Graphs of the Sine and Cosine Functions

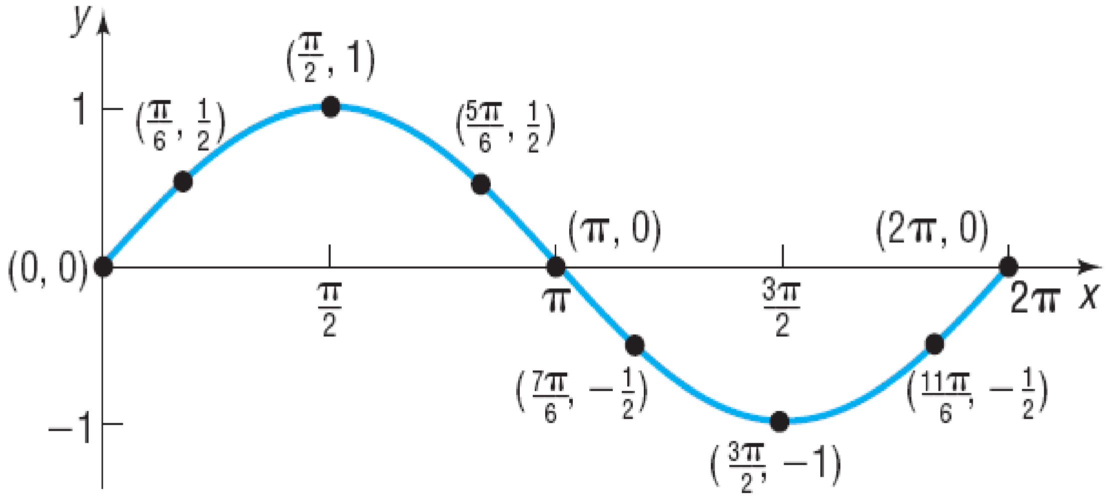
$$y = f(x) = \sin x \qquad y = f(x) = \cos x \qquad y = f(x) = \tan x$$

$$y = f(x) = \csc x \qquad y = f(x) = \sec x \qquad y = f(x) = \cot x$$

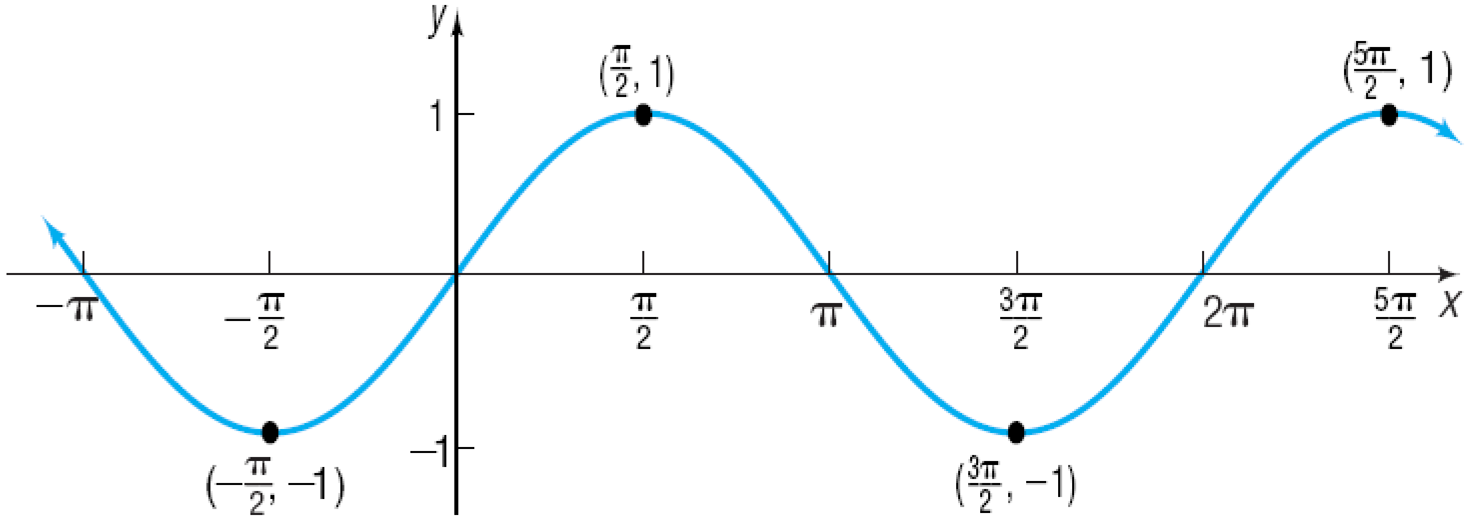
OBJECTIVE 1

1 Graph Transformations of the Sine Function

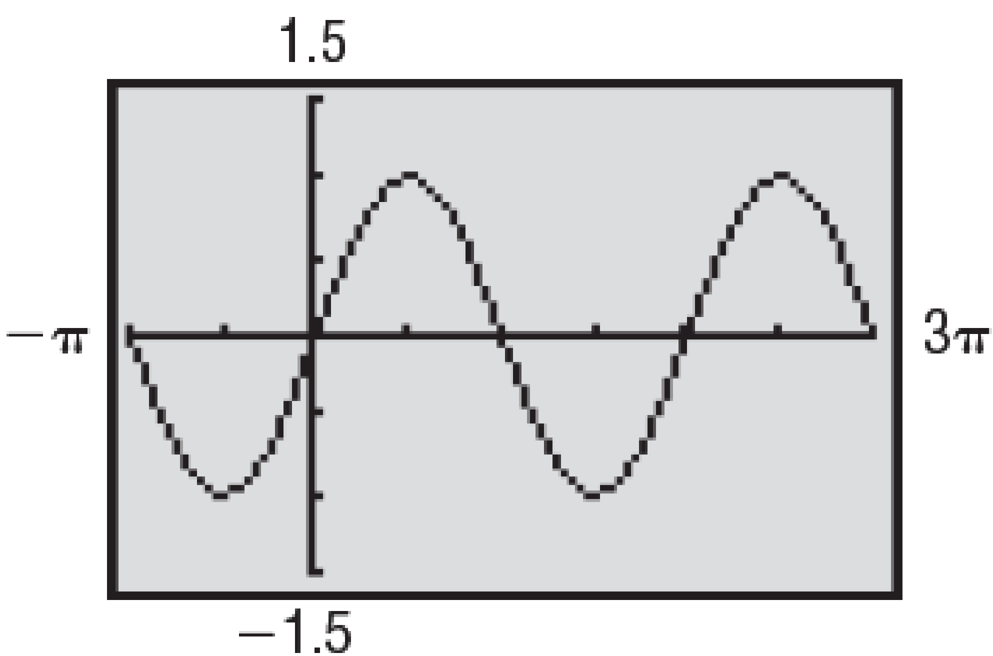
x	y = sin x	(x, y)
0	0	$(0, 0)$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\left(\frac{\pi}{6}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	1	$\left(\frac{\pi}{2}, 1\right)$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$
π	0	$(\pi, 0)$
$\frac{7\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	-1	$\left(\frac{3\pi}{2}, -1\right)$
$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$
2π	0	$(2\pi, 0)$



$y = \sin x, 0 \leq x \leq 2\pi$



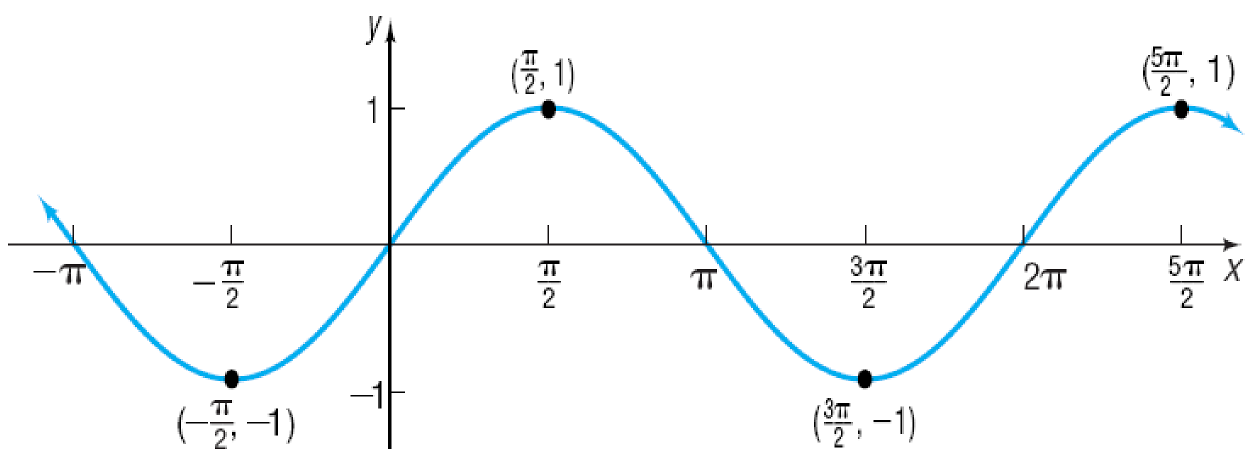
$$y = \sin x, -\infty < x < \infty$$



Properties of the Sine Function

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The sine function is an odd function, as the symmetry of the graph with respect to the origin indicates.
4. The sine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$; the y-intercept is 0.
6. The maximum value is 1 and occurs at $x = \ldots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$;

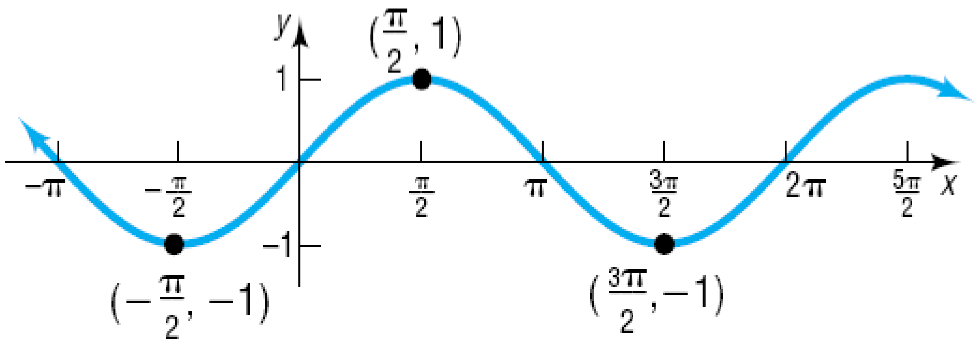
   the minimum value is -1 and occurs at $x = \ldots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$.



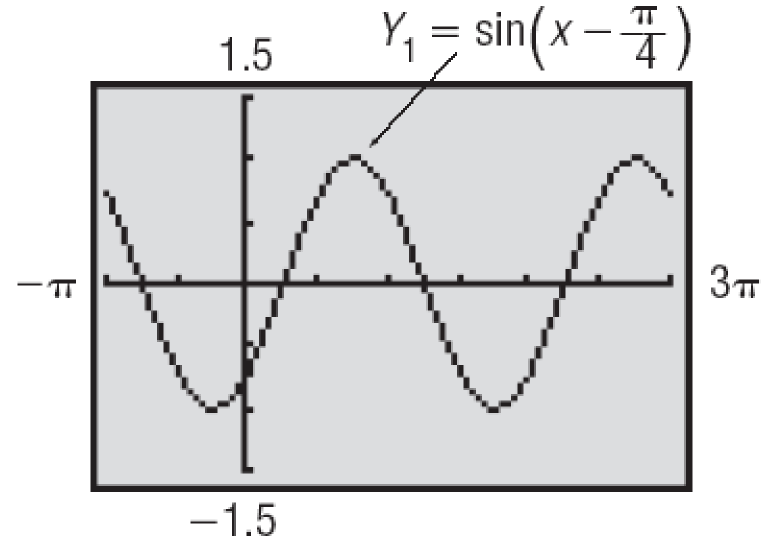
EXAMPLE

Graphing Variations of $y = \sin x$ Using Transformations

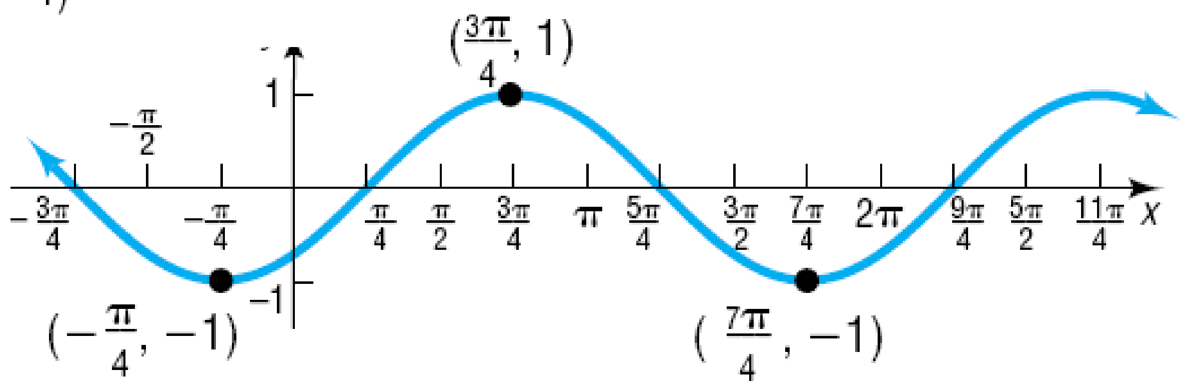
Use the graph of $y = \sin x$ to graph $y = \sin\left(x - \frac{\pi}{4}\right)$.



$y = \sin x$



Replace x by $x - \frac{\pi}{4}$; horizontal shift to the right $\frac{\pi}{4}$ units.

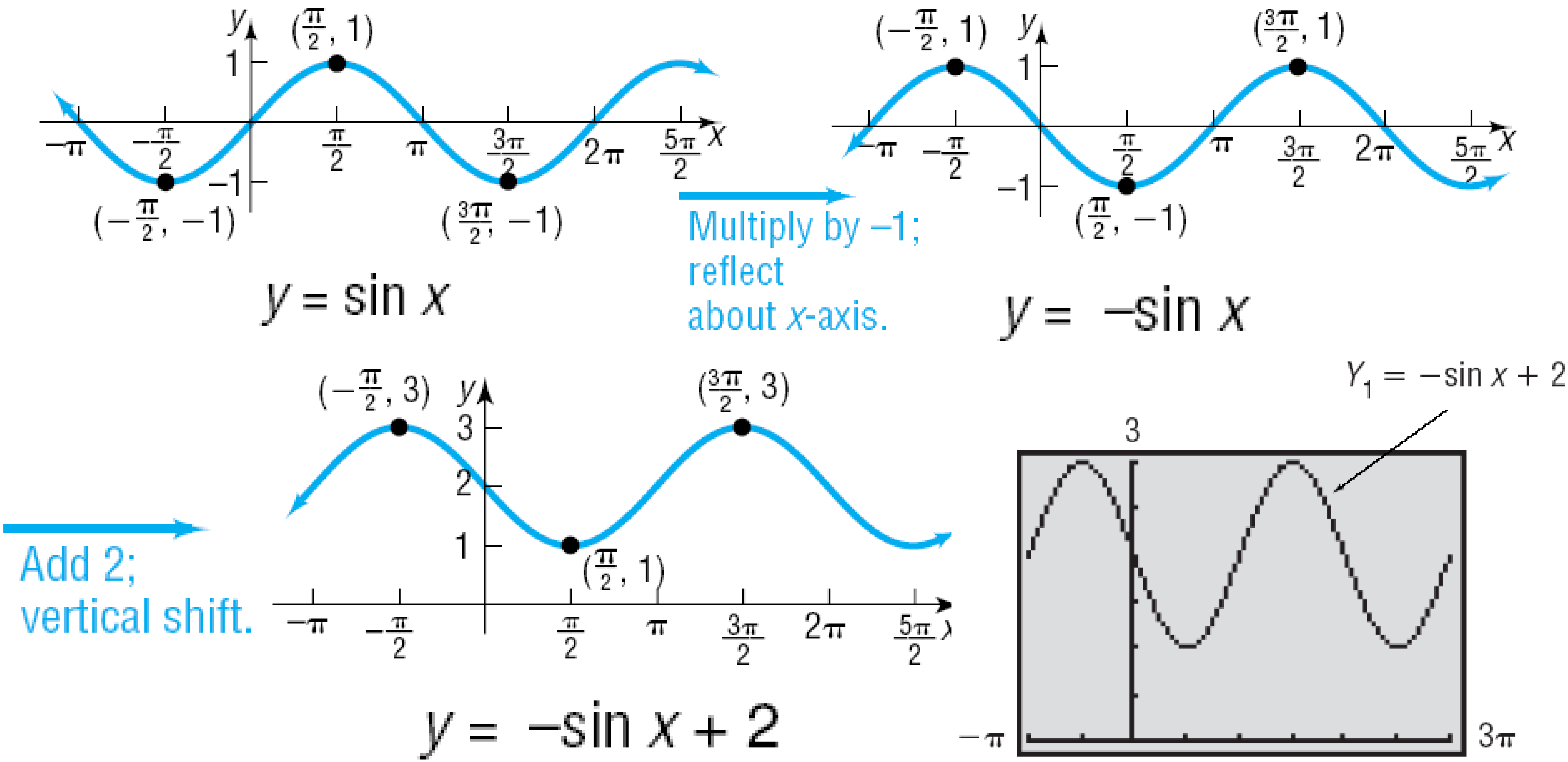


$y = \sin\left(x - \frac{\pi}{4}\right)$

EXAMPLE

Graphing Variations of $y = \sin x$ Using Transformations

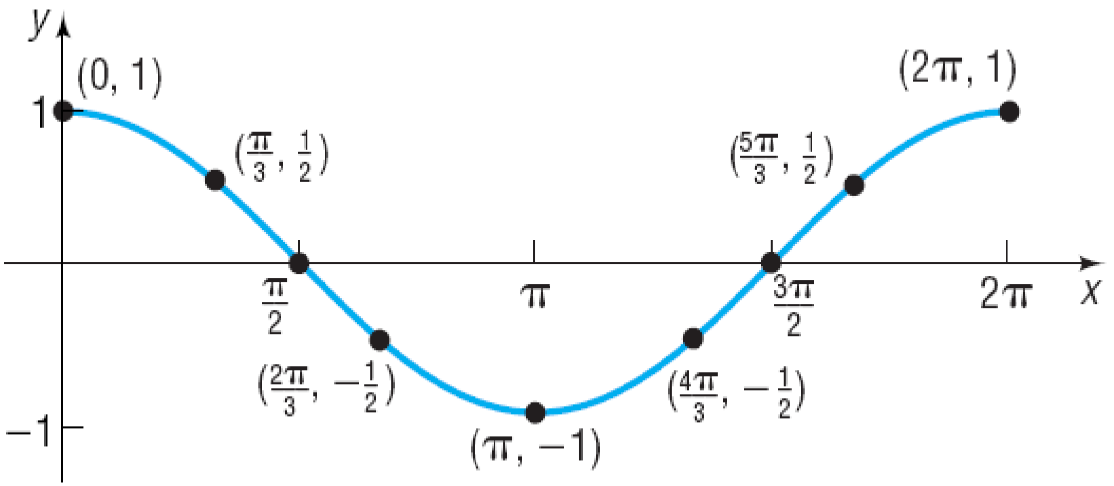
Use the graph of $y = \sin x$ to graph $y = -\sin x + 2$.



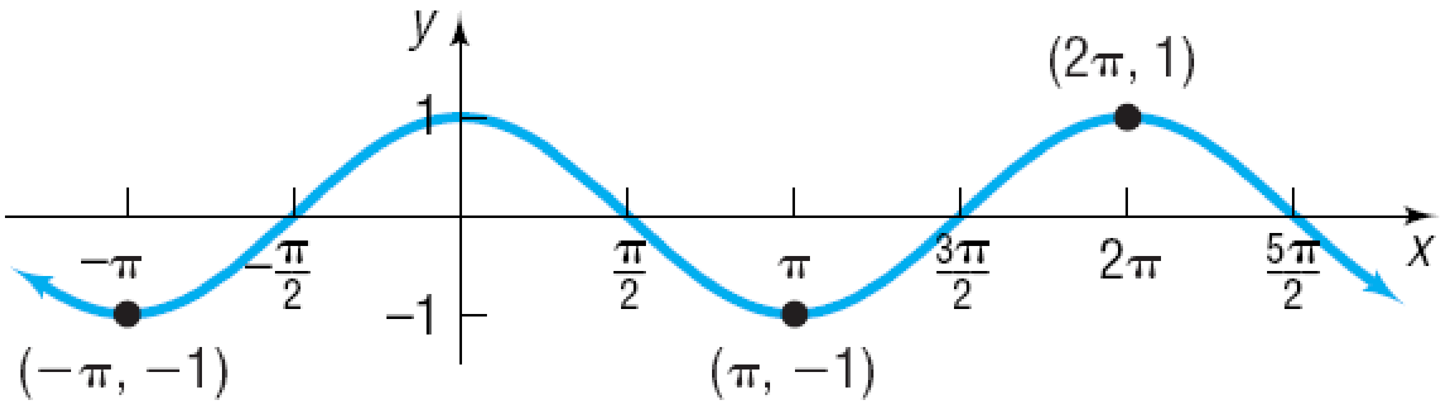
OBJECTIVE 2

2 Graph Transformations of the Cosine Function

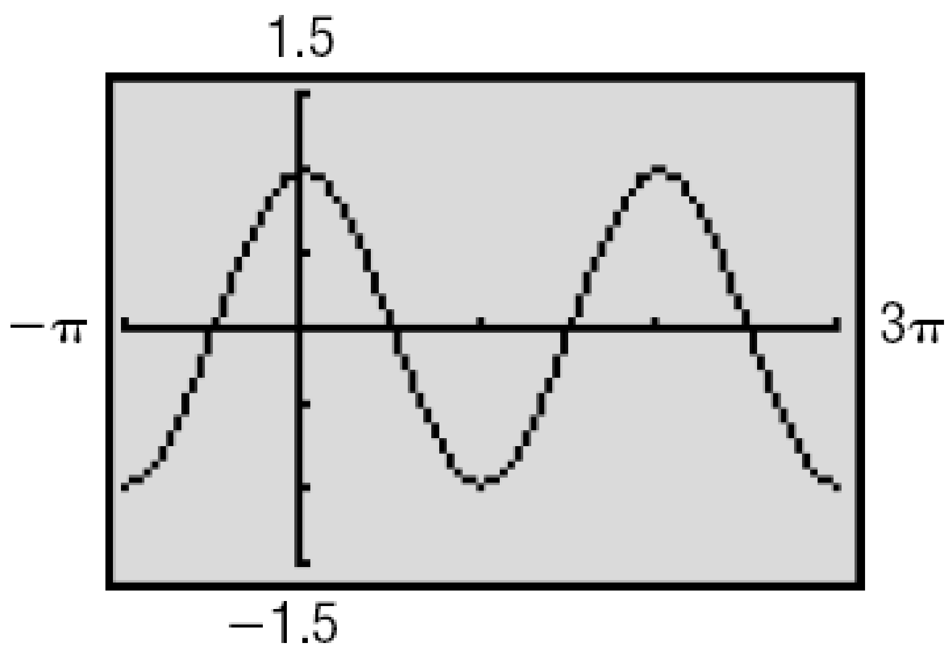
x	y = cos x	(x, y)
0	1	$(0, 1)$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\left(\frac{\pi}{3}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	0	$\left(\frac{\pi}{2}, 0\right)$
$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{2\pi}{3}, -\frac{1}{2}\right)$
π	-1	$(\pi, -1)$
$\frac{4\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{4\pi}{3}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	0	$\left(\frac{3\pi}{2}, 0\right)$
$\frac{5\pi}{3}$	$\frac{1}{2}$	$\left(\frac{5\pi}{3}, \frac{1}{2}\right)$
2π	1	$(2\pi, 1)$



$y = \cos x, 0 \leq x \leq 2\pi$

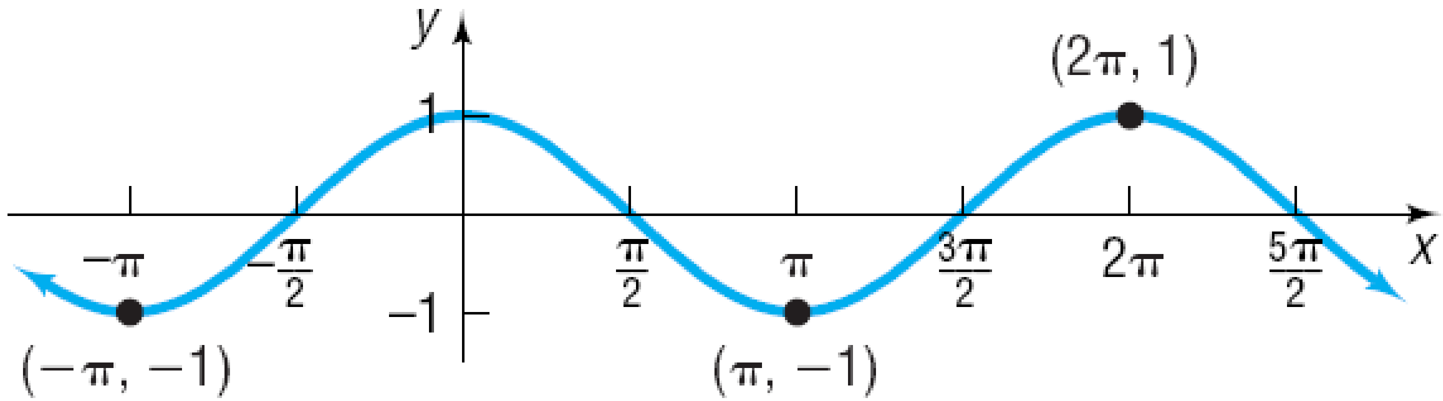


$$y = \cos x, -\infty < x < \infty$$



Properties of the Cosine Function

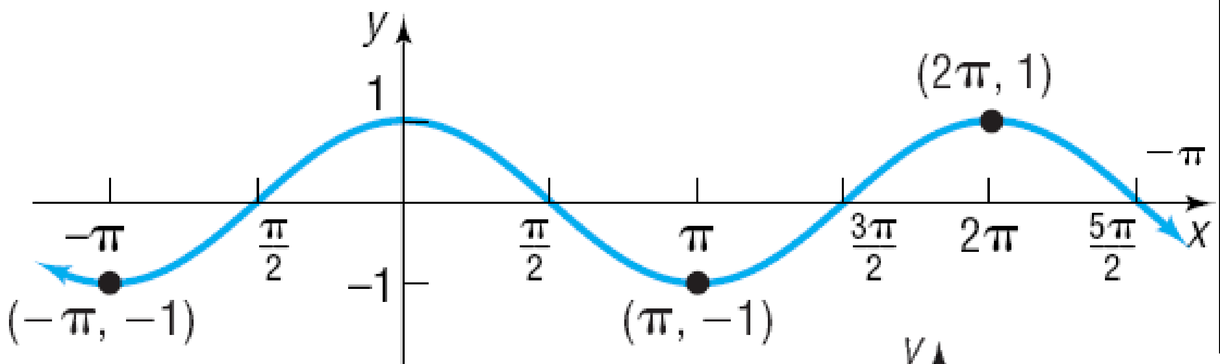
1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The cosine function is an even function, as the symmetry of the graph with respect to the y-axis indicates.
4. The cosine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$; the y-intercept is 1.
6. The maximum value is 1 and occurs at $x = \ldots, -2\pi, 0, 2\pi, 4\pi, 6\pi, \ldots$; the minimum value is -1 and occurs at $x = \ldots, -\pi, \pi, 3\pi, 5\pi, \ldots$.

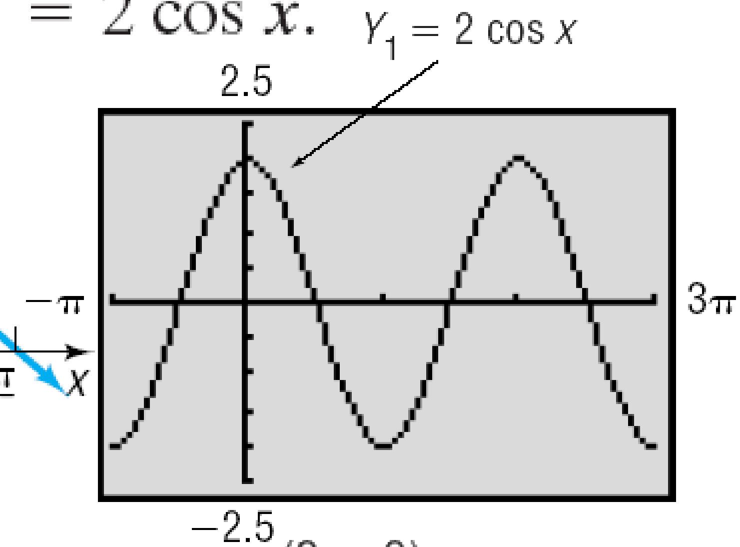


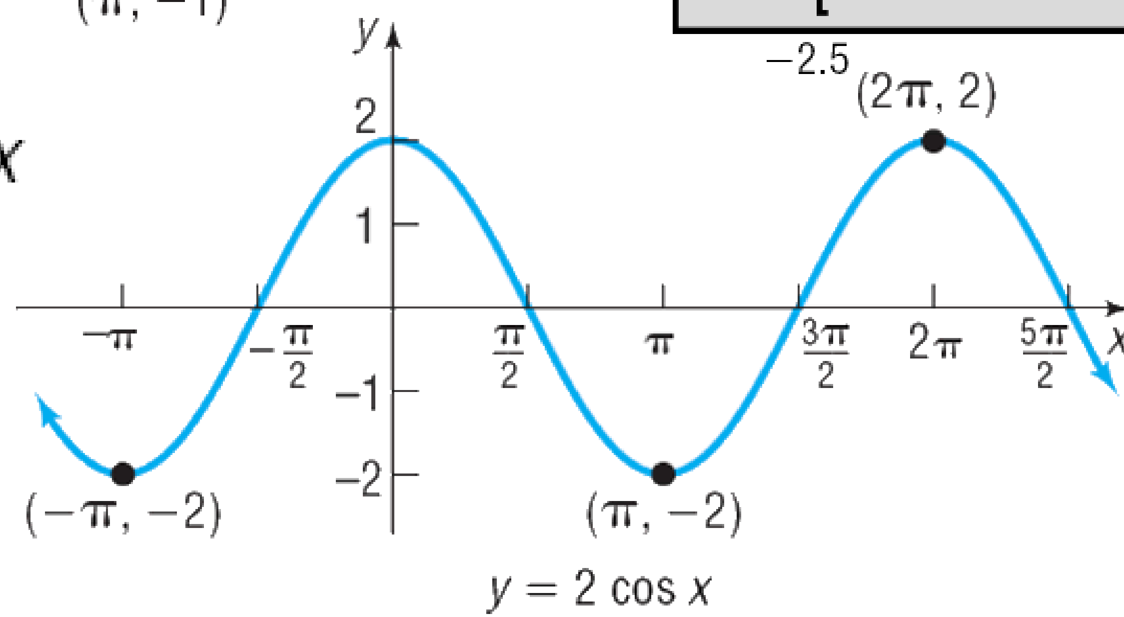
EXAMPLE

Graphing Variations of y = cos x Using Transformations

Use the graph of $y = \cos x$ to graph $y = 2 \cos x$.



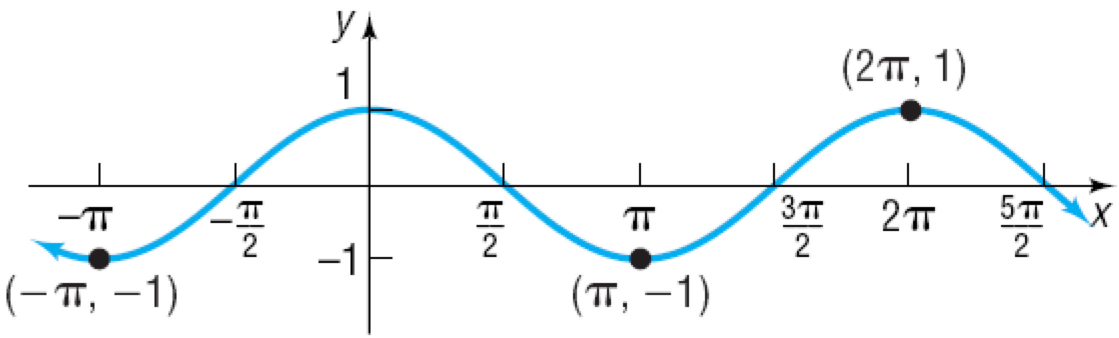




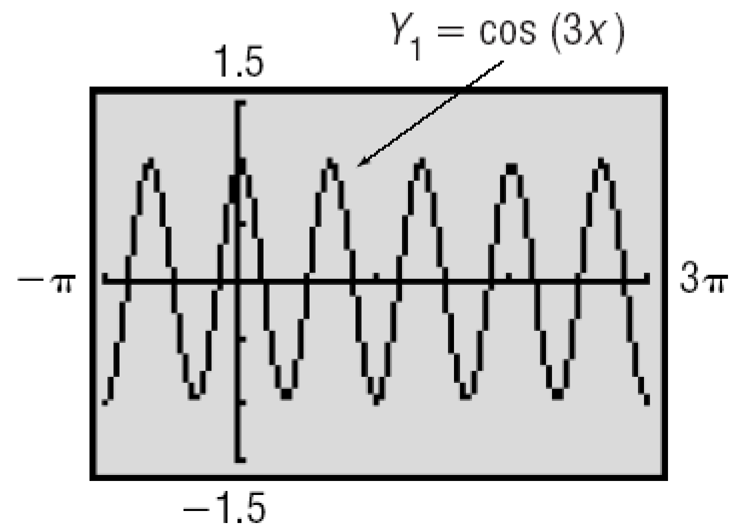
EXAMPLE

Graphing Variations of $y = \cos x$ Using Transformations

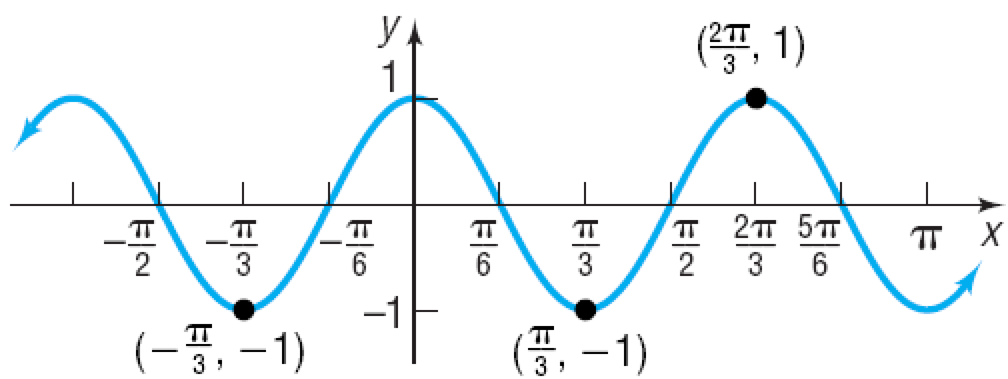
Use the graph of $y = \cos x$ to graph $y = \cos(3x)$.



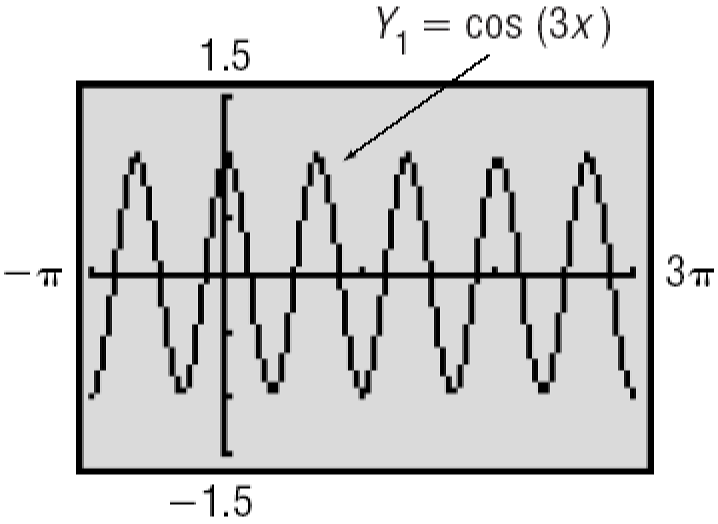
$y = \cos x$

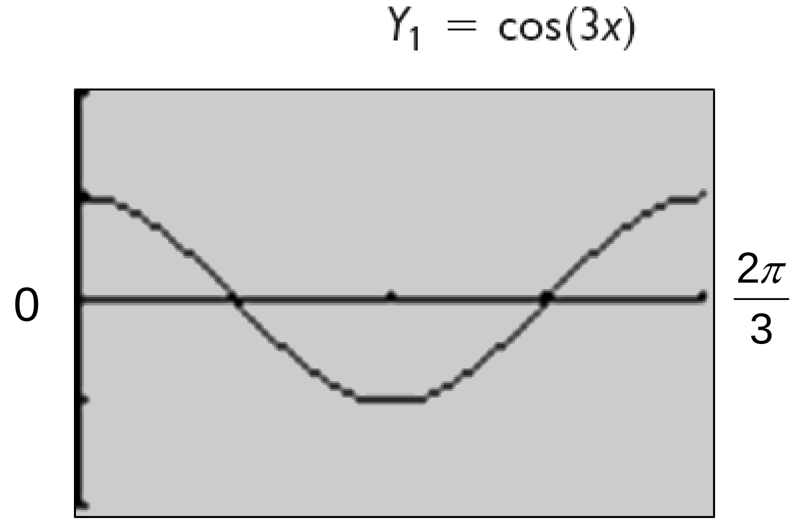


Replace x by $3x$; horizontal compression by a factor of $\frac{1}{3}$.



$y = \cos(3x)$

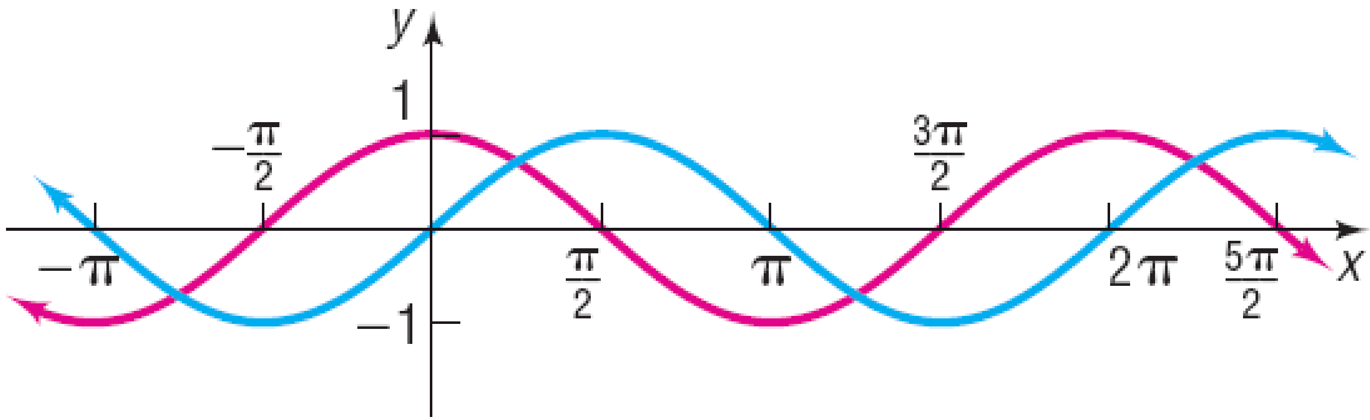




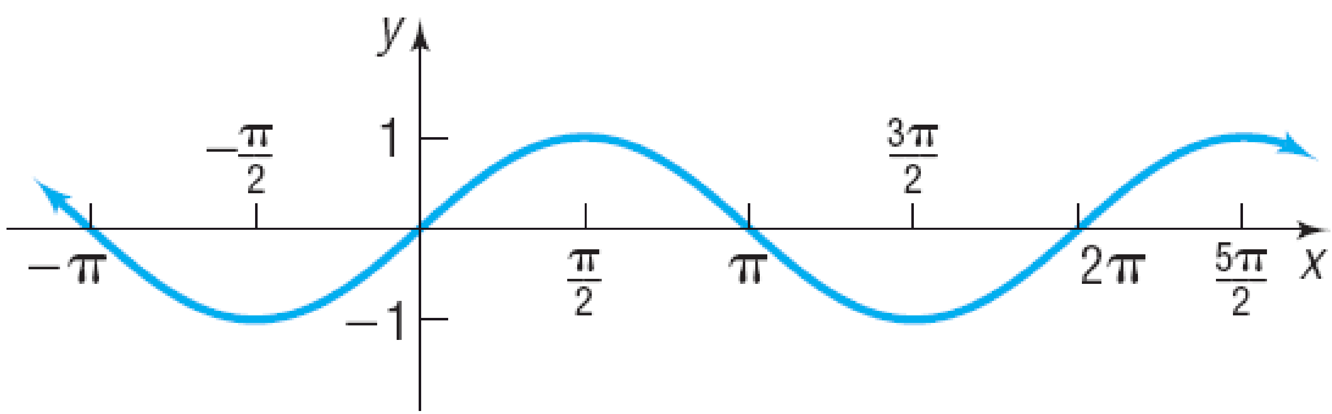
— Seeing the Concept —

Graph $Y_1 = \cos(3x)$ with Xmin $= 0$, Xmax $= \frac{2\pi}{3}$, and Xscl $= \frac{\pi}{6}$ to verify that the period is $\frac{2\pi}{3}$.

Sinusoidal Graphs



(a) $y = \cos x$ $y = \cos\left(x - \frac{\pi}{2}\right)$



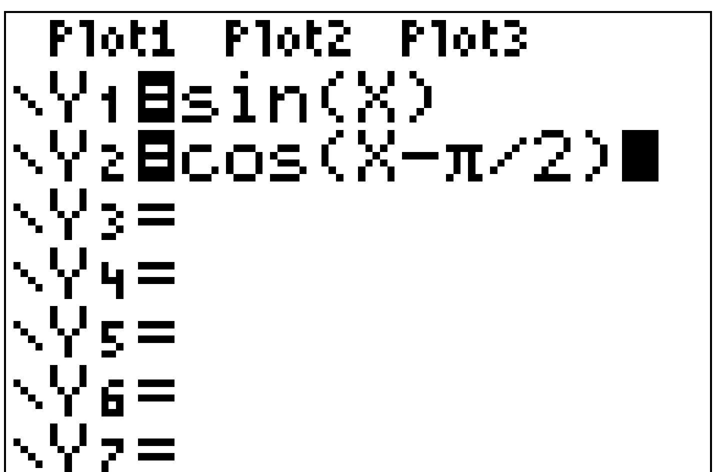
(b) $y = \sin x$

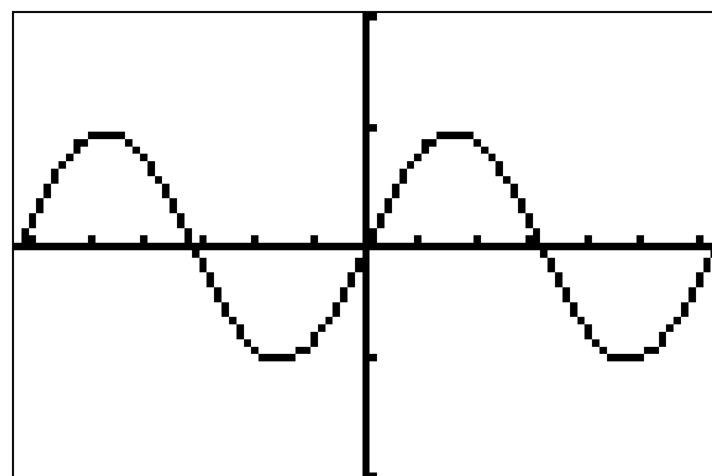
$$\sin x = \cos\left(x - \frac{\pi}{2}\right)$$

— Seeing the Concept —

Graph $Y_1 = \sin x$ and $Y_2 = \cos\left(x - \frac{\pi}{2}\right)$.

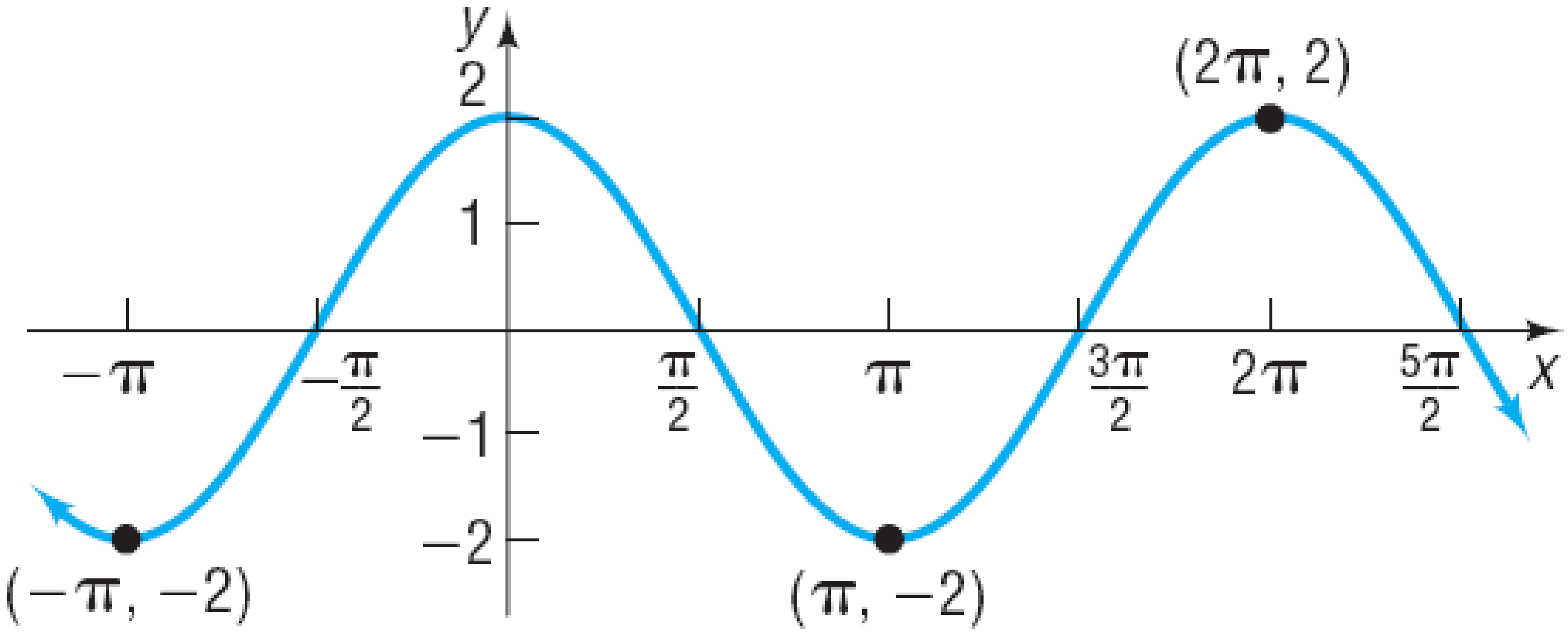
How many graphs do you see?



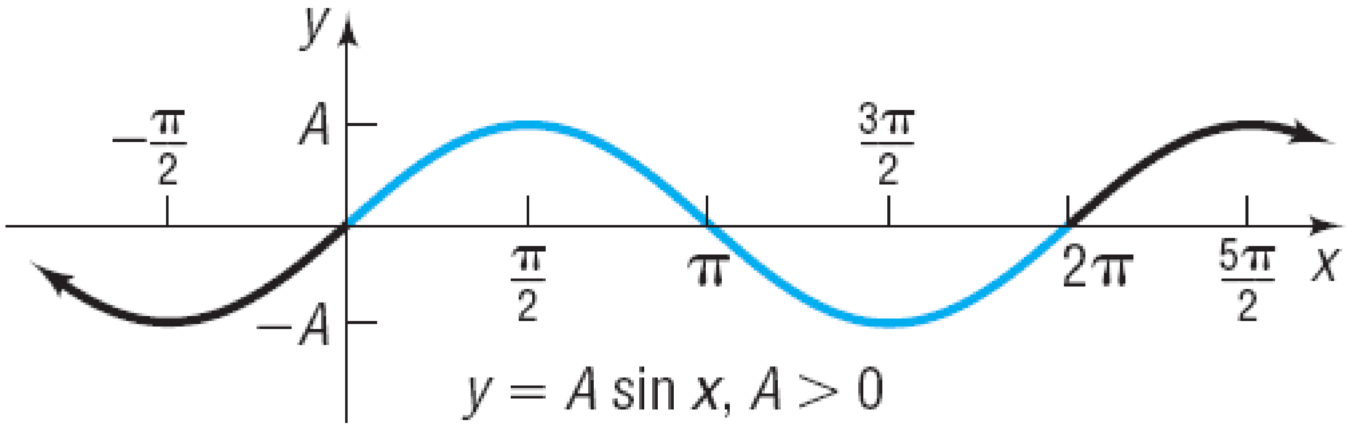


OBJECTIVE 3

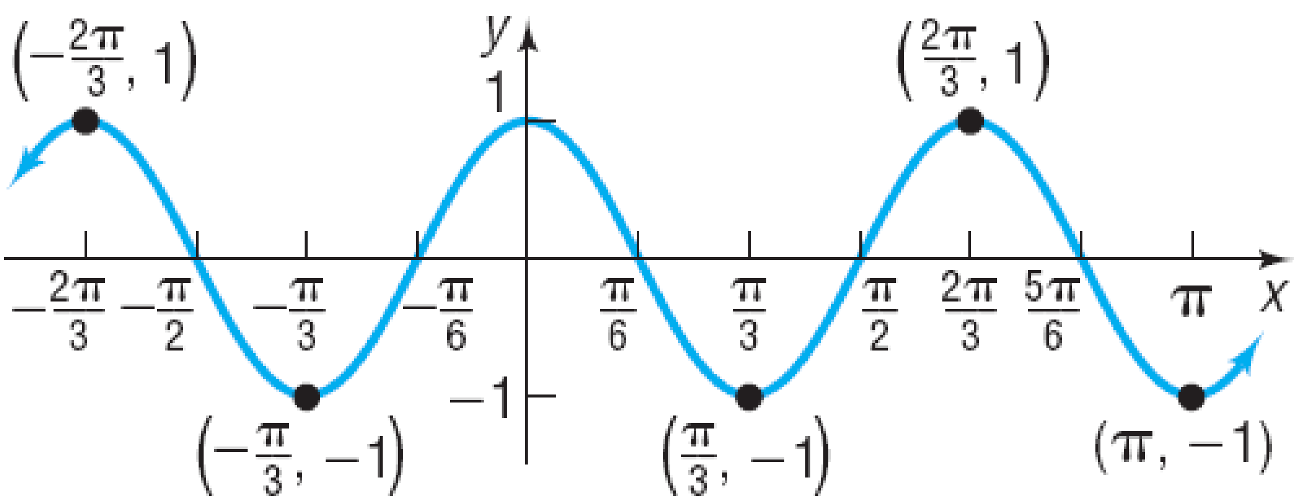
3 **Determine the Amplitude and Period of Sinusoidal Functions**



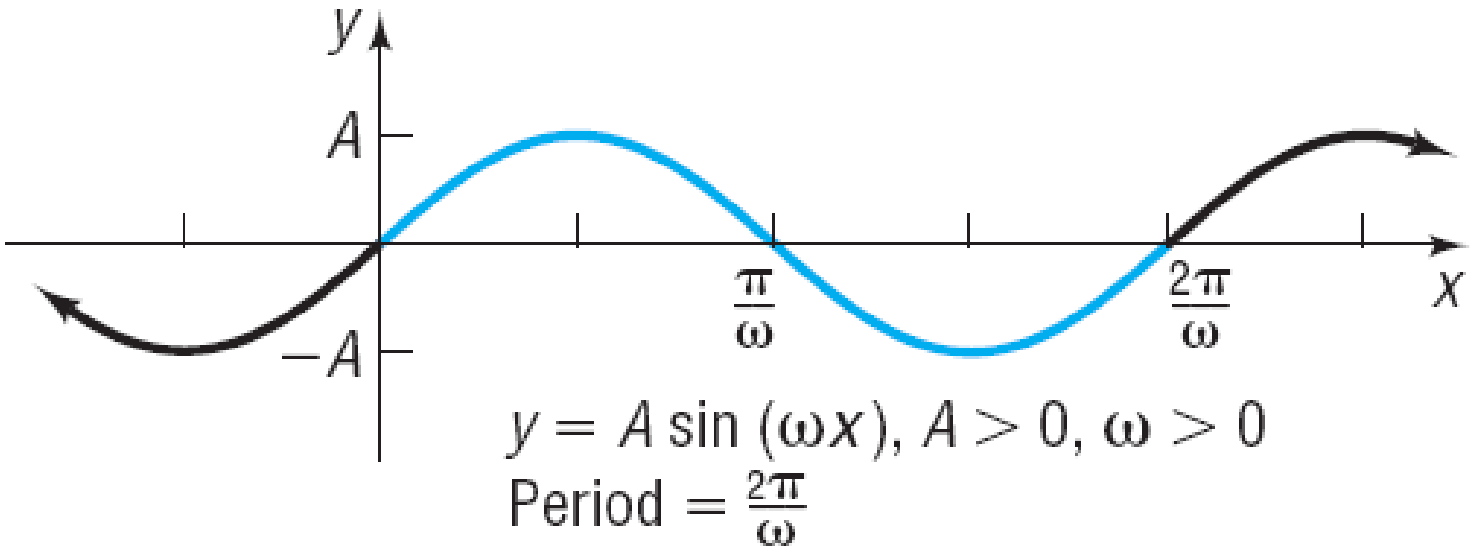
$y = 2\cos x$



$y = A \sin x,\ A > 0$
Period $= 2\pi$



$y = \cos(3x)$



Theorem

If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are

$$\text{Amplitude} = |A| \qquad \text{Period} = T = \frac{2\pi}{\omega}$$

EXAMPLE

Finding the Amplitude and Period of a Sinusoidal Function

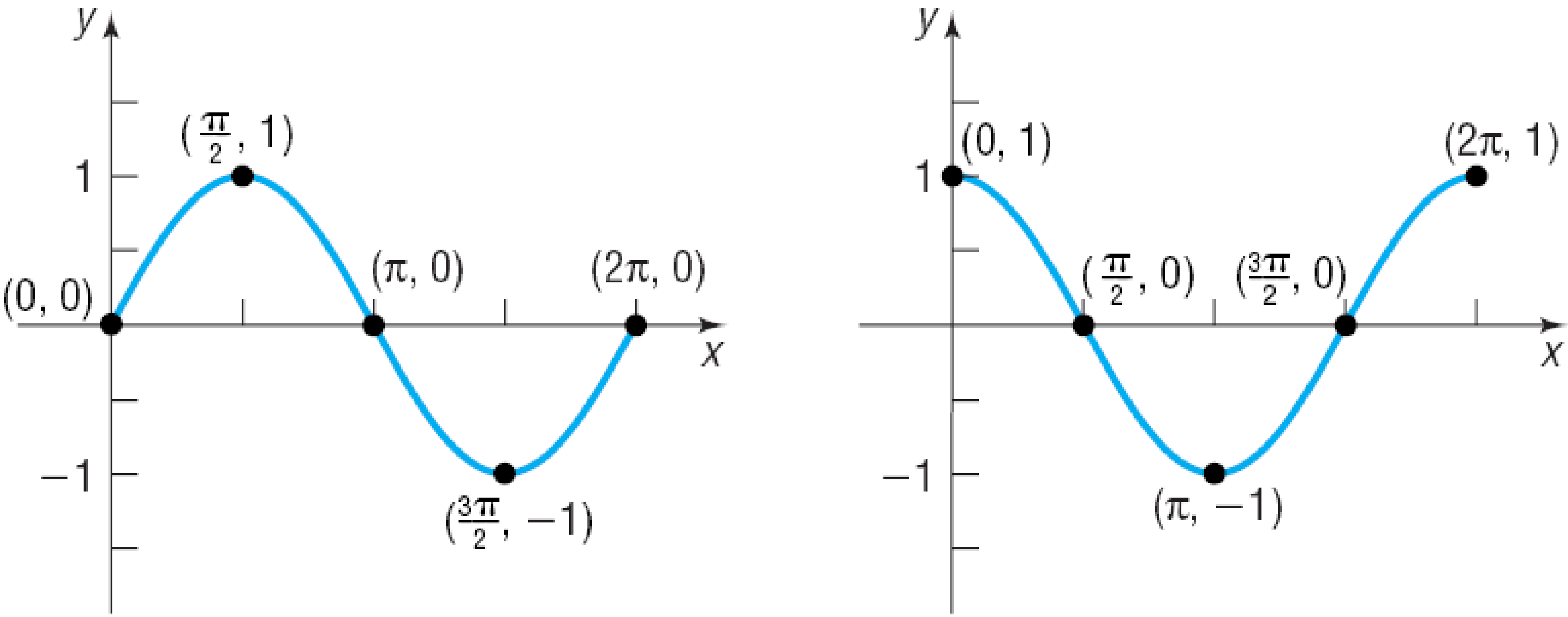
Determine the amplitude and period of $y = -4\cos(3x)$

If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are

$$\text{Amplitude} = |A| \qquad \text{Period} = T = \frac{2\pi}{\omega}$$

OBJECTIVE 4

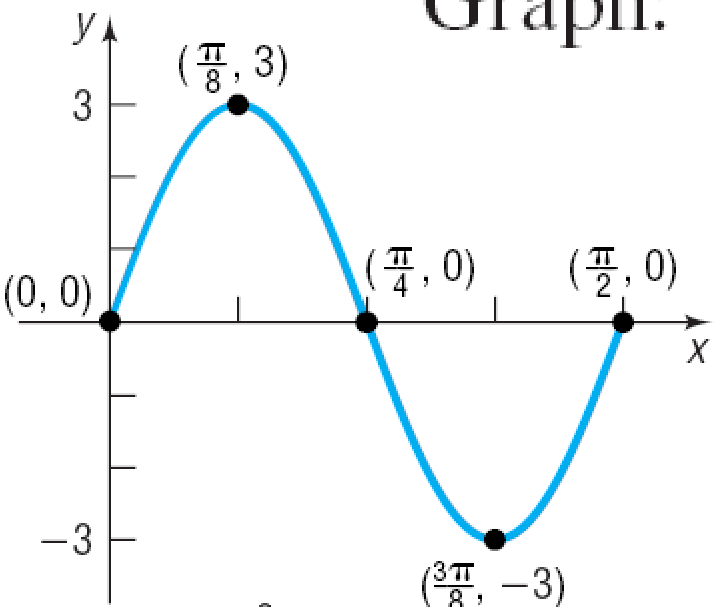
4 Graph Sinusoidal Functions Using Key Points

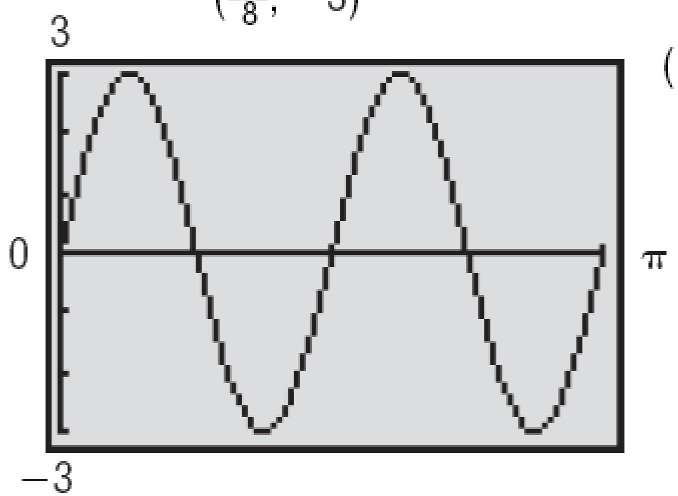

y
(π/2, 1)
1
(0, 0)
(π, 0)
(2π, 0)
x
−1
(3π/2, −1)
y
(0, 1)
(2π, 1)
1
(π/2, 0)
(3π/2, 0)
x
−1
(π, −1)

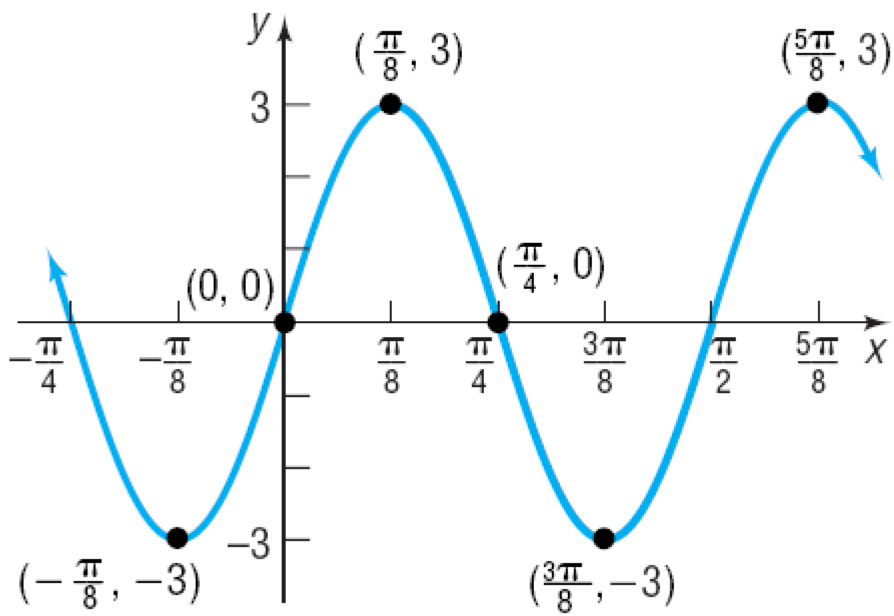
EXAMPLE

Graphing a Sinusoidal Function Using Key Points

Graph: $y = 3\sin(4x)$



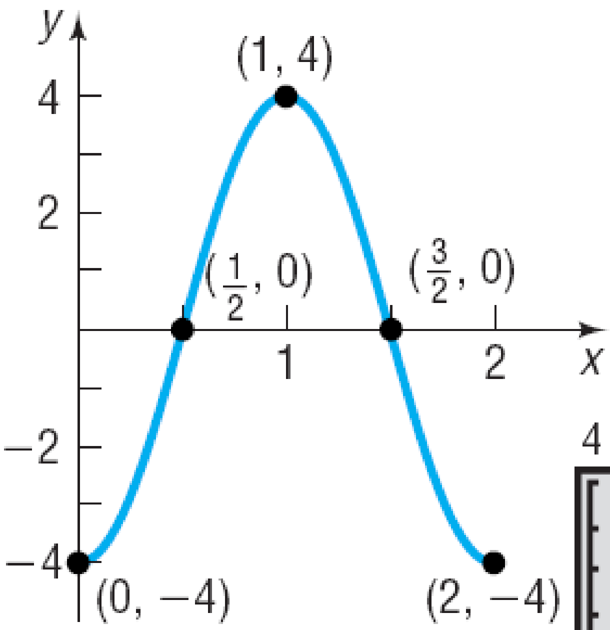


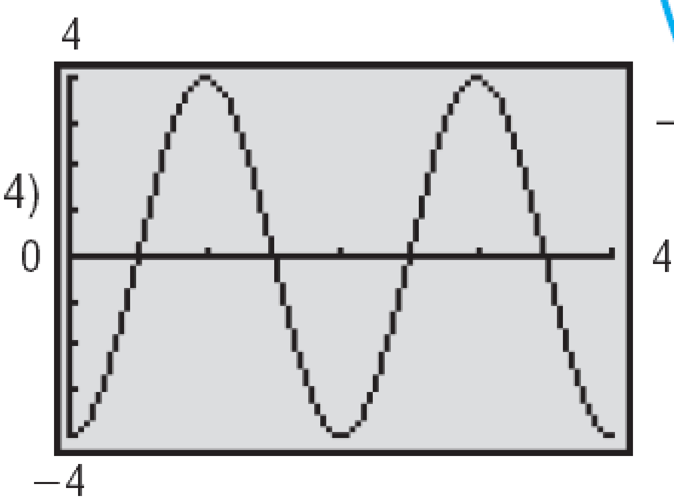


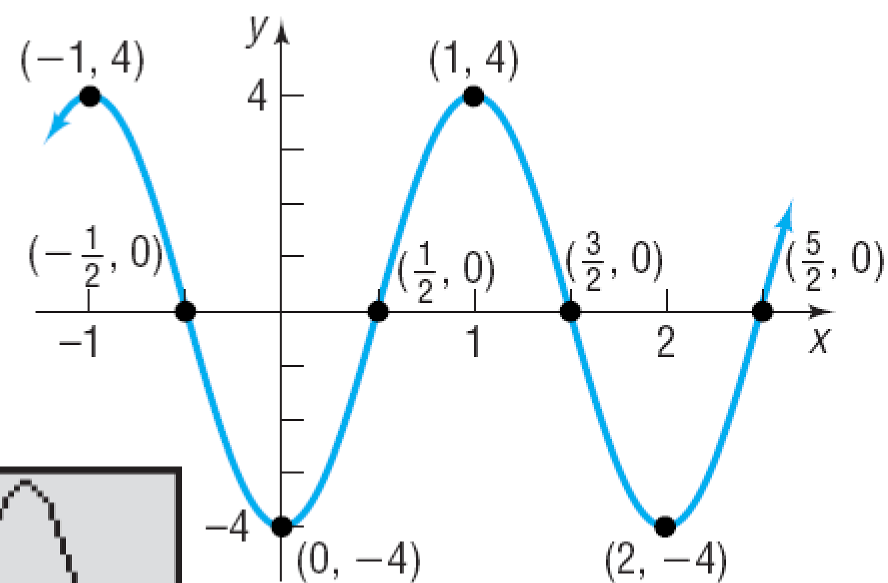
EXAMPLE

Finding the Amplitude and Period of a Sinusoidal Function and Graphing It Using Key Points

Determine the amplitude and period of $y = -4\cos(\pi x)$, and graph the function.



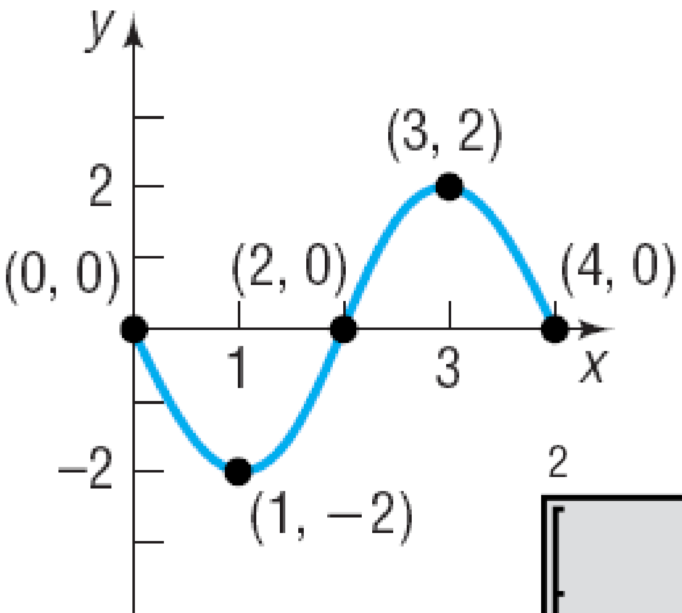


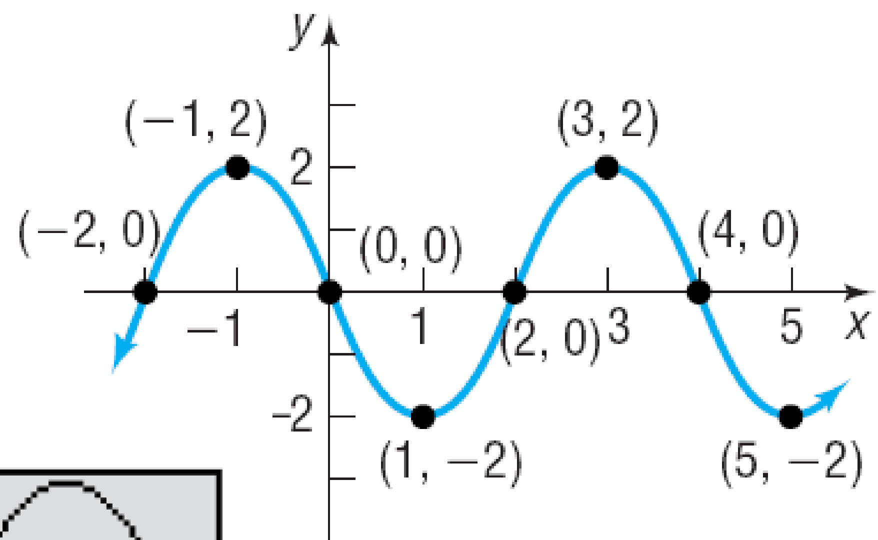


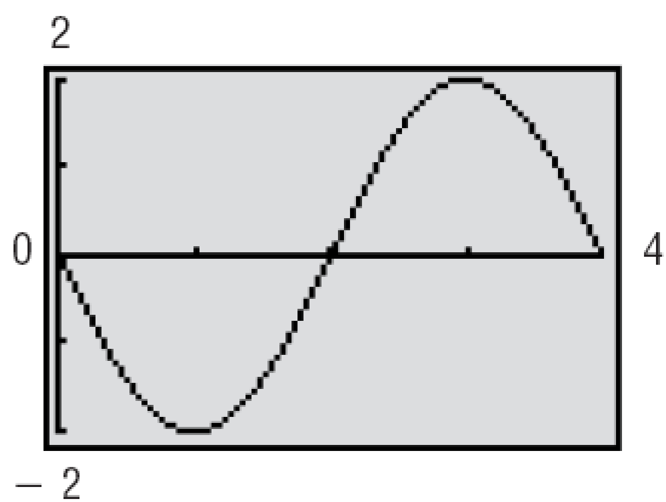
EXAMPLE

Finding the Amplitude and Period of a Sinusoidal Function and Graphing It Using Key Points

Determine the amplitude and period of $y = 2\sin\left(-\frac{\pi}{2}x\right)$, and graph the function.







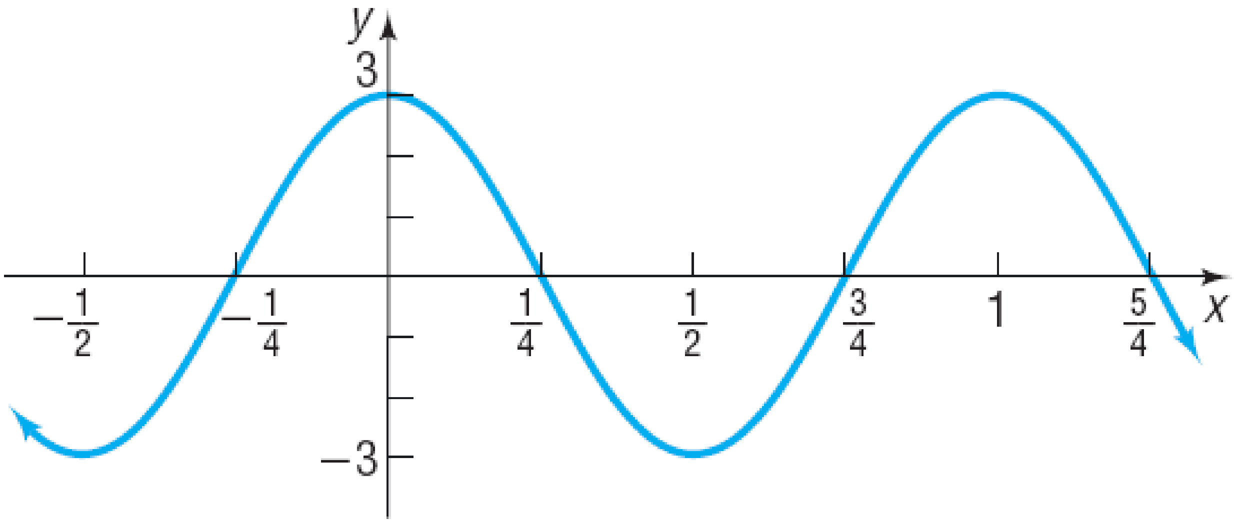
OBJECTIVE 5

5 Find an Equation for a Sinusoidal Graph

EXAMPLE

Finding an Equation for a Sinusoidal Graph

Find an equation for the graph shown



EXAMPLE

Finding an Equation for a Sinusoidal Graph

Find an equation for the graph shown