

**MATH 096 Dr. Katiraie Chapter 5 Polynomials, Factoring, and Solving Equations
Solutions to Practice Problems**

1) Perform the indicated operations on the following polynomials:

$$\begin{aligned} \text{a) } & (3x^2 - 7x - 9) + (5x^2 - 9x + 8) \\ & = 3x^2 - 7x - 9 + 5x^2 - 9x + 8 \\ & = 8x^2 - 16x - 1 \end{aligned}$$

$$\begin{aligned} \text{b) } & (6x^2 - 8x) - (4x^2 - 2x - 5) \\ & = 6x^2 - 8x - 4x^2 + 2x + 5 \\ & = 2x^2 - 6x + 5 \end{aligned}$$

$$\begin{aligned} \text{c) } & (4a^2 - 3ab + 5b^2) - (3a^2 - 3ab + 4b^2) \\ & = 4a^2 - 3ab + 5b^2 - 3a^2 + 3ab - 4b^2 \\ & = a^2 + b^2 \end{aligned}$$

$$\begin{aligned} \text{d) } & -2x^3(3x^4 - 2x^2 + 4) \\ & = -6x^7 + 4x^5 - 8x^3 \end{aligned}$$

2) Perform the indicated operations:

$$\begin{aligned} \text{a) } & (3x - 5)(2x + 3) \\ & = 6x^2 + 9x - 10x - 15 \\ & = 6x^2 - x - 15 \end{aligned}$$

$$\begin{aligned} \text{b) } & (3x + 2)^2 \\ & = (3x + 2)(3x + 2) \\ & = 9x^2 + 6x + 6x + 4 \\ & = 9x^2 + 12x + 4 \end{aligned}$$

$$\begin{aligned} \text{c) } & (3x - 2)^2 \\ & = (3x - 2)(3x - 2) \\ & = 9x^2 - 6x - 6x + 4 \\ & = 9x^2 - 12x + 4 \end{aligned}$$

$$\begin{aligned} \text{d) } & (2x - 5)(2x + 5) \\ & = 4x^2 + 10x - 10x - 25 \\ & = 4x^2 - 25 \end{aligned}$$

3) Perform the indicated operation: $(3x + 1)(9x^2 - 3x + 1)$

$$\begin{aligned}(3x+1)(9x^2-3x+1) &= 27x^3 - 9x^2 + 3x + 9x^2 - 3x + 1 \\ &= 27x^3 + 1\end{aligned}$$

4) Factor each of the following:

a) $5x^4 - 15x^3$
 $= 5x^3(x-3)$

b) $a(x+2) - 3(x+2)$
 $= (x+2)(a-3)$

c) $3x^2 + 6xy - 5x - 10y$
 $= 3x(x+2y) - 5(x+2y)$
 $= (x+2y)(3x-5)$

d) $5x^3 + 20x^2 - 105x$
 $= 5x(x^2 + 4x - 21)$
 $= 5x(x+7)(x-3)$

5) Factor the following (if a polynomial cannot be factored, write "prime"):

a) $x^2 - 6x - 27$
 $= (x-9)(x+3)$

b) $x^2 + 7x + 10$
 $= (x+5)(x+2)$

c) $x^2 - 3x + 9$
prime

d) $x^2 - 10x + 25$
 $= (x-5)(x-5)$

e) $x^3 - 27y^3$
 $= (x-3y)(x^2 + 3xy + 9y^2)$

6) Factor the following (if a polynomial cannot be factored, write "prime"):

$$\begin{aligned} \text{a) } x^2 - 81 \\ = (x+9)(x-9) \end{aligned}$$

$$\begin{aligned} \text{b) } x^2 + 81 \\ \text{prime} \end{aligned}$$

$$\begin{aligned} \text{c) } x^2 + 18x + 81 \\ = (x+9)(x+9) \end{aligned}$$

$$\begin{aligned} \text{d) } x^2 - 18x + 81 \\ = (x-9)(x-9) \end{aligned}$$

7) Solve the equations:

a)

$$x^2 + 4x - 3 = 0$$

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{16 - 4(1)(-3)}}{2} \\ &= \frac{-4 \pm \sqrt{16 + 12}}{2} \\ &= \frac{-4 \pm \sqrt{28}}{2} \\ &= \frac{-4 \pm 2\sqrt{7}}{2} \\ &= \cancel{2} \frac{(-2 \pm \sqrt{7})}{\cancel{2}} \\ &= -2 \pm \sqrt{7} \end{aligned}$$

b)

$$\begin{aligned} (x-3)^2 &= 16 \\ x-3 &= \pm 4 \\ x &= 4+3 \text{ or } x = -4+3 \\ x &= 7 \text{ or } x = -1 \end{aligned}$$

c)

$$2y^2 - 6y = 1$$

$$2y^2 - 6y - 1 = 0$$

$$\begin{aligned} y &= \frac{6 \pm \sqrt{36 - 4(2)(-1)}}{4} \\ &= \frac{6 \pm \sqrt{44}}{4} \\ &= \frac{6 \pm 2\sqrt{11}}{4} \\ &= \frac{2(3 \pm \sqrt{11})}{4} \\ &= \frac{3 \pm \sqrt{11}}{2} \end{aligned}$$

d)

$$3x^2 - 2x - 7 = 0$$

$$\begin{aligned} x &= \frac{2 \pm \sqrt{4 - 4(3)(-7)}}{6} \\ &= \frac{2 \pm \sqrt{4 + 84}}{6} \\ &= \frac{2 \pm \sqrt{88}}{6} \\ &= \frac{2 \pm 2\sqrt{22}}{6} \\ &= \frac{2(1 \pm \sqrt{22})}{6} \\ &= \frac{1 \pm \sqrt{22}}{3} \end{aligned}$$

8) If $f(x) = x^3 - x^2 + 1$, find each of the following values:

a)

$$\begin{aligned} f(2) &= 2^3 - 2^2 + 1 \\ &= 8 - 4 + 1 \\ &= 5 \end{aligned}$$

b)

$$\begin{aligned} f(-2) &= (-2)^3 - (-2)^2 + 1 \\ &= -8 - 4 + 1 \\ &= -11 \end{aligned}$$

c)

$$\begin{aligned} f\left(\frac{1}{2}\right) &= \left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + 1 \\ &= \frac{1}{8} - \frac{1}{4} + 1 \\ &= \frac{1}{8} - \frac{2}{8} + \frac{8}{8} \\ &= \frac{7}{8} \end{aligned}$$

d) Find x when $f(x) = 1$

$$f(x) = x^3 - x^2 + 1$$

$$x^3 - x^2 + 1 = 1 \quad \text{subtract 1 from both sides of the equation}$$

$$x^3 - x^2 = 0 \quad \text{factor the GCF}$$

$$x^2(x-1) = 0$$

$$x^2 = 0 \quad \text{or} \quad (x-1) = 0$$

$$x = 0 \quad \text{or} \quad x = 1$$