure)** (7 points)

$m_{\perp} = \frac{1}{2}$

$y = mx + b$

$-2 = \frac{1}{2}(-4) + b$

$-2 = -2 + b$

$0 = b$

$y = \frac{1}{2}x$

3. Solve the following system of equations using Elimination method. (5 points)

$$2\begin{cases} -2x + 4y = 6 \\ x - 2y = -4 \end{cases} \Rightarrow \begin{cases} -2x + 4y = 6 \\ 2x - 4y = -8 \end{cases}$$

$0 = -2$

No Solutions

X = Amount of \$ in 9% Account

y = Amount of \$ in 6% Account

4. A student takes out two loans to help pay for college. One loan is at 9% simple interest, and the other is at 6% simple interest. The total amount borrowed is \$5000, and the interest after 1 year for both loans is \$384. Find the amount of each loan. (10 points)

$$\begin{cases} x+y=5000 \\ 0.09x+0.06y=384 \end{cases} \xrightarrow{-9} \begin{cases} x+y=5000 \\ 9x+6y=38400 \end{cases} \Rightarrow \begin{cases} -9x-9y=-45000 \\ 9x+6y=38400 \end{cases}$$

ADD

$$-3y=-6600$$

$$y=2200 \text{ \$}$$

$$x+y=5000 \Rightarrow x=2800 \text{ \$}$$

5. SOLVE FOR X: $\dfrac{x}{x+2}+\dfrac{7}{x-6}=\dfrac{14}{x^2-4x-12}$ (5 points)

$$\frac{x}{x+2}+\frac{7}{x-6}=\frac{14}{(x+2)(x-6)}$$

multiply Both sides by $(x+2)(x-6)$

$$x(x-6)+7(x+2)=14$$

$$x^2-6x+7x+14=14$$

$$x^2+x=0$$

$$x(x+1)=0$$

$x=0$ $x=-1$

6. SOLVE FOR X: *Check for extraneous solutions.* (5 points)

$$\frac{12}{x-5}-\frac{3}{x-2}=\frac{5}{x-2} \qquad \frac{12}{x-5}=\frac{5}{x-2}+\frac{3}{x-2}$$

$$\frac{12}{x-5}=\frac{8}{x-2}$$

$$12x-24=8x-40$$

$$4x=-16$$

$x=-4$

X = No of Dimes

y = No of Nickels

7. A vending machine will only accept nickels and dimes. When the coins are collected, the machine has 215 coins worth \$20.75. How many nickels were there? How many dimes were there? Show your work! (10 points)

$$\begin{cases} 0.10X + 0.05y = 20.75 \\ X + y = 215 \end{cases} \xrightarrow{-10} \begin{cases} 10X + 5y = 2075 \\ X + y = 215 \end{cases}$$

$$\Rightarrow \begin{cases} 10X + 5y = 2075 \\ -10X - 10y = -2150 \end{cases} \quad \text{ADD}$$

$$-5y = -75$$

$$y = 15$$

there are 15 Nickels

$X + y = 215$

$X = 215 - y$

$X = 200$ Dimes

8. Assume \$2500 is deposited in an account that earns 8 % interest compounded annually. (10 points)

a. Find a formula for g(t) where t is time and g(t) is the amount of money in the account after **t years**.

$$g(t) = 2500(1 + 0.08)^t = 2500(1.08)^t$$

b. How long will it take for the money to double.

$$5000 = 2500(1.08)^t$$

$$2 = 1.08^t$$

$$t = \log_{1.08} 2 = \frac{\log 2}{\log 1.08} = 9.006 \text{ years}$$

9. Find the inverse of the following functions (10 points)

a) $f(x) = 7x - 105$

$y = 7x - 105$

$x = 7y - 105$

$7y = x + 105$

$y = \frac{x + 105}{7}$

$f^{-1}(x) = \frac{x + 105}{7}$

b) $f(x) = x^3 - 8$

$y = x^3 - 8$

$x = y^3 - 8$

$y^3 = x + 8$

$y = \sqrt[3]{x + 8}$

$f^{-1}(x) = \sqrt[3]{x + 8}$

10. Assume that the growth of the population of bacteria triples every hour. The colony of bacteria start out with 50 bacteria. Let f(t) represent the population of bacteria at time t, where **t is in hours**.

(10 points)

a. Find the formula for f(t) $= 50(3)^t$

b. Predict when there will be 250,000 bacteria. $250000 = 50(3)^t$

$5000 = 3^t$

$\log_3 5000 = t \Rightarrow t = 7.75 \text{ HRS}$

11. For problems a through g, algebraically find all solutions, real and non real. Complex solutions should be written in the form $a+bi$ (5 points each)

a. $x(x+2)=x-4$

$x^2+2x-x+4=0$

$x^2+x+4=0$

$x=\frac{-1\pm\sqrt{1-16}}{2}=\frac{-1\pm\sqrt{15}\,i}{2}$

d. $4x^2+x-5=0$

$x=\frac{-1\pm\sqrt{1-4(4)(-5)}}{8}=\frac{-1\pm\sqrt{81}}{8}$

$=\frac{-1\pm 9}{8}=$ $x=1$, $x=-1.25$

e. $x^2+2x+5=0$

$x=\frac{-2\pm\sqrt{4-4(5)}}{2}=\frac{-2\pm\sqrt{-16}}{2}$

$=-1\pm 2i$

f. $x^2-x=72$

$x^2-x-72=0$

$(x-9)(x+8)=0$

$x=9$ $x=-8$

Solve for x. (10 points)

12. $\frac{x}{3x+1}-\frac{1-x}{6x}=\frac{1}{6x}$

$x(6x)-(1-x)(3x+1)=1(3x+1)$

$6x^2-(3x+1-3x^2-x)=3x+1$

$6x^2+3x^2-2x-1=3x+1$

$9x^2-5x-2=0$

$x=\frac{5\pm\sqrt{25-4(9)(-2)}}{18}=\frac{5\pm\sqrt{97}}{18}$

13. The height of a thrown math book is given by the formula $h(t) = -16t^2 + 32t + 4$
Where, h(t) is the height measured in feet and t is time measured in seconds. (15 points)

a. When does the book reach its maximum height?

$t = \frac{-b}{2a} = \frac{-32}{2(-16)} = 1$ seconds later

b. What is the maximum height of the book?

$h(1) = -16 + 32 + 4 = 20$ feet

c. How long does it take for the book to hit the ground?

$-16t^2 + 32t + 4 = 0$

$t = 2.12$ seconds

14. Solve the following system by substitution method. (10 points)

$\begin{cases} y + x - 2 = 0 \\ x^2 - y = 4 \end{cases}$ $y = -x + 2$

$x^2 - (-x+2) = 4$

$x^2 + x - 2 - 4 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3$ $\quad$ $x = 2$

$y = 5$ $\quad$ $y = 0$

$(-3, 5)$ & $(2, 0)$ ← Solutions

15. The following table represents an exponential function of the form $y = ab^x$. Find the value of a and b, and write the formula for the function in the form $y = ab^x$.
(Please very clearly show all of the mathematical steps) (10 points)

x	0	1	2	3	4
$f(x)$	64	16	4	1	0.25

Note $a = 64$

$16 = 64(b)^1 \Rightarrow b = \frac{16}{64} = \frac{1}{4}$

$f(x) = 64\left(\frac{1}{4}\right)^x$

16. Let $f(x)=\log(x)$ Evaluate f at the indicated values. (15 points)

a. $f(1000) = \log(1000) = 3$

b. $f^{-1}(5)$ $\quad 5=\log(x) \Rightarrow 10^5 = x$

$x = 100000$

c. Find x when $f(x) = -4$

$-4 = \log x \quad \rightarrow \quad 10^{-4} = x$

$x = 0.0001$

17. Solve $5x^4 = x^4 + 108$ for x analytically. (5 points)

$4x^4 = 108$

$x^4 = 27 \Rightarrow x = 27^{1(1\div 4)} \approx 2.28$

18. Some values for the function f is shown in the table below. (2.5 points each)

x	0	1	2	3
$f(x)$	3	2	1	0

x	0	1	2	3
$g(x)$	1	2	3	0

a) Find $(f \circ g)(0) = f(1) = 2$

b) Find $(g \circ f)(1)$

$g(2) = 3$

c) Find $(f \circ g^{-1})(3)$

$= f(2) = 1$

d) Find $(g \circ f^{-1})(1)$

$g(2) = 3$

Evaluate the following. (2.5 points each)

19a) Write the equation $b^M = c$ in logarithmic form.

$$\log_b C = M$$

19b) Write the equation $\ln(X) = 3$ in exponential form.

$$e^3 = X$$

20. Perform the indicated operations. Simplify your answers. (10 pts)

a. $(3-5\sqrt{7})(4+4\sqrt{7})$

$12 + 12\sqrt{7} - 20\sqrt{7} - 20(7)$

$= 12 - 8\sqrt{7} - 140$

$= -128 - 8\sqrt{7}$

b. $(3-2i)(4+7i)$

$12 + 21i - 8i - 14i^2$

$12 + 13i + 14$

$26 + 13i$

21. Solve for x (algebraically). (10 points)

a. $x+5 = \sqrt{x}$

$(x+5)^2 = x$

$x^2 + 10x + 25 = x$

$x^2 + 9x + 25 = 0$

$x = \frac{-9 \pm \sqrt{81 - 4(1)(25)}}{2}$

$x = \frac{-9 \pm \sqrt{-19}}{2}$

$x = \frac{-9 \pm \sqrt{19}\, i}{2}$

b. $\sqrt{x+6} = x$

$x + 6 = x^2$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$

$x = -2$

Extra Credits:

22. Write the vertex form of a parabola that satisfies the following condition.

Vertex (5, -2) and $a = -\frac{1}{2}$ (2 points)

$y = -\frac{1}{2}(x-5)^2 - 2$

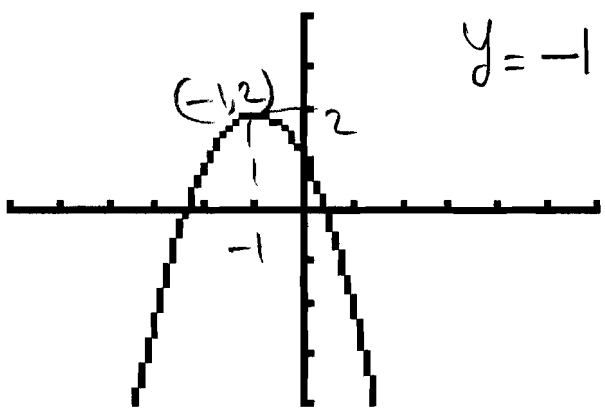
b) Write the above equation in the form of $y = ax^2 + bx + c$ (2 points)

$y = -\frac{1}{2}(x^2 - 10x + 25) - 2$

$= -\frac{1}{2}x^2 + 5x - \frac{25}{2} - 2$

$= -\frac{1}{2}x^2 + 5x - \frac{29}{2}$

23. Write the vertex form of the parabola shown in the following graph.
Assume $a = \pm 1$ (but you have to choose either +1 or -1) (2 points)



$y = -1(x+1)^2 + 2$

24. Solve the following equations symbolically (i.e. algebraically) (2 points each)
Check your results.

a) $\sqrt[4]{t+1} = 2$

$t+1 = 2^4$

$t = 16 - 1$

$t = 15$

b) $\sqrt[3]{2x-4} = -2$

$2x - 4 = -8$

$2x = -4$

$x = -2$