**MATH 099** **NAME: __Solutions______________________**

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**Find all numbers for which the rational expression is not defined.**

1) $\dfrac{x^3+2}{x^2+8x}$

**Solution:**

The domain of a rational function is all real numbers except those values that make the denominator Zero.

We find domain by setting the denominator equal to Zero and solve for the variable, (in our case, solve for x).

$x^2+8x$ Noticed the denominator

$x^2+8x=0$ Set the denominator equal to zero

x(x + 8) = 0 Factored the left side of the equation

Either x = 0 or (x + 8) = 0 Applied the zero factor property

The values of x for which the rational expression would be undefined are x = 0 and x = -8

In other words the domain is all real numbers except x = 0 and x = - 8.

In set-builder notation we state the domain as follows:

$$x \mid x \text{ is all real numbers and } x \neq 0 \; and \; x \neq -8$$

2) **Find all numbers not in the domain of the function.**

**F (x) =**

$$\frac{x^2-16}{x^2+4x-32}$$

**Solution:**

The domain of a rational function is all real numbers except those values that make the denominator Zero.

We find domain by setting the denominator equal to Zero and solve for the variable, (in our case, solve for x).

$x^2 + 4x - 32$ Noticed the denominator

$x^2 + 4x - 32 = 0$ Set the denominator equal to zero

$(x+8)(x-4) = 0$ Factored the left side of the equation

Either (x + 8) = 0 or (x – 4) = 0 Applied the zero factor property

The values of x for which the rational expression would be undefined are x = -8 and x = 4

In other words the domain is all real numbers except x =- 8 and x = 4.

In set-builder notation we state the domain as follows:

$$x \mid x \text{ is all real numbers and } x \neq -8 \text{ } and \text{ } x \neq 4$$

**3) Find all numbers not in the domain of the function.**

$$f(x) = \frac{x^2 - 64}{x^2 - 2x - 63}$$

**Solution:**

The domain of a rational function is all real numbers except those values that make the denominator Zero.

We find domain by setting the denominator equal to Zero and solve for the variable, (in our case, solve for x).

$x^2 - 2x - 63$ Noticed the denominator

$x^2 - 2x - 63 = 0$ Set the denominator equal to zero

$(x-9)(x+7) = 0$ Factored the left side of the equation

Either (x - 9) = 0 or (x + 7) = 0

The values of x for which the rational expression would be undefined are x = 9 and x = - 7

In other words the domain is all real numbers except x = 9 and x = - 7.

In set-builder notation we state the domain as follows:

$$x \mid x \text{ is all real numbers and } x \neq 9 \text{ } and \text{ } x \neq -7$$

**4) Multiply and simplify (Please see Example 6 in Section 6.1)**

$$\frac{k^2 + 11k + 18}{k^2 + 15k + 54} \quad \frac{k^2 + 6k}{k^2 + 7k + 10}$$

**Solution:**

First we have to factor all of the numerators and denominators of the above fractions.

$$\frac{k^2+11k+18}{k^2+15k+54} \bullet \frac{k^2+6k}{k^2+7k+10}$$
$$= \frac{(k+9)(k+2)}{(k+9)(k+6)} \bullet \frac{k(k+6)}{(k+2)(k+5)}$$

Factored the numerators and denominators of both equations

Noticed the following and reduced the above expression

$$\frac{(k+9)}{(k+9)} = 1$$

$$\frac{(k+2)}{(k+2)} = 1$$

$$\frac{(k+6)}{(k+6)} = 1$$

*Therefore*

$$\frac{(k+9)(k+2)}{(k+9)(k+6)} \bullet \frac{k(k+6)}{(k+2)(k+5)} = \frac{k}{(k+5)}$$

**5) Divide and simplify**

$$\frac{z^2+8z+12}{z^2+9z+14} \div \frac{z^2+6z}{z^2+11z+28}$$

**Solution:**

First we have to factor all of the numerators and denominators of the above fractions, change division into multiplication, and flip the second fraction.

$$\frac{z^2+8z+12}{z^2+9z+14} \div \frac{z^2+6z}{z^2+11z+28}$$
$$= \frac{(z+6)(z+2)}{(z+7)(z+2)} \bullet \frac{(z+4)(z+7)}{z(z+6)}$$

Factored the numerators and denominators of both equations

Noticed the following and reduced the above expression

$$\frac{(z+6)}{(z+6)}=1$$
$$\frac{(z+2)}{(z+2)}=1$$
$$\frac{(z+7)}{(z+7)}=1$$
*therefore,*
$$\frac{z^2+8z+12}{z^2+9z+14}\div\frac{z^2+6z}{z^2+11z+28}=\frac{(z+4)}{z}$$

**6) Perform the following operation (if possible, simplify your answer)**

$$\frac{3}{y^2-3y+2}+\frac{7}{y^2-1}$$

**Solution:**

First we have to factor all of the denominators of the above fractions,

$$\frac{3}{y^2-3y+2}+\frac{7}{y^2-1}=\frac{3}{(y-2)(y-1)}+\frac{7}{(y-1)(y+1)}$$

Now, we have to find the lowest common denominator which is $(y-2)(y-1)(y+1)$ then, we set up fractions with the proper common denominator, and multiply the numerators by the needed terms. Finally we add the fractions and simplify the results.

$$=\frac{3(y+1)}{(y-2)(y-1)(y+1)}+\frac{7(y-2)}{(y-2)(y-1)(y+1)}$$
$$=\frac{3(y+1)+7(y-2)}{(y-2)(y-1)(y+1)}$$
$$=\frac{3y+3+7y-14}{(y-2)(y-1)(y+1)}$$
$$=\frac{10y-11}{(y-2)(y-1)(y+1)}$$

**7) Solve the following equation**

$$\frac{3}{2x+3}-\frac{1}{2x-3}=\frac{4}{4x^2-9}$$

**Solution:**

First we have to factor the denominators of the above fractions, to find the Lowest Common Denominator

$$\frac{3}{2x+3}-\frac{1}{2x-3}=\frac{4}{4x^2-9}$$

$$\frac{3}{2x+3}-\frac{1}{2x-3}=\frac{4}{(2x+3)(2x-3)}$$

Now, we have to find the lowest common denominator which is $(2x+3)(2x-3)$

Then, we multiply both sides (every terms) of the equation $\frac{3}{2x+3}-\frac{1}{2x-3}=\frac{4}{4x^2-9}$ by

$(2x+3)(2x-3)$

$$(2x+3)(2x-3)\left[\frac{3}{2x+3}-\frac{1}{2x-3}=\frac{4}{(2x+3)(2x-3)}\right](2x+3)(2x-3)$$

Which Equals to:

$$(2x+3)(2x-3)\frac{3}{2x+3}-(2x+3)(2x-3)\frac{1}{2x-3}=\frac{4}{(2x+3)(2x-3)}(2x+3)(2x-3)$$

Now, we have to reduce the above equation, and find

$$3(2x-3)-(2x+3)=4$$

$$6x-9-2x-3=4$$

$$4x-12=4$$

$$4x=16$$

$$x=4$$

**Solve.**
8) Melissa can clean the house in 4 hours, whereas her husband, Zack, can do the same job in 6 hours. They have agreed to clean the house together so that they can finish in time to watch a movie on TV. How long will it take them to clean the house together?

**Solution:**

The key behind solving problems like these is to understand the relationship between the *time* (in hours) that it takes to complete the job and the *part of the job* completed in 1 unit of time (in this case 1 hour).

For example, if it takes Melissa alone 4 hours to complete the job, then the part of the job that she can complete in one hour is ¼. Similarly, since it takes her husband 6 hours to complete the job, then the part of the job that he can complete in one hour is 1/6.

If we let t = *time in hours* it takes for both Melissa and her husband to clean the house working together, then the part of the job completed in one hour is 1/t. We can summarize the given information in the following table:

| | Hours to Clean the House | Part of job completed in 1 hour. |
|---|---|---|
| Melissa alone | 4 | ¼ |
| Her Husband alone | 6 | 1/6 |
| Melissa and her husband working at the same time | t | 1/t |

$$\frac{1}{4}+\frac{1}{6}=\frac{1}{t}$$
$$12t\left(\frac{1}{4}+\frac{1}{6}\right)=12t\left(\frac{1}{t}\right)$$
$$3t+2t=12$$
$$5t=12$$
$$t=\frac{12}{5}=2.4 \text{ hours}$$

**Solve.**
9) Jeff takes 5 hr longer to build a fence than it takes Bill. When they work together, it takes them 6 hours. How long would it take Bill to do the job alone?

Here again, the key behind solving problems like these is to understand the relationship between the *time* (in hours) that it takes to complete the job and the *part of the job* completed in 1 unit of time (in this case 1 hour).

For example, if it takes Bill x hours to build the fence, then it takes Jeff x + 5 hours to build the fence. Then the part of the job that Bill can complete in one hour is 1/x. Similarly, since it takes Jeff x+5 hours to complete the job, then the part of the job that he can complete in one hour is 1/(x+5).

And we know that if Bill and Jeff work together, then it takes them 6 hours to complete the job, then the part of the job completed in one hour is 1/6. We can summarize the given information in the following table:

| | Hours to Build a Fence | Part of job completed in 1 hour. |
|---|---|---|
| Bill | x | 1/x |
| Jeff | x + 5 | 1/(x + 5) |
| Bill and Jeff working at the same time | 6 | 1/6 |

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$$\frac{1}{x}+\frac{1}{x+5}=\frac{1}{6}$$

$$6x(x+5)\left(\frac{1}{x}+\frac{1}{x+5}\right)=6x(x+5)\left(\frac{1}{6}\right)$$

$$\frac{6x(x+5)}{x}+\frac{6x(x+5)}{(x+5)}=\frac{6x(x+5)}{6}$$

$$6(x+5)+6x=x(x+5)$$

$$6x+30+6x=x^2+5x$$

$$12x+30=x^2+5x$$

$$0=x^2+5x-12x-30$$

$$0=x^2-7x-30$$

$$0=(x-10)(x+3)$$

$$x-10=0 \text{ or } x+3=0$$

$$x=10hours \text{ or } x=-3hours$$

Since time can not be negative

*Therefore,* $x=10$ *hours.*

**Find an equation of variation if the following conditions exist.**

10) Suppose that y varies directly as z and y = 45 when z = 270.

**Solution:**

y = k z — Translated the statement that y varies directly as z

45 = k (270) — Substituted 45 for y and 270 for z

45/270 = k (270) / 270 — Divided both sides of the equation by 270

$\frac{1}{6}$ = k — Simplified

y = k z — Recalled the original equation

y = $\frac{1}{6}$z — Substituted $\frac{1}{6}$ for k.