Professor Katiraie MA 103 Test Three Form A (Chapters 7, 8, 9, and 10.3)

Name Solution (1 POINT)

Total Possible Points = 200 Points + 10 Points Extra Credits

Note: Show all your work.

1. Simplify: $\left(\dfrac{x^3y^{-7}z^2}{x^{-5}yz^{-4}}\right)^3$ (Assume no variables are equal to zero.) (5 points)

(Must Show Procedure)

$= \left(x^8y^{-8}z^{+6}\right)^3$

$= x^{24}y^{-24}z^{+18} = \boxed{\dfrac{x^{24}z^{18}}{y^{24}}}$

2. A car is rented for a day. It costs \$45 plus \$.37 per mile. (9 points)

a. Write a formula for a linear function f that calculates the cost of renting the car when the car is driven x miles. **(Must Show Procedure)**

$\boxed{Cost(x) = 0.37x + \$45}$

*b. How much does it cost to rent the car for a day and drive 137 miles.

$= 0.37(137) + 45 = \boxed{\$95.69}$

c) If it costs \$63.50 to rent this car for one day, how many miles was it driven?

$63.50 = 0.37x + 45$

$18.50 = 0.37x$

$\dfrac{18.50}{0.37} = x \Longrightarrow \boxed{x = 50 \text{ miles}}$

3. The monthly fees for a condo association can be modeled by the following formula:

$$f(x) = 50x + 100$$

where x is the number of years since the condo association was built in 1990.

(Must Show Procedure) (9 points)

*a. What were the monthly fees in 2002? $2002 - 1990 = 12$

$f(12) = 50(12) + 100 = \$700$

b. Determine the year when the monthly fees were $410?

$\$410 = 50X + 100 \Rightarrow X = 6.2$ year $1990 + 6.2 \approx 1996$

c. Interpret the slope as a rate of change.

slope is $\frac{\$50}{\text{year}}$, this means that every year the monthly condo association fee increases by $50.

4. Find the equation of a line perpendicular to $-2x - 5y = 10$ and passing through $(4, -2)$

(Must Show Procedure) (6 points)

$-2X - 10 = 5y$

$y = -\frac{2}{5}x - 2$ $m = \frac{-2}{5}$

$m\perp = \frac{5}{2}$

$y = mX + b$

$-2 = \frac{5}{2}(4) + b$

$-2 = 10 + b \Rightarrow b = -12 \longrightarrow y = \frac{5}{2}X - 12$

5. A student takes out two loans to help pay for college. One loan is at 6% simple interest, and the other is at 7% simple interest. The total amount borrowed is $7500, and the interest after 1 year for both loans is $495. Find the amount of each loan.

(Must Show Procedure) (10 points)

X at 6%

y at 7%

$$\begin{cases} x + y = 7500 \\ 0.06X + 0.07y = 495 \end{cases}$$

$$-6\begin{cases} X + y = 7500 \\ 6x + 7y = 49500 \end{cases} \Rightarrow \begin{cases} -6x - 6y = -45000 \\ 6X + 7y = 49500 \end{cases}$$

$y = \$4500$ at 7%

$X = \$3000$ at 6%

6. State the domain of the following functions. (10 points)
Write your answer in set-builder notation:

a) $h(x) = \dfrac{1}{3x-9}$

$\{X \mid X \text{ is all Reals and } X \neq 3\}$

b) $f(x) = \dfrac{1}{x-2}$

$\{X \mid X \text{ is all Reals and } X \neq 2\}$

c) $g(x) = \dfrac{1}{x^2-4}$

$\{X \mid X \text{ is all reals and } X \neq \pm 2\}$

d) $f(x) = x^2 - 3x + 2$

$\{X \mid X \text{ is all Reals}\}$

7. Assume \$2000 is deposited in an account that earns 7 % interest compounded annually. (10 points)

a. Find a formula for g(t) where t is time and g(t) is the amount of money in the account after **t years**.

$g(t) = 2000(1+0.07)^t = 2000(1.07)^t$

b. How long will it take for the money to double.

$4000 = 2000(1.07)^t$

$2 = 1.07^t \implies t = \log_{1.07} 2 = \dfrac{\log 2}{\log 1.07} = \boxed{10.24 \text{ years}}$

8. Find the inverse of the following functions (10 points)

a. $f(x) = -3x - 7$

$y = -3x - 7$

$-3x = y + 7 \Rightarrow x = \frac{y+7}{-3}$

$f^{-1}(x) = \frac{x+7}{-3}$

b. $g(x) = \log_8 x$

$\boxed{g^{-1}(x) = 8^x}$

9. Assume that the growth of the population of bacteria triples every hour. The colony of bacteria start out with 100 bacteria. Let f(t) represent the population of bacteria at time t, where **t is in hours**. (10 points)

a. Find the formula for f(t) $= 100(3)^t$

b. Predict when there will be 100,000 bacteria.

$100000 = 100(3)^t$

$1000 = 3^t \Rightarrow t = \log_3 1000 = \frac{\log 1000}{\log 3} = \boxed{6.29 \text{ Hours}}$

10. For problems a through g, algebraically find all solutions, real and non real. Complex solutions should be written in the form $a + bi$ (5 points each)

a. $16x^2 - 49 = 0$

$16x^2 = 49$

$x^2 = \frac{49}{16} \Rightarrow \boxed{x = \pm\frac{7}{4}}$

d. $4x^2 + x - 5 = 0$

-20

$-4 \cdot +5 = -20$

$-4 + +5 = 1$

$4x^2 - 4x + 5x - 5 = 0$

$4x(x-1) + 5(x-1) = 0$

$(4x+5)(x-1) = 0$ $x = \frac{-5}{4}$ $\boxed{x = 1}$

e. $x^2 + x + 7 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(7)}}{2} = \frac{-1 \pm \sqrt{-27}}{2}$

$= \frac{-1}{2} \pm \frac{3\sqrt{3}\,i}{2}$

f. $x^2 - x = 42$

$x^2 - x - 42 = 0$

$(x-7)(x+6) = 0$

$\boxed{x = 7}$ and $\boxed{x = -6}$

g. $x(-3x + 3) = 2$

$-3x^2 + 3x - 2 = 0$

$x = \frac{-3 \pm \sqrt{9 - 4(-3)(-2)}}{-6} = \frac{-3 \pm \sqrt{-15}}{-6} = \frac{1}{2} \mp \frac{\sqrt{15}\,i}{6}$

Solve for x. (10 points)

11. $\frac{x}{2x+1}-\frac{1-x}{5x}=\frac{1}{5x}$ Multiply by $5x(2x+1)$

$x(5x)-(1-x)(2x+1)=1(2x+1)$

$5x^2-(2x+1-2x^2-x)=2x+1$

$5x^2-x+2x^2\cancel{+1}-2x\cancel{-1}=0$

$7x^2-3x-2=0$

$\rightarrow x=\frac{3\pm\sqrt{9-4(7)(-2)}}{14}$

$\boxed{x=\frac{3\pm\sqrt{65}}{14}}$

12. The height of a thrown math book is given by the formula $h(t)=-16t^2+32t+4$
Where, h(t) is the height measured in feet and t is time measured in seconds. (15 points)

a. When does the book reach its maximum height?

$t=\frac{-b}{2a}=\frac{-32}{2(-16)}=\boxed{1 \text{ Second}}$

b. What is the maximum height of the book?

$h(1)=-16(1)^2+32(1)+4=\boxed{20 \text{ feet}}$

c. How long does it take for the book to hit the ground?

$-16t^2+32t+4=0$

$\boxed{t=2.118 \text{ Seconds}}$

13. Solve the following system by substitution method. (10 points)

$$\begin{cases} y=x^2-3 \\ 2x^2-y=1-3x \end{cases}$$

$2x^2-(x^2-3)=1-3x$

$2x^2-x^2+3=1-3x$

$x^2+3x+3-1=0$

$x^2+3x+2=0$

$\rightarrow (x+2)(x+1)=0$

$x=-2$
$y=1$

$x=-1$
$y=-2$

Solutions $(-2,1)$ and $(-1,-2)$

14. The following table represents an exponential function of the form $y = ab^x$. Find the value of a and b, and write the formula for the function in the form $y = ab^x$.
(Please very clearly show of all the mathematical steps) (10 points)

x	y
1	12
2	48
3	192
4	768

$48 = ab^2$

$12 = ab^1$

Now divide these:

$\frac{48}{12} = \frac{ab^2}{ab} \Rightarrow \boxed{b = 4}$

$12 = a(4)^1 \Rightarrow \boxed{a = 3}$ $\boxed{y = 3(4)^x}$

15. Let $f(x) = (7)^x$ Evaluate f at the indicated values. (15 points)

a. $f(3) = 7^3 = \boxed{343}$

b. $f^{-1}(240) \Rightarrow 240 = 7^x \Rightarrow x = \frac{\log 240}{\log 7} \boxed{\cong 2.82}$

c. Find x when $f(x) = \frac{1}{343}$

$\frac{1}{343} = 7^x \Rightarrow x = \log_7 \frac{1}{343} = \frac{\log\left(\frac{1}{343}\right)}{\log 7} \boxed{= -3}$

16. Solve $6x^3 = x^3 + 108$ for x analytically. (5 points)

$5x^3 = 108$

$x^3 = \frac{108}{5}$

$x = \sqrt[3]{\frac{108}{5}}$ $\boxed{\cong 2.785}$

17. Some values for the function f is shown in the table below. (5 points each)

x	0	1	2	3
$f(x)$	3	2	1	0

x	0	1	2	3
$g(x)$	1	2	3	0

a. Find $(f \circ g)(2) = f(3)$

$\boxed{= 0}$

b. Find $(g \circ f)(1)$

$= g(2) \boxed{= 3}$

c. Find $(f \circ g^{-1})(3)$

$= f(2) \boxed{= 1}$

d. Find $(g \circ f^{-1})(2)$

$= g(1) \boxed{= 2}$

Evaluate the following. (2.5 points each)

18 a) Write the equation $a^M = c$ in logarithmic form.

$\boxed{\log_a c = M}$

18b) Write the equation $\log_7(W) = 3$ in exponential form.

$\boxed{7^3 = W}$

Extra Credits:

19. Perform the indicated operations. Simplify your answers. (6 pts)

c. $(3-5\sqrt{7})(4+4\sqrt{7})$

$= 12 + 12\sqrt{7} - 20\sqrt{7} - 20(7)$

$\boxed{= -128 - 8\sqrt{7}}$

d. $(3-2i)(4+7i)$

$= 12 + 21i - 8i - 14i^2$

$= 12 + 13i + 14$

$\boxed{= 26 + 13i}$

20. Solve for x (algebraically). (4 points)

a. $2x - 1 = \sqrt{7-x}$

Square Both Sides

$(2x-1)^2 = 7 - x$

$4x^2 - 4x + 1 = 7 - x$

$+x \quad -7 \quad -7 \quad +x$

$4x^2 - 3x - 6 = 0$

$x = \dfrac{3 \pm \sqrt{9-4(4)(-6)}}{8} = \dfrac{3 \pm \sqrt{105}}{8}$

$x \cong 1.656$ and $\cancel{x \cong -0.906}$

Extraneous Solution

b. $\sqrt{2x} = x - 4$

Square Both Sides

$2x = (x-4)^2$

$2x = x^2 - 8x + 16$

$-2x \quad -2x$

$x^2 - 10x + 16 = 0$

$(x-8)(x-2) = 0$

$x = 8$ $\quad$ $\cancel{x = 2}$ Extraneous Solution