MA 103 CHAPTER 9: LOGARITHMIC FUNCTIONS
SECTION 9.1: INVERSE FUNCTIONS

The following table gives values of a function f(x) for six inputs 0, 1, 2, 3, 4, and 5.

x	f(x)
0	12
1	4
2	3
3	10
4	0
5	8

Read the table to find:

1. f(4) = _____ 2. f(0) = _____

3. f(2) = _____ 4. f(5) = _____

The inverse of f, written f^{-1}, and read "f inverse" sends outputs of f to inputs of f.

For example: f sends 5 to 8 and f^{-1} sends 8 to 5.
The statement f(5) = 8 and $f^{-1}(8) = 5$ are **equivalent**. (See bottom p.203)

Find:

5. $f^{-1}(10)$ = _____ 6. $f^{-1}(0)$ = _____

7a. $f^{-1}(3)$ = _____ 7b. $f^{-1}(12)$ = _____

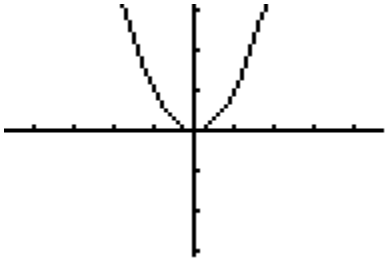
Note: f^{-1} "undoes" f.

8. The inverse of a function is not necessarily a function. For example, $g(x) = x^2$ is a function. We know that g(2) = 4 and g(-2) = 4. However, how do we answer $g^{-1}(4)$?

Note:

9. **Invertible functions:** When the inverse of a function, f, is also a function, we say that f is invertible. f and f^{-1} are inverse functions of each other. In general, linear functions of the form y = mx + b with $m \neq 0$, are invertible. Futhermore, only functions that are **one-to-one** are invertible. A function is one-to-one if each output is used only once. A one-to-one function will pass both the vertical line test and the horizontal line test.

Recall $g(x) = x^2$, which is sketched to the right. g passes the vertical line test, but fails the horizontal line test. g is a function, but not a one-to-one function. g is not invertible.

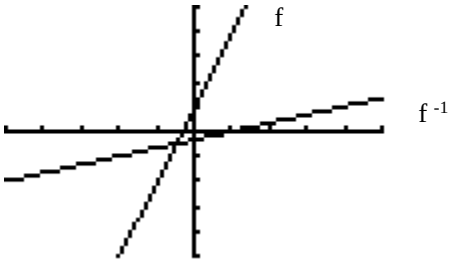


10. **Graphing an inverse function.**

For f(x) = 3x + 2, complete the table.

x	f(x)	x	$f^{-1}(x)$
-2		-4	
-1		-1	
0		2	
1		5	
2		8	

axes scaled by 2



11a. **Reflection Property:** For an invertible function f, the graph of f^{-1} is the reflection of the graph of f across the line y = x.

11b. **Finding the inverse equation of a function.**

f(x) = x - 3 (f(x) subtracts 3 from x, **then $f^{-1}(x)$ should add 3 to x to "undo" f**.)

$$\mathbf{f^{-1}(x) = x + 3}$$

$g(x) = \frac{x}{4}$ (g(x) divides x by 4, **then $g^{-1}(x)$ should multiply x by 4 to "undo" g**.)

$$\mathbf{g^{-1}(x) = 4x}$$

Note: To check $g^{-1}(x)$, graph g and $g^{-1}(x)$ to see if they are reflections of each other about the line y = x. **DRAW the functions.**

FINDING THE INVERSE FUNCTION OF A LINEAR FUNCTION ALGEBRACIALLY:

(Four-Step Process, Please See page 606 of our textbook)

We Want to Find the Inverse Function of **f(x) = x - 3**	*Now You Try to Find the Inverse Function of* **g(x) = 2x + 5**
STEP 1: Replace f(x) with y $y = x - 3$	
STEP 2: Now, Solve for x $x - 3 = y$ $x = y + 3$	
STEP 3: Replace x with $f^{-1}(y)$ $f^{-1}(y) = y + 3$	
STEP 4: Write in terms of x $f^{-1}(x) = x + 3$	

12. **Making interpretations using the inverse function.**

Let n = f(t) = .25 t – 1.67 represent the number of people (in millions) undergoing laser eye surgery in the year that is t years since 1990.

A. Find & interpret f(10).

B. Find an equation for f^{-1}.

C. Find & interpret f^{-1} (3).

D. What is the slope of f^{-1}? What does it mean in the context of this problem?