Dr. Katiraie MA 099 Test Two Practice Test (Chapters 1,2,3,4,5)

Name Solutions

When asked for the equation of a line, the equation should be given in slope-intercept form.

1. Simplify: $\left(\dfrac{x^3y^{-7}z^2}{x^{-5}yz^{-4}}\right)^3$ (Assume no variables are equal to zero.) (5 points)

(Must Show Procedure)

$$\left(x^{8}y^{-8}z^{6}\right)^3 = x^{24}y^{-24}z^{18}$$

$$= \boxed{\frac{x^{24}z^{18}}{y^{24}}}$$

2. Let $g(x) = -x^2 + 6x + 7$ and $f(x) = -4x + 7$ (9 points)

a. Find $f(4) = -4(4)+7 = -16+7 = \boxed{-9}$

b. Find $g(-3) = -(-3)^2+6(-3)+7 = -9-18+7 = \boxed{-20}$

c. Find x when $f(x) = 6$

$$6 = -4x+7$$
$$-7 \qquad -7$$
$$-1 = -4x$$
$$x = \frac{-1}{-4} = \boxed{\frac{1}{4}}$$

*3. If $y = -x^3 + x + 5$ use your calculator. (10 points)

a) Find the x-intercept

let $Y_1 = -x^3+x+5$

$Y_2 = 0$

$\boxed{(1.904, 0)}$

b) Find the y-intercept

let $x = 0 \Rightarrow y = 5$

$\boxed{(0,5)}$

4. A car is rented for a day. It costs $45 plus $.37 per mile. (10 points)

a. Write a formula for a linear function f that calculates the cost of renting the car when the car is driven x miles. **(Must Show Procedure)**

$\boxed{C(x) = 0.37x + 45}$

*b. How much does it cost to rent the car for a day and drive 137 miles.

$C(137) = 0.37(137) + 45 = \boxed{\$95.69}$

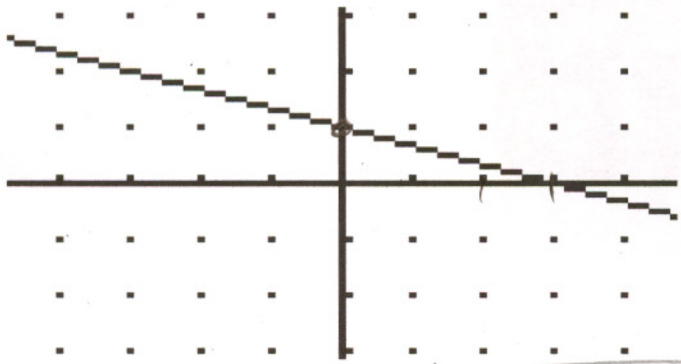
c) If it costs $63.50 to rent this car for one day, how many miles was it driven?

$63.50 = 0.37x + 45 \Rightarrow 63.50 - 45 = 0.37x$

$18.50 = 0.37x$

$\boxed{x = 50 \text{ miles}}$

5. Find the equation of the line shown below on the graph. (8 points)
(Must Show Procedure)



$(0,1)$
$(3,0)$

$m = \frac{0-1}{3-0} = \frac{-1}{3}$

y intercept is $(0,1)$

$y = -\frac{1}{3}x + 1$

6. Find the equation of a line perpendicular to $y = -3x + 7$ $\Longrightarrow$ slope $= -3$
and passing through $(4, -2)$ **(Must Show Procedure)** (7 points)

slope of Perpendicular line $= \frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y - -2 = \frac{1}{3}(x - 4)$

$y = \frac{1}{3}x - \frac{4}{3} - 2$

$\Longrightarrow y = \frac{1}{3}x - \frac{10}{3}$

7. Solve the following system of equations using Elimination method.

(10 points)

$$-3\begin{cases}2x+y=2\\6x+3y=6\end{cases} \Rightarrow \begin{cases}-6x-3y=-6\\6x+3y=6\end{cases}$$

$$0+0=0$$

$\boxed{\text{The system has infinitely Many Solutions}}$

let x be the amount at 8% and y be the amount at 9%

8. A student takes out two loans to help pay for college. One loan is at 8% simple interest, and the other is at9% simple interest. The total amount borrowed is \$4200, and the interest after 1 year for both loans is \$363. Find the amount of each loan.

(10 points)

$$\begin{cases}x+y=4200\\0.08x+0.09y=363\end{cases} \Rightarrow -8\begin{cases}x+y=4200\\8x+9y=36300\end{cases}$$

$$\begin{cases}-8x-8y=-33600\\8x+9y=36300\end{cases}$$

$$y=\$2700$$

$$\Rightarrow x+y=4200$$

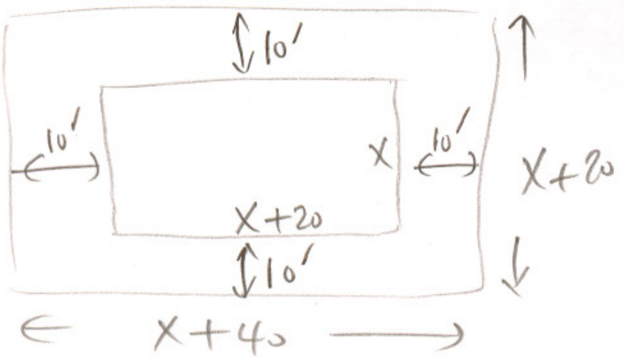
$$x+2700=4200$$

$$x=\$1500$$

$\boxed{\begin{array}{l}\$1500 \text{ borrowed at } 8\%\\ \$2700 \text{ borrowed at } 9\%\end{array}}$

9. A 10-foot wide sidewalk around a rectangular swimming pool has a total area of 2400 square feet. Find the dimensions of the swimming pool if the pool is 20 feet longer than it is wide.

(10 points)



Area of Yard = Area of Pool + Area of Sidewalk

$$(x+20)(x+40) = x(x+20) + 2400$$

$$\cancel{x^2}+40x+\cancel{20x}+800=\cancel{x^2}+\cancel{20x}+2400$$

$$40x=1600$$

$$x=40 \text{ feet}$$

$\boxed{\text{So, the Pool is 40 feet by 60 feet}}$

(+) 10. State the domain of the following functions. (10 points)
Write your answer in set-builder notation:

a) $h(x)=\frac{1}{3x-9}$ let $3X-9=0$ and solve

$3X=9$

$X=3$

$\boxed{\{X \mid X \text{ is } \mathbb{R} \text{ and } x \neq 3\}}$

Interval Notation $(-\infty,3)\cup(3,\infty)$

b) $f(x)=\frac{1}{x-2}$

$\boxed{\{X \mid X \text{ is } \mathbb{R} \text{ and } X \neq 2\}}$

Interval Notation $(-\infty,2)\cup(2,\infty)$

c) $g(x)=\frac{1}{x^2-4}$

$x^2-4=0$

$(x+2)(x-2)=0$

$x=-2 \quad x=2$

$\boxed{\{x \mid x \text{ is } \mathbb{R} \text{ and } x\neq 2, X\neq -2\}}$

Int. Notation $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

d) $f(x)=\sqrt{-x+2}$

$-X+2\geq 0$

$-X\geq -2$

$X\leq 2$

$\boxed{\{X \mid X\leq 2\}}$

$(-\infty,2]$

11. Solve the following system of equations using Elimination method. (10 points)

$$-2\begin{cases}2x+3y=6\\ x-2y=-4\end{cases} \Rightarrow \begin{cases}\cancel{2X}+3y=6\\ \cancel{-2X}+4y=8\end{cases}$$

$7y=14$

$y=2$

$x-2y=-4$

$x-2(2)=-4 \Rightarrow x-4=-4$

$+4 \quad +4$

$x=0$

$\boxed{\text{Solution is } (0,2)}$

12. A vending machine will only accept quarters and dimes. When the coins are collected, the machine has 226 coins worth \$24.10. How many quarters were there? How many dimes? Show your work! (10 points)

$$\begin{cases}Q+D=226\\ 0.25Q+0.10D=24.10\end{cases} \xRightarrow{} \begin{matrix}-10\\ \times 100\end{matrix}\begin{cases}Q+D=226\\ 25Q+10D=2410\end{cases}$$

$$\begin{cases}-10Q-10D=-2260\\ 25Q+10D=2410\end{cases}$$ ADD

$15Q=150 \Rightarrow Q=10$

$Q+D=226 \Rightarrow D=216$

So, there are 10 Quarters & 216 Dimes

13. For problems a through f, algebraically find all solutions (20 points)

a. $16x^2 - 25 = 0$

$(4x+5)(4x-5)=0$

$x=\frac{-5}{4}$ and $x=\frac{5}{4}$

d. $4x^2 + 11x - 3 = 0$

$(4x-1)(x+3)=0$

$\boxed{x=\frac{1}{4} \text{ and } x=-3}$

e. $x^2 + 9x = -20$

$x^2+9x+20=0 \Rightarrow (x+5)(x+4)=0$

$\boxed{x=-5 \text{ \& } x=-4}$

f. $4s^2 + 36 = 24s$

$4s^2-24s+36=0$

$4(s^2-6s+9)=0 \Rightarrow 4(s-3)(s-3)=0$

$\boxed{s=3}$

g. $4x^2 = 20x$

$4x^2-20x=0$

$4x(x-5)=0$

$\boxed{x=0 \text{ \& } x=5}$

h. $4x^2 = 25$

$4x^2-25=0$

$(2x+5)(2x-5)=0 \Longrightarrow \boxed{x=\frac{-5}{2} \text{ \& } x=\frac{5}{2}}$

i. $a^2 + 7a = 8$

$a^2+7a-8=0$

$(a+8)(a-1)=0 \Longrightarrow \boxed{a=-8 \text{ \& } a=1}$

14. Solve $\begin{cases} x-3y=-6 \\ x+7y=14 \end{cases}$ by Elimination Method. (10 points)

-1

$\begin{cases} -x+3y=6 \\ x+7y=14 \end{cases}$

ADD

$10y=20 \Rightarrow \boxed{y=2} \Rightarrow$

$x-3y=-6$

$x-3(2)=-6$

$\boxed{x=0}$

$\boxed{\text{final Solution is } (0,2)}$

15. The enrollment (in millions) at U. S. colleges for men and women are modeled by the following equations. (10 points)

$$M(t) = 0.083t + 4.60 \quad \text{and} \quad W(t) = 0.14t + 4.52$$

Where t is the number of years since 1970 and M & W represent the enrollment of men & women that year.

A. What is the slope of W(t)? Slope is 0.14

B. What does the slope mean in the context of years and college enrollment?

Every year the enrollment of women in colleges increases by 0.14 millions

C. Where there more men or women enrolled in college in 1970?

$M(0) = 4.60$ million But $W(0) = 4.52$ million; So there were more men

D. Estimate the year in which women's and men's enrollments are approximately equal.

$0.083t + 4.60 = 0.14t + 4.52 \Rightarrow 0.08 = 0.057t$

$t = 1.4 \Rightarrow$ $\boxed{\text{About } 1971}$

16. In 2000, the enrollment at college A was 15,600 students and enrollment at college B was 12,100 students. Each year, the enrollment at college A has been increasing by 200 students and the enrollment at college B has been increasing by 350 students. (10 points)

A. Write a model A(t) that represents the enrollment of college A, t years since the 2000.

$A(t) = 200t + 15600$

B. Write a model B(t) that represents the enrollment of college B, t years since the year 2000.

$B(t) = 350t + 12100$

C. Predict in what years the enrollment at college B will be greater than the enrollment at college A.

$350t + 12100 > 200t + 15600$

$150t + 12100 > 15600 \Rightarrow 150t > 3500 \Rightarrow t > 23.33$

After 2023

17. Find the zeros of the following functions. (10 points)

a. $f(x) = x^3 - 2x^2 - 8x$

$= x(x^2 - 2x - 8)$

$= x(x-4)(x+2)$

$x = 0$ $x = 4$ $x = -2$

b. $f(x) = x^2 - 2x - 8$

$(x-4)(x+2) = 0$

$x = 4$ $x = -2$

c. $f(x) = 2x^2 - 9x - 5$

$(2x+1)(x-5) = 0$

$x = -1/2$ $x = 5$

d. $f(x) = x^2 + 16x + 39$

$(x+13)(x+3) = 0$

$x = -13$ $x = -3$

18. Your calculator gives an answer of $x = 1.1578074156$, $y = 2E\text{-}13$ when using **ROOT** or **ZERO** in the graphing mode. Give the coordinates of the x-intercept your calculator has given you to the nearest thousandth. (4 points)

$\boxed{(1.158, 0)}$

19. What is the domain of $f(x)$ if $f(x) = \dfrac{2x-3}{4x+5}$? Write your answer in interval notation. (4 points)

$\boxed{(-\infty, -\frac{5}{4}) \cup (-\frac{5}{4}, \infty)}$

20. Write a polynomial function that has the following zeros: - 4, and 6. The coefficient of the highest degree term must be 1. (8 points)

$= (x - -4)(x-6)$

$= (x+4)(x-6) = \boxed{x^2 - 2x - 24}$

21. Write a polynomial function that has the following zeros: 0, 3 ,and 4. The coefficient of the highest degree term must be 1. (8 points)

$= x(x-3)(x-4)$

$= x(x^2 - 7x + 12)$

$= \boxed{x^3 - 7x^2 + 12x}$

22. Factor the following completely: (8 points)

a) $6b^2+4b-16$

$=2(3b^2+2b-8)$

$=\boxed{2(3b-4)(b+2)}$

b) $9r^2+18r+8$

$=\boxed{(3r+4)(3r+2)}$

b) $4c^2-9c+5$

$=\boxed{(4c-5)(c-1)}$

d) w^3+18w^2+81w

$=w(w^2+18w+81$

$=\boxed{w(w+9)(w+9)}$

23. Attendance A, in million, at auto races is approximately by the polynomial function $A(x)=0.003x^3-0.005x^2+0.3x+3$ where x is the number of years since 1982.

a) Use the above function to find the attendance in 1985. (4 points)

$1985-1982=3$

$A(3)=0.003(3)^3-0.005(3)^2+0.3(3)+3$

$=\boxed{3.936 \text{ Million}}$

b) In what year the attendance reached 8.5 million? (4 points)

$8.5=0.003x^3-0.005x^2+0.3x+3$

$Y_1=0.003x^3-0.005x^2+0.3x+3$

$Y_2=8.5$

$\Rightarrow x=10$

1982+10

Year 1992

24. A ball is thrown upward with an initial velocity of 48 ft/sec from a height 640 feet. Its height h, in feet, after t seconds is given by $h(t)=-16t^2+48t+640$

a) After how long will the ball reach the ground? (4 points)

$-16t^2+48t+640=0$

$-16(t^2-3t-40)=0$

$-16(t-8)(t+5)=0$

$\boxed{t=8 \text{ second}}$

b) How high above the ground was ball after 1.5 seconds? (4 points)

$h(1.5)=-16(1.5)^2+48(1.5)+640=\boxed{676 \text{ feet}}$