MA 103 Some Applications of Quadratic Functions
Sections 8.3, 8.8

1. The profit made by a small business on the manufacture and sale of x items is
$P(x) = -x^2 + 126x - 1100$.

(a) What is the value of $P(0)$? Write a sentence explaining what this number means in the context of this problem.

(b) Algebraically find the number of items that should be manufactured and sold in order to realize the maximum profit.

(c) What is the amount of the maximum profit?

(d) What is a graphing window on your calculator that displays the vertex and all intercepts for this profit function? Using this window, graph the function.

(e) Suppose that the company is making a profit of $1600. How many items (to the nearest whole number) are being manufactured and sold?

(f) When profit is zero, the company is said to break even. Determine the number of items (to the nearest whole number) that must be manufactured and sold so that the company breaks even.

2. If an object is tossed downward with an initial speed (velocity) of v_0, then it will travel a distance of *s* meters, where $s = 4.9t^2 + v_0 t$ and t is measured in seconds.

(a) Suppose an object is tossed downward with an initial speed of 40 m/sec.

(i) How far will it travel in 2 seconds?

(ii) After how many seconds will the object have traveled 200 meters?

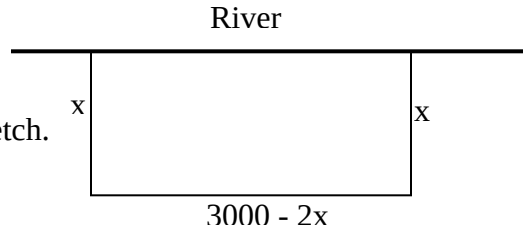
(b) Suppose an object falls from a helicopter that has an altitude of 400 m. How long will it take for the object to hit the ground?

3. The function $h(t) = -16t^2 + 80t + 5$ gives the vertical position, in feet above ground level, of a baseball *t* seconds after it has been hit.

(a) Algebraically determine how high above the ground the ball will rise.

(b) If no one catches the ball, after how many seconds will it hit the ground? Compute an *exact* answer to this question symbolically and then check your result graphically. Hint: What is the height of the object above the ground when the object is on the ground?

4. A rancher has 3000 feet of fencing available to enclose a rectangular field that borders on a river. There will be fencing only on 3 sides of the field, as indicated in the sketch.



(a) The area of a rectangle is $A = (length) \cdot (width)$ Write a function $A(x)$ that represents the area of the field.

(b) Estimate the dimensions of a field that has an area of $500,000\ ft^2$. Round answers to the nearest foot.

(c) What are the dimensions of the field that encloses the largest area?

(d) What is the area of the largest field?

5. The number of inmates in custody in US prisons and jails can be modeled by the quadratic function $p(x) = -x^2 + 93x + 1128$, where $p(x)$ is the number of inmates in thousands, and x is the number of years after 1990.

(a) According to this model, in what year will the number of prison and jail inmates in custody in the United States be at its maximum?

(b) What is that maximum number of inmates?

OVER

6. Methane is a gas produced by landfills, natural gas systems, and coal mining operations that contributes to the greenhouse effect and global warming. Based on data from the US EPA, projected methane emissions in this country can be modeled by the quadratic function $f(x) = -0.072x^2 + 1.93x + 173.9$, where $f(x)$ is the amount of methane produced in million metric tons, and x is the number of years after 2000.

(a) According to this model, what will US methane emissions be in 2009?

(b) In what year will methane emissions in the US be at their maximum? Round to the nearest year.

(c) What will be that maximum amount of methane emissions?

(d) What is a graphing window on your calculator that displays the vertex and all intercepts for this function?

Answers

1. (a) -1100 The company will have a loss of \$1100 if no items are manufactured and sold.
   (b) 63 items (c) \$2869 (d) $[0, 150] \times [-2000, 3000]$ (e) 27 or 99 items (f) 10
2. (a) (i) 99.6 meters (ii) 3.5 seconds (b) about 9 seconds
3. (a) Find the vertex of the parabola: $t = 2.5$ sec., height = $h(2.5) = 105$ ft
   (b) Use the quadratic formula to find the positive t-axis intercept: $t = \dfrac{80 + \sqrt{6720}}{32} \approx 5.06$ sec
4. (a) $A(x) = x(3000 - 2x)$
   (b) $x = 191$ ft., width = 2618 ft; or $x = 1309$ ft., width = 382 ft
   (c) $x = 750$ ft., width = 1500 ft (d) 1125000 square feet
5. (a) 2036 (b) 3,290,250
6. (a) 185.44 million metric tons (b) 2013
   (c) 186.82 million metric tons (d) $[-50, 120] \times [-50, 200]$