MA 103 Simplifying Radical Expressions
Sections 7.3, 7.4, 7.5

I. Radical expressions can be written in simplified form by making use of the properties

- If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are defined, then $\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$ and $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$
- For nonnegative a, $\sqrt[n]{a^n} = a$; $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ or $\left(\sqrt[n]{a}\right)^m$

1. Simplify

(a). $\sqrt{2}\cdot\sqrt{32}$ $= \sqrt{64} = 8$	(b) $\sqrt[3]{3}\,\sqrt[3]{9}$ $\sqrt[3]{27} = 3$	(c) $\sqrt{50}\,\sqrt{2}$ $\sqrt{100} = 10$
(d) $\dfrac{\sqrt{50}}{\sqrt{2}} = \sqrt{\dfrac{50}{2}}$ $= \sqrt{25} = 5$	(e) $\sqrt{\dfrac{16}{81}} = \dfrac{\sqrt{16}}{\sqrt{81}}$ $= \dfrac{4}{9}$	(f) $\sqrt{\dfrac{5}{8}}\,\sqrt{\dfrac{5}{2}}$ $= \sqrt{\dfrac{25}{16}} = \dfrac{5}{4}$

2. Rewrite in simplified form. Assume all variables are nonnegative.

(a) $\sqrt{700}$ $= \sqrt{100}\,\sqrt{7} = 10\sqrt{7}$	(b) $\sqrt{45}$ $\sqrt{9}\,\sqrt{5} = 3\sqrt{5}$	(c) $\sqrt{48}$ $\sqrt{16}\,\sqrt{3} = 4\sqrt{3}$
(d) $7\sqrt{18}$ $7\sqrt{9}\sqrt{2} = 21\sqrt{2}$	(e) $\sqrt{x^8}$ $= x^4$	(f) $\sqrt{x^{11}}$ $\sqrt{x^{10}}\,\sqrt{x}$ $= x^5\sqrt{x}$
(g) $\sqrt[3]{a^6} = a^2$	(h) $\sqrt{20x^5}$ $\sqrt{4}\sqrt{5}\sqrt{x^4}\sqrt{x}$ $\boxed{= 2x^2\sqrt{5x}}$	(i) $\sqrt{x}\,\sqrt{x^7} = \sqrt{x^8}$ $\boxed{= x^4}$
(j) $\sqrt{5a^3}\,\sqrt{8a^7}$ $= \sqrt{40a^{10}}$ $= \sqrt{4}\,\sqrt{10}\,\sqrt{a^{10}}$ $= 2a^5\sqrt{10}$	(k) $\sqrt[3]{x^2}\,\sqrt[3]{x^4}$ $\sqrt[3]{x^6}$ $= \boxed{x^2}$	(l) $\dfrac{\sqrt{9x^3y}}{\sqrt{xy}} = \sqrt{\dfrac{9x^3y}{xy}}$ $= \sqrt{9x^2} = \boxed{3x}$

OVER→

II. To ***rationalize the denominator*** of a fraction means to rewrite the fraction in an equivalent form so that the denominator is rational, that is, so that it does not contain a radical. If the denominator contains one term, multiply the numerator and denominator of the fraction by the radical in the denominator and then simplify the fraction.

3. Rationalize the denominator and simplify:

(a) $\frac{10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5}$	(b) $\frac{6}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{6\sqrt{7}}{7}$
(c) $\sqrt{\frac{5}{3}} = \frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{15}}{3}$	(d) $\frac{2}{3\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{2\sqrt{10}}{30} = \frac{\sqrt{10}}{15}$

III. Radical expressions can be simplified by addition or subtraction if and only if they are like radicals, that is, if they have the same index and the same radicand. To add or subtract like radicals, add or subtract the coefficients. If the radicals are not like radicals, then they cannot be combined by addition or subtraction.

4. Simplify each expression.

(a) $7\sqrt{3}-2\sqrt{3} = 5\sqrt{3}$	(b) $3\sqrt{20}+\sqrt{45}$ $3\sqrt{4}\sqrt{5}+\sqrt{9}\sqrt{5}$ $=6\sqrt{5}+3\sqrt{5}=9\sqrt{5}$
(c) $4\sqrt[3]{6}-\sqrt[3]{6} = 3\sqrt[3]{6}$ $4\sqrt[3]{6}+\sqrt[3]{6}=5\sqrt[3]{6}$	(d) $\sqrt{32}+5\sqrt{18}$ $\sqrt{16}\sqrt{2}+5\sqrt{9}\sqrt{2}$ $=4\sqrt{2}+(5)(3)\sqrt{2}=\boxed{19\sqrt{2}}$

Answers

1(a) 8	(b) 3	(c) 10	(d) 5	(e) 4/9	(f) 5/4	
2 (a) $10\sqrt{7}$	(b) $3\sqrt{5}$	(c) $4\sqrt{3}$	(d) $21\sqrt{2}$	(e) x^4	(f) $x^5\sqrt{x}$	(g) a^2
2 (h) $2x^2\sqrt{5x}$	(i) x^4	(j) $2a^5\sqrt{10}$	(k) x^2	(l) $3x$		
3 (a) $2\sqrt{5}$	(b) $\frac{6\sqrt{7}}{7}$	(c) $\frac{\sqrt{15}}{3}$	(d) $\frac{\sqrt{10}}{15}$			
4 (a) $5\sqrt{3}$	(b) $9\sqrt{5}$	(c) $5\sqrt[3]{6}$	(d) $19\sqrt{2}$			

MA 103 Solving Quadratic Equations
Sections 8.1, 8.2

I. Using the Principle of Square Roots; Completing the Square

1. The generalized principle of square roots states that for any real number k, if $X^2 = k,\ then\ X = \pm\sqrt{k}$, where X is an algebraic expression.

   Use this principle to solve each of the following equations. Answers involving square roots of negative numbers should be written in the form $a + bi$.

(a) $6x^2 = 13$ $x^2 = \frac{13}{6}$ $x = \pm\sqrt{\frac{13}{6}} = \frac{\pm\sqrt{13}}{\sqrt{6}} = \boxed{\frac{\pm\sqrt{78}}{6}}$	(b) $x^2 = -16$ $x = \pm\sqrt{-16}$ $x = \pm 4i$
(c) $(x-3)^2 = 17$ Take Square Root of Both sides of the equation $x - 3 = \pm\sqrt{17}$ Now Add 3 to both sides of eqn $x = 3 \pm \sqrt{17}$	

2. Use the method of completing the square and the generalized principle of square roots to solve each of the following problems.

(a) Solve the equation: $x^2 + 8x + 3 = 0$ $x^2 + 8x = 3$ $\left(\frac{8}{2}\right)^2 = 16$ $x^2 + 8x + 16 = 3 + 16$ $(x+4)^2 = 19$ $x + 4 = \pm\sqrt{19}$ $x = -4 \pm \sqrt{19}$	(b) Solve the equation: $2x^2 - 10x - 1 = 0$ $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$ $2x^2 - 10x = 1$ $2(x^2 - 5x\ ) = 1$ $2(x^2 - 5x + \frac{25}{4}) = 1 + \frac{25}{2}$ $2(x - \frac{5}{2})^2 = \frac{27}{2}$ $(x - \frac{5}{2})^2 = \frac{27}{4}$ $x - \frac{5}{2} = \pm\sqrt{\frac{27}{4}}$

OVER→

$\frac{-(-10) \pm \sqrt{100 - 4(2)(-1)}}{2(2)}$ $\frac{5 \pm 3\sqrt{3}}{2}$ → $x = \frac{5}{2} \pm \frac{3\sqrt{3}}{2}$

$\frac{10 \pm \sqrt{36}\sqrt{3}}{4} = \frac{10 \pm 6\sqrt{3}}{4}$

DON'T PANIC ; You Can Also Use Quad formula

$x^2-4x-7=0$

(c) Find the x-intercepts of the function $f(x)=x^2-4x-7$ $\quad a=1 \quad b=-4 \quad c=-7$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-7)}}{2}=\frac{4\pm\sqrt{44}}{2}$$

$$=\frac{4\pm\sqrt{4}\sqrt{11}}{2}=\frac{4\pm2\sqrt{11}}{2}$$

$$\boxed{=2\pm\sqrt{11}}$$

II. **Using the Quadratic Formula**

Any quadratic equation $ax^2+bx+c=0,\ a\neq 0$, can be solved using the quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

3. Use this formula to solve each of the following problems. Write answers in exact form (using radicals). For any answer involving real numbers, use your calculator to approximate the answer correct to three decimal places. Any answer involving square roots of negative numbers should be written in the form $a+bi$.

(a) Solve the equation: $2x^2-3x-7=0$

$$x=\frac{3\pm\sqrt{(-3)^2-4(2)(-7)}}{2(2)}=$$

$$=\boxed{\frac{3\pm\sqrt{65}}{4}}$$

(b) Solve the equation: $x^2-5x+7=0$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(7)}}{2}$$

$$x=\frac{+5\pm\sqrt{-3}}{2}=\boxed{\frac{5\pm\sqrt{3}\,i}{2}}$$

(c) Let $f(x)=\frac{x+2}{x}$ and $g(x)=\frac{x-4}{2}$. Find all x such that $f(x)=g(x)$.

$$\frac{x+2}{x}=\frac{x-4}{2}$$ Cross Multiply

$$2(x+2)=x(x-4)$$

$$2x+4=x^2-4x$$

$$x^2-4x-2x-4=0$$

$$x^2-6x-4=0$$

$$x=\frac{6\pm\sqrt{(6)^2-4(1)(-4)}}{2}$$

$$x=\frac{6\pm\sqrt{36+16}}{2}$$

$$x=\frac{6\pm\sqrt{52}}{2}=\frac{6\pm\sqrt{4}\sqrt{13}}{2}$$

$$x=\frac{6\pm2\sqrt{13}}{2}$$

$$x=3\pm\sqrt{13}$$

MA 103 Some Applications of Quadratic Functions

Sections 8.3, 8.8

1. The profit made by a small business on the manufacture and sale of x items is
$$P(x) = -x^2 + 126x - 1100.$$

(a) What is the value of $P(0)$? Write a sentence explaining what this number means in the context of this problem.

$P(0) = -0^2 + 126(0) - 1100 = -1100$

The company will have a loss of \$1100 if no items are manufactured & sold.

(b) Algebraically find the number of items that should be manufactured and sold in order to realize the maximum profit.

$X = \frac{-b}{2a} = \frac{-126}{2(-1)} = \frac{126}{2} = 63$ items

(c) What is the amount of the maximum profit?

$P(63) = -63^2 + 126(63) - 1100 = \$2869$

(d) What is a graphing window on your calculator that displays the vertex and all intercepts for this profit function? Using this window, graph the function.

$[0, 150] \times [-2000, 3000]$

(e) Suppose that the company is making a profit of \$1600. How many items (to the nearest whole number) are being manufactured and sold?

$1600 = -x^2 + 126x - 1100$

$x^2 - 126x + 1100 + 1600 = 0$

$x^2 - 126x + 2700 = 0$

use Quad formula

$x = \frac{126 \pm \sqrt{(126)^2 - 4(1)(2700)}}{2}$

$\boxed{x = 98.62 \approx 99 \text{ or } x = 27.38 \approx 27 \text{ items}}$

(f) When profit is zero, the company is said to break even. Determine the number of items (to the nearest whole number) that must be manufactured and sold so that the company breaks even.

$0 = -x^2 + 126x - 1100$

$x^2 - 126x + 1100 = 0$

$x = \frac{126 \pm \sqrt{(-126)^2 - 4(1)(1100)}}{2} =$

$\boxed{\begin{array}{l} \to 116.56 \approx 117 \text{ items} \\ \to 9.44 \approx 9 \text{ items} \end{array}}$

2. If an object is tossed downward with an initial speed (velocity) of v_0, then it will travel a distance of s meters, where $s = 4.9t^2 + v_0 t$ and t is measured in seconds.

(a) Suppose an object is tossed downward with an initial speed of 40 m/sec.

(i) How far will it travel in 2 seconds? $S = 4.9t^2 + 40t$

$S(2) = 4.9(2)^2 + 40(2) = 99.6$ meters

(ii) After how many seconds will the object have traveled 200 meters? let $4.9t^2 + 40t = 200$

$4.9t^2 + 40t - 200 = 0$ Use Quadratic formula

$t = \dfrac{-40 \pm \sqrt{40^2 - 4(4.9)(200)}}{2(4.9)} = \boxed{3.5 \text{ seconds}}$ Ignore the Neg. Ans.

(b) Suppose an object falls from a helicopter that has an altitude of 400 m. How long will it take for the object to hit the ground? Note: $V_0 = 0$

$4.9t^2 = 400$

$t^2 = \dfrac{400}{4.9} \Rightarrow t = \sqrt{\dfrac{400}{4.9}} = 9.035$ seconds $\approx \boxed{9 \text{ seconds}}$

3. The function $h(t) = -16t^2 + 80t + 5$ gives the vertical position, in feet above ground level, of a baseball t seconds after it has been hit.

(a) Algebraically determine how high above the ground the ball will rise.

$t = \dfrac{-b}{2a} = \dfrac{-80}{2(-16)} = 2.5$ seconds

Now plug in 2.5 for t

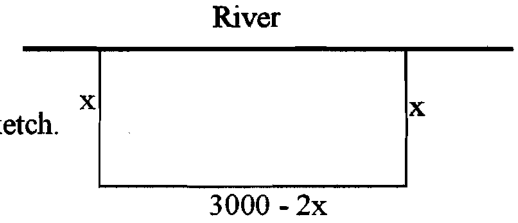
$h(2.5) = -16(2.5)^2 + 80(2.5) + 5 = \boxed{105 \text{ feet}}$

(b) If no one catches the ball, after how many seconds will it hit the ground? Compute an *exact* answer to this question symbolically and then check your result graphically. Hint: What is the height of the object above the ground when the object is on the ground?

$-16t^2 + 80t + 5 = 0$ $a = -16$ $b = 80$ $c = 5$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-80 \pm \sqrt{80^2 - 4(-16)(5)}}{2(-16)} = \begin{cases} \cancel{-0.06} & \text{time cannot be Negative} \\ \boxed{5.06 \text{ seconds}} \end{cases}$

4. A rancher has 3000 feet of fencing available to enclose a rectangular field that borders on a river. There will be fencing only on 3 sides of the field, as indicated in the sketch.



(a) The area of a rectangle is $A = (length)\cdot(width)$ Write a function $A(x)$ that represents the area of the field.

$$A = x(3000-2x) = \boxed{-2x^2+3000x}$$

(b) Estimate the dimensions of a field that has an area of 500,000 ft^2. Round answers to the nearest foot.

$$500000 = -2x^2+3000x$$

$$2x^2-3000x+500000=0$$

$$x = \frac{3000 \pm \sqrt{(3000)^2-4(2)(500000)}}{4} = \boxed{1309 \text{ feet}}$$

$$3000-2x = 3000-2(1309) = \boxed{382 \text{ feet}}$$

(c) What are the dimensions of the field that encloses the largest area?

Here we are looking for the vertex of $A = -2x^2+3000x$

$$x = \frac{-b}{2a} = \frac{-3000}{2(-2)} = \frac{-3000}{-4} = \boxed{750 \text{ feet}}$$ & length $= 3000-2x = 3000-2(750) = \boxed{1500 \text{ feet}}$

(d) What is the area of the largest field?

$$A = -2x^2+3000x$$

Plug in $x = 750$ we get $\Rightarrow A = -2(750)^2+3000(750) = \boxed{1125000 \text{ ft}^2}$

5. The number of inmates in custody in US prisons and jails can be modeled by the quadratic function $p(x) = -x^2+93x+1128$, where $p(x)$ is the number of inmates in thousands, and x is the number of years after 1990.

(a) According to this model, in what year will the number of prison and jail inmates in custody in the United States be at its maximum?

$$x = \frac{-b}{2a} = \frac{-93}{2(-1)} = 46.5$$

In the middle of the year 2036

(b) What is that maximum number of inmates?

$$P(46.5) = -46.5^2+93(46.5)+1128$$

$= 3290.25$ thousand inmates

OVER→

$= \boxed{3290250 \text{ inmates}}$

6. Methane is a gas produced by landfills, natural gas systems, and coal mining operations that contributes to the greenhouse effect and global warming. Based on data from the US EPA, projected methane emissions in this country can be modeled by the quadratic function $f(x) = -0.072x^2 + 1.93x + 173.9$, where $f(x)$ is the amount of methane produced in million metric tons, and x is the number of years after 2000.

(a) According to this model, what will US methane emissions be in 2009? $X = 2009 - 2000 = 9$

$f(9) = -0.072(9)^2 + 1.93(9) + 173.9 = \boxed{185.44}$ Million Metric Tons

(b) In what year will methane emissions in the US be at their maximum? Round to the nearest year.

$X = \frac{-b}{2a} = \frac{-1.93}{2(-0.072)} = 13.40$

the year will be $2000 + 13 = \boxed{2013}$

(c) What will be that maximum amount of methane emissions?

Plug in $X = 13.4$ in the $f(x) = -0.072X^2 + 1.93X + 173.9$

$f(13.4) = \boxed{186.8 \text{ Million Metric Tons}}$

(d) What is a graphing window on your calculator that displays the vertex and all intercepts for this function?

Xmin = −50

Xmax = 120

Xscl = 1

Ymin = −50 ; Ymax = 200

Plug in $Y_1 = -0.072X^2 + 1.93X + 173.9$

then use ZoomFit and adjust WINDOW

$[-50, 120] \times [-50, 200]$

Answers

1. (a) -1100 The company will have a loss of \$1100 if no items are manufactured and sold.
   (b) 63 items (c) \$2869 (d) $[0, 150] \times [-2000, 3000]$ (e) 27 or 99 items (f) ~~10~~ 9, 117
2. (a) (i) 99.6 meters (ii) 3.5 seconds (b) about 9 seconds
3. (a) Find the vertex of the parabola: $t = 2.5$ sec., height $= h(2.5) = 105$ ft

   (b) Use the quadratic formula to find the positive t-axis intercept: $t = \dfrac{80 + \sqrt{6720}}{32} \approx 5.06$ sec
4. (a) $A(x) = x(3000 - 2x)$
   (b) $x = 191$ ft., width = 2618 ft; or $x = 1309$ ft., width = 382 ft
   (c) $x = 750$ ft., width = 1500 ft (d) 1125000 square feet
5. (a) 2036 (b) 3,290,250
6. (a) 185.44 million metric tons (b) 2013
   (c) 186.82 million metric tons (d) $[-50, 120] \times [-50, 200]$