Professor Katiraie MA 103 Test Three Form C (Chapters 7, 8, 9, and 10.3)

Name _____(1 POINT)

Total Possible Points = 200 Points + 10 Points Extra Credits

Note: Show all your work.

1. Simplify: $\dfrac{18x^{-4}y^{8}z^{2}}{6xy^{-3}}$ (Assume no variables are equal to zero.) (5 points)

(Must Show Procedure)

$= 3x^{-4-1}y^{8+3}z^{2}$

$= 3x^{-5}y^{11}z^{2} = \dfrac{3y^{11}z^{2}}{x^{5}}$

2. Find the equation of a line passing through $(9,-3)$ and $(5,-7)$ (9 points)

(Must Show Procedure)

$m = \dfrac{-7-(-3)}{5-9} = \dfrac{-4}{-4} = 1$

$y = mx + b$

$-3 = 1(9) + b \Rightarrow b = -12$

$y = 1x - 12$

3. The monthly fees for a condo association can be modeled by the following formula:

$f(x) = 40x + 90$

where x is the number of years since the condo association was built in 1998.

(Must Show Procedure) (10 points)

*a. What were the monthly fees in 2002? $2002 - 1998 = 4$

$f(4) = 40(4) + 90 = \$250$

b. Determine the year when the monthly fees were $410?

$410 = 40x + 90 \Rightarrow 40x = 320 \Rightarrow x = 8$

Year 2006

c. Interpret the slope as a rate of change.

Every year the monthly fee increases by $\$40$

4. Find the equation of a line parallel to $-2x-5y=10$ and passing through $(4,-2)$
(Must Show Procedure) (5 points)

$-5y = 2x+10$

$y = \frac{-2}{5}x - 2$ $\quad m = \frac{-2}{5}$

$y = mx+b$

$-2 = \frac{-2}{5}(4)+b \Rightarrow b = \frac{-2}{5}$

$y = \frac{-2}{5}x - \frac{2}{5}$

5. A student takes out two loans to help pay for college. One loan is at 8% simple interest, and the other is at9% simple interest. The total amount borrowed is $4200, and the interest after 1 year for both loans is $363. Find the amount of each loan.
(10 points)

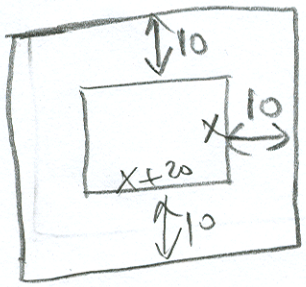
$\begin{cases} x+y = 4200 \\ 0.08x+0.09y = 363 \end{cases} \xrightarrow{-8} \begin{cases} x+y = 4200 \\ 8x+9y = 36300 \end{cases}$

$\begin{cases} -8x-8y = -33600 \\ 8x+9y = 36300 \end{cases}$

$y = \$2700$

\$2700 at 9%
\$1500 at 8%

6. A 10-foot wide sidewalk around a rectangular swimming pool has a total area of 2400 square feet. Find the dimensions of the swimming pool if the pool is 20 feet longer than it is wide.
(10 points)



$(x+40)(x+20) - x(x+20) = 2400$

$x^2+60x+800 - x^2-20x = 2400$

$40x = 1600$

$x = 40$ feet

$x+20 = 40+20 = 60$ feet

7. Assume $500 is deposited in an account that earns 4% interest compounded annually. (10 points)

a. Find a formula for g(t) where t is time and g(t) is the amount of money in the account after **t years**.

$g(t) = 500(1+0.04)^t$

$g(t) = 500\,(1.04)^t$

b. How long will it take for the money to double.

$1000 = 500\,(1.04)^t$

$2 = 1.04^t$ $\quad \log_{1.04} 2 = t \quad \Rightarrow t = 17.67$ yrs

8. Find the inverse of the following functions (10 points)

a. $f(x) = 3x + 5$

$y = 3x + 5$

$3x = y - 5 \Rightarrow x = \frac{y-5}{3}$

$f^{-1}(x) = \frac{x-5}{3}$

b. $g(x) = 7^x$

$g^{-1}(x) = \log_7 x$

9. Assume that the growth of the population of bacteria doubles every hour. The colony of bacteria start out with 1000 bacteria. Let f(t) represent the population of bacteria at time t, where **t is in hours**. (10 points)

a. Find the formula for f(t) $= 1000\,(2)^t$

b. Predict when there will be 100,000 bacteria. $100000 = 1000\,(2)^t$

$100 = 2^t$

$\log_2 100 = t \quad \Rightarrow t = \frac{\log 100}{\log 2} = 6.64$ HRS

10. For problems a through g, algebraically find all solutions, real and non real. Complex solutions should be written in the form $a+bi$ (5 points each)

a. $16x^2-81=0$

$16x^2=81$

$x^2=\frac{81}{16}$

$x=\pm\frac{9}{4}$

d. $4x^2+11x-3=0$

$(4x-1)(x+3)=0$

$4x-1=0$ $\quad x+3=0$

$x=\frac{1}{4}$ $\quad x=-3$

e. $x^2+x+5=0$

$x=\frac{-1\pm\sqrt{1-4(1)(5)}}{2}=\frac{-1\pm\sqrt{-19}}{2}$

$x=\frac{-1\pm\sqrt{19}\,i}{2}$

f. $x^2-5x=50$

$x^2-5x-50=0$

$(x-10)(x+5)=0$

$x=10$ $\quad x=-5$

g. $x(-3x+4)=2$

$-3x^2+4x-2=0$

$x=\frac{-4\pm\sqrt{16-4(-3)(-2)}}{-6}=\frac{-4\pm\sqrt{-8}}{-6}=\frac{2}{3}\mp\frac{2\sqrt{2}\,i}{6}$

$=\frac{2}{3}\pm\frac{1}{3}\sqrt{2}\,i$

Solve for x. (10 points)

11. $\frac{4}{x-3}-\frac{3}{x-1}=\frac{2}{x-3}$ Multiply by $(x-3)(x-1)$

$4(x-1)-3(x-3)=2(x-1)$

$4x-4-3x+9=2x-2$

$x+5=2x-2$

$7=x$

12. The height of a thrown math book is given by the formula $h(t) = -16t^2 + 44t + 4$
Where, h(t) is the height measured in feet and t is time measured in seconds. (15 points)

a. When does the book reach its maximum height?

Max occurs at Vertex $t = \frac{-44}{2(-16)} = 1.375$ seconds

b. What is the maximum height of the book?

$h(1.375) = -16(1.375)^2 + 44(1.375) + 4 = 34.25$ feet

c. How long does it take for the book to hit the ground?

$-16t^2 + 44t + 4 = 0$

$t = 2.84$ seconds

13. Solve the following system by substitution method. (10 points)

$$\begin{cases} y = 2x \\ x^2 + y^2 = 45 \end{cases}$$

$x^2 + (2x)^2 = 45$

$x^2 + 4x^2 = 45$

$5x^2 = 45$

$x^2 = 9$

$x = \pm 3$ and $y = \pm 6$ if $y = 2x$ then

$(3, +6)$

$(-3, -6)$

14. The following table represents an exponential function of the form $y = ab^x$. Find the value of a and b, and write the formula for the function in the form $y = ab^x$.
(Please very clearly show of all the mathematical steps) (10 points)

x	y
1	16
2	4
3	1
4	1/4

$\frac{4 = ab^2}{16 = ab^1}$ Now divide we get

$\frac{1}{4} = b$ $\quad 16 = a\left(\frac{1}{4}\right)$

$a = 64$

$y = 64\left(\frac{1}{4}\right)^x$

15. Let $f(x) = (6)^x$ Evaluate f at the indicated values. (15 points)

a. $f(0) = 6^0 = 1$

b. $f^{-1}(1296)$ $\quad 1296 = 6^x \Rightarrow x = \frac{\log 1296}{\log 6} = 4$

c. Find x when $f(x) = \frac{1}{216}$

$\frac{1}{216} = 6^x \Rightarrow x = \frac{\log\left(\frac{1}{216}\right)}{\log 6} = -3$

16. Solve $6x^3 = 108$ for x analytically. (5 points)

$x^3 = \frac{108}{6}$

$x^3 = 18 \Rightarrow x = 18^{\wedge(1\div 3)} \cong 2.621$

17. Some values for the function f is shown in the table below. (5 points each)

x	0	1	2	3
$f(x)$	3	2	1	0

x	0	1	2	3
$g(x)$	1	2	3	0

a. Find $(f \circ g)(2) = f(3) = 0$

b. Find $(g \circ f)(1) = g(2) = 3$

c. Find $(f \circ g^{-1})(3) = f(2) = 1$

d. Find $(g \circ f^{-1})(2)$

$= g(1) = 2$

18. Evaluate the following. (2.5 points each)

a. $\log_b\left(\sqrt[5]{b}\right) = \frac{1}{5}$

b. $\log_2\left(2^{1000000}\right) = 1000\,000$

c. $\log_b 1 = 0$

d. $\ln\left(e^R\right) = R$

Extra Credits:

19. Simplify (6 points)

a. $\sqrt{162}$

$= \sqrt{81}\sqrt{2}$

$= 9\sqrt{2}$

b. $\frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{3\sqrt{5}}{5}$

c. $3\sqrt{12} - 7\sqrt{75} + \sqrt{3}$

$= 3\sqrt{4}\sqrt{3} - 7\sqrt{25}\sqrt{3} + \sqrt{3}$

$= 6\sqrt{3} - 35\sqrt{3} + \sqrt{3}$

$= -28\sqrt{3}$

20. Solve for x (algebraically). (4 points)

a. $\sqrt[3]{x-1} = 2$ Cube Both sides

$x - 1 = 2^3$

$x - 1 = 8$

$x = 9$

b. $\sqrt{2x} = x - 4$ Square Both sides

$2x = (x-4)^2$

$2x = x^2 - 8x + 16$

$x^2 - 10x + 16 = 0$

$(x-8)(x-2) = 0$

$x = 8$ $x = 2$