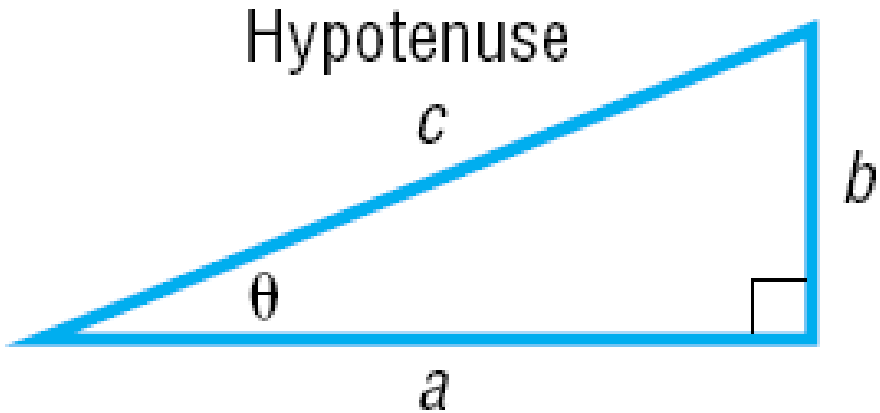
Section 7.1

Right Triangle Trigonometry;

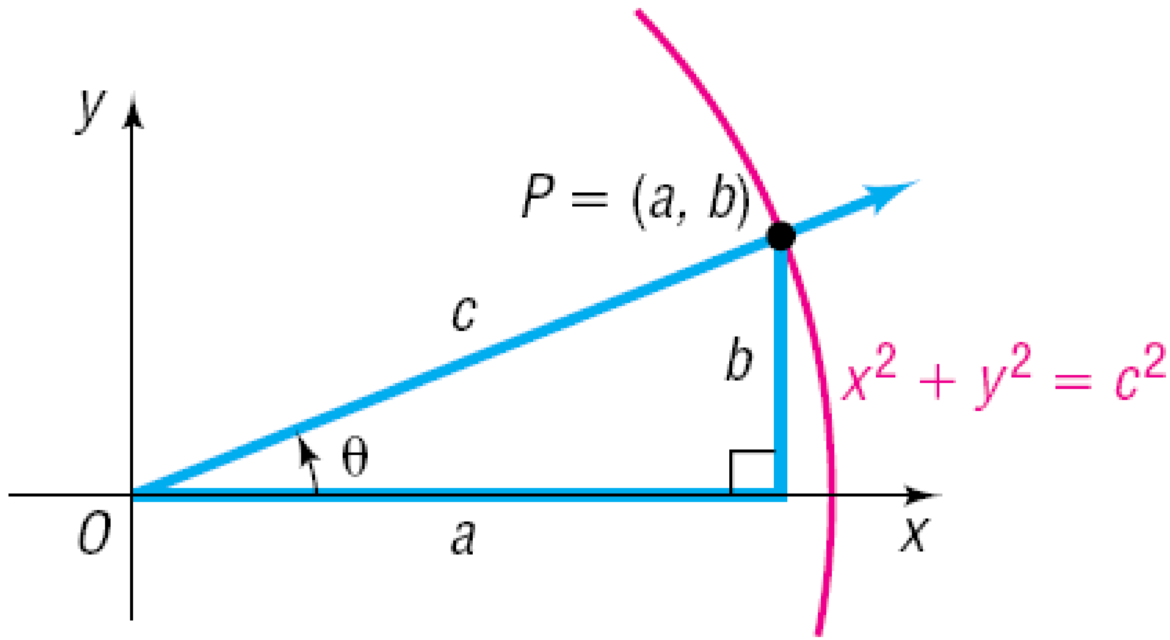
Applications

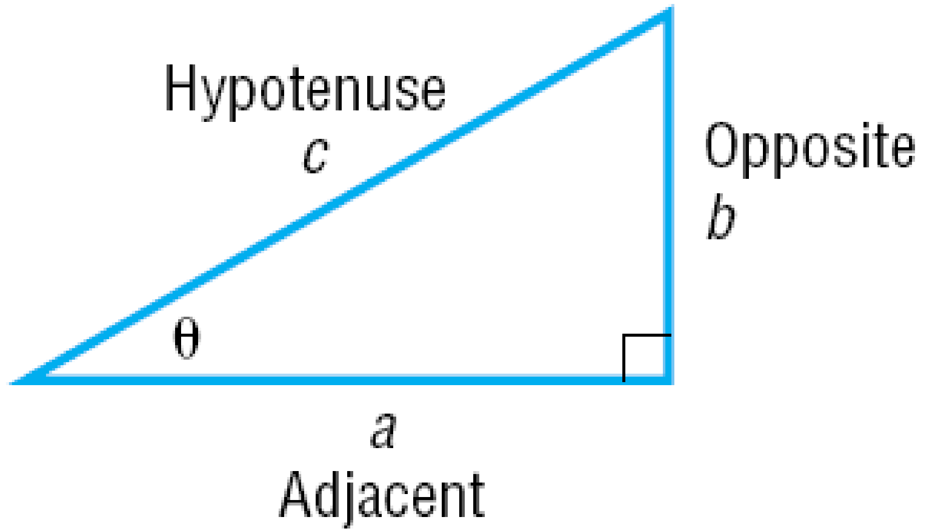
OBJECTIVE 1

1 Find the Value of Trigonometric Functions of Acute Angles



$$a^2 + b^2 = c^2$$





$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{b}{c} \qquad \csc\theta = \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{c}{b}$$

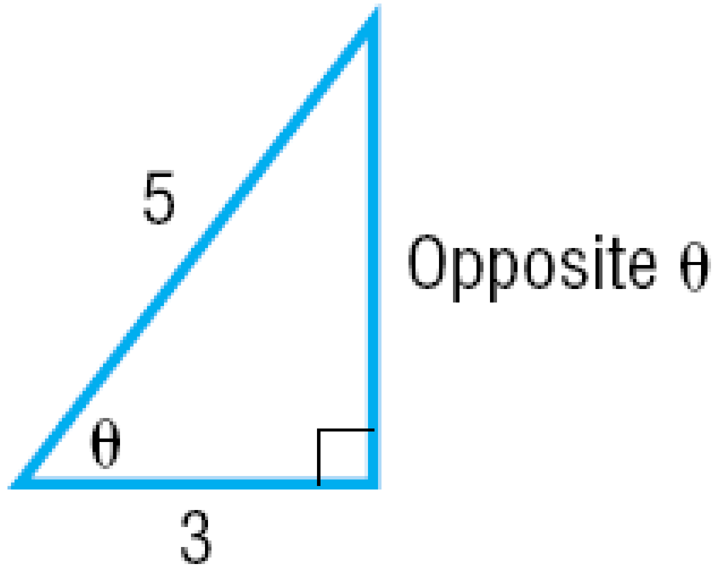
$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{a}{c} \qquad \sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{c}{a}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{b}{a} \qquad \cot\theta = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{a}{b}$$

EXAMPLE

Finding the Value of Trigonometric Functions from a Right Triangle

Find the value of each of the six trigonometric functions of the angle θ.



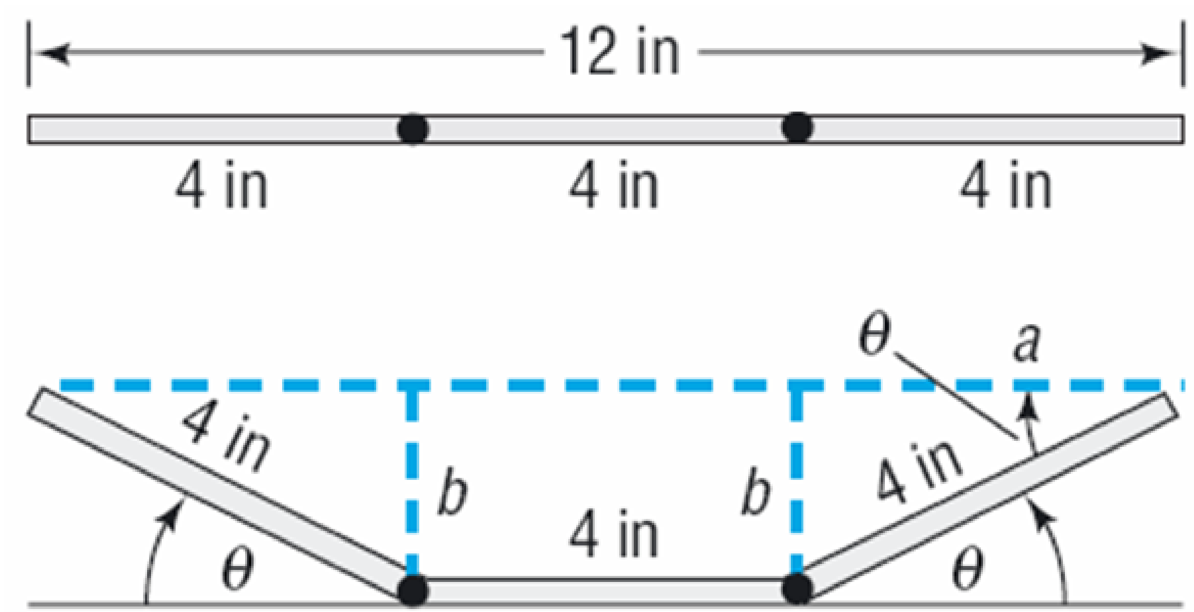
EXAMPLE Constructing a Rain Gutter

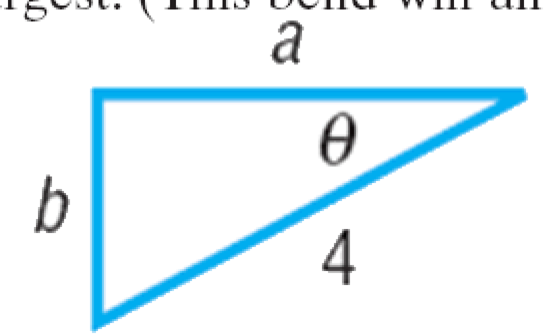
A rain gutter is to be constructed of aluminum sheets 12 inches wide. After marking off a length of 4 inches from each edge, this length is bent up at an angle θ. See Figure 34.

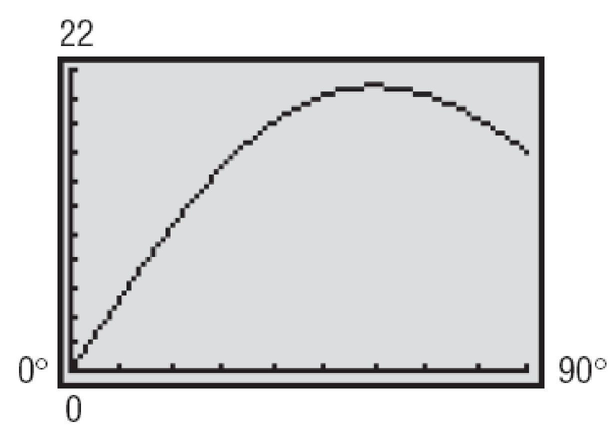
(a) Express the area A of the opening as a function of θ.
[**Hint:** Let b denote the vertical height of the bend.]

(b) Find the area A of the opening for $\theta = 30°$, $\theta = 45°$, $\theta = 60°$, and $\theta = 75°$.

(c) Graph $A = A(\theta)$. Find the angle θ that makes A largest. (This bend will allow the most water to flow through the gutter.)

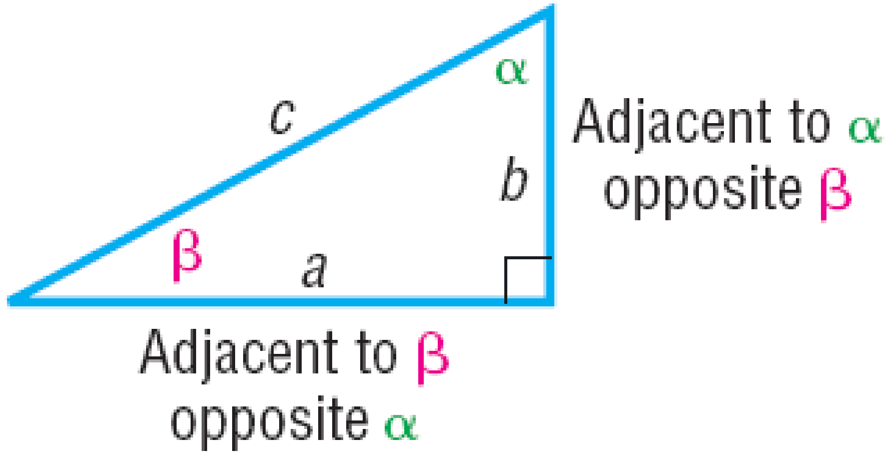






OBJECTIVE 2

2 Use the Complementary Angle Theorem



$$\sin\beta = \frac{b}{c} = \cos\alpha \qquad \cos\beta = \frac{a}{c} = \sin\alpha \qquad \tan\beta = \frac{b}{a} = \cot\alpha$$

$$\csc\beta = \frac{c}{b} = \sec\alpha \qquad \sec\beta = \frac{c}{a} = \csc\alpha \qquad \cot\beta = \frac{a}{b} = \tan\alpha$$

Complementary Angle Theorem

Cofunctions of complementary angles are equal.

Complementary angles

$$\sin 30^\circ = \cos 60^\circ$$

Cofunctions

Complementary angles

$$\tan 40^\circ = \cot 50^\circ$$

Cofunctions

Complementary angles

$$\sec 80^\circ = \csc 10^\circ$$

Cofunctions

EXAMPLE

Using the Complementary Angle Theorem

(a) $\sin 62^\circ = \cos(90^\circ - 62^\circ) = \cos 28^\circ$

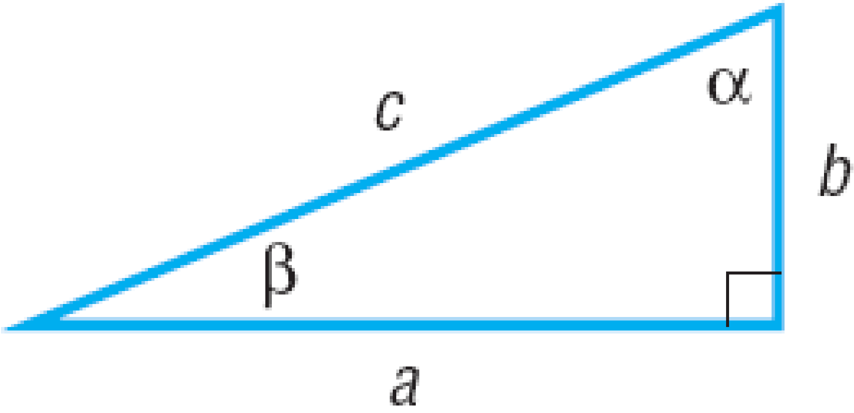
(b) $\tan \frac{\pi}{12} = \cot\left(\frac{\pi}{2} - \frac{\pi}{12}\right) = \cot \frac{5\pi}{12}$

(c) $\cos \frac{\pi}{4} = \sin\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \sin \frac{\pi}{4}$

(d) $\csc \frac{\pi}{6} = \sec\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \sec \frac{\pi}{3}$

OBJECTIVE 3

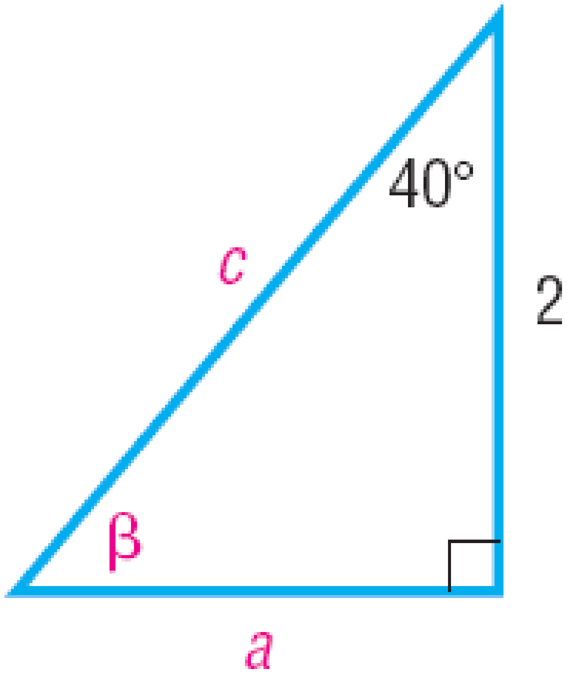
3 Solve Right Triangles



$$c^2 = a^2 + b^2, \qquad \alpha + \beta = 90^\circ$$

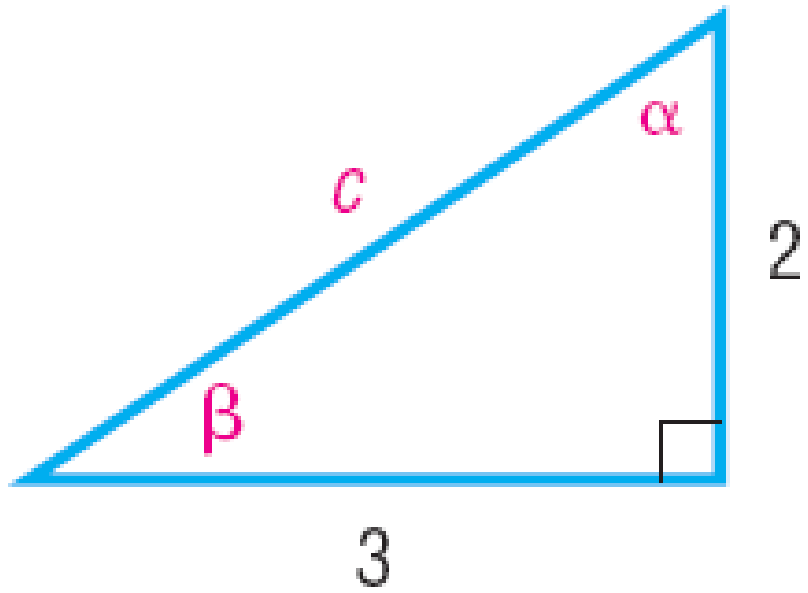
EXAMPLE Solving a Right Triangle

If $b = 2$ and $\alpha = 40°$, find a, c, and β.



EXAMPLE **Solving a Right Triangle**

If $a = 3$ and $b = 2$, find c, α, and β.

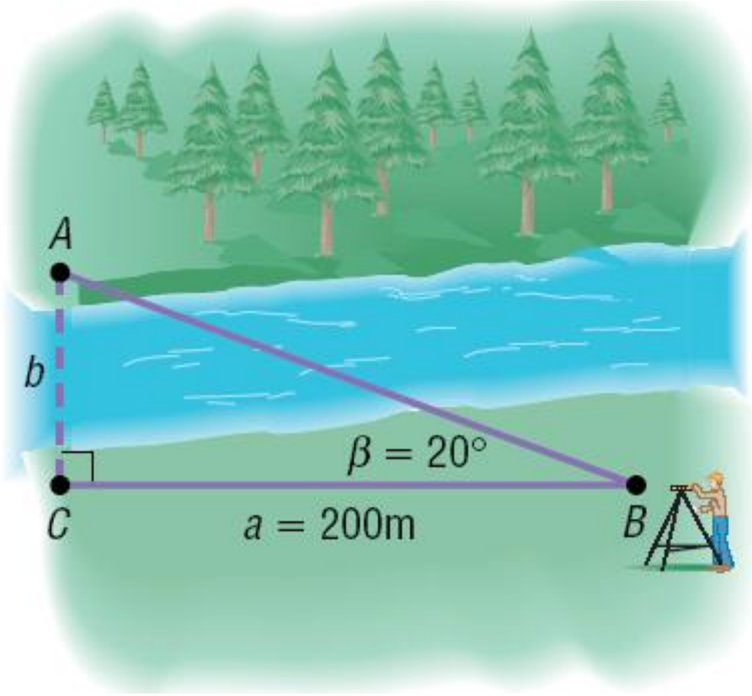


OBJECTIVE 4

4 Solve Applied Problems

EXAMPLE Finding the Width of a River

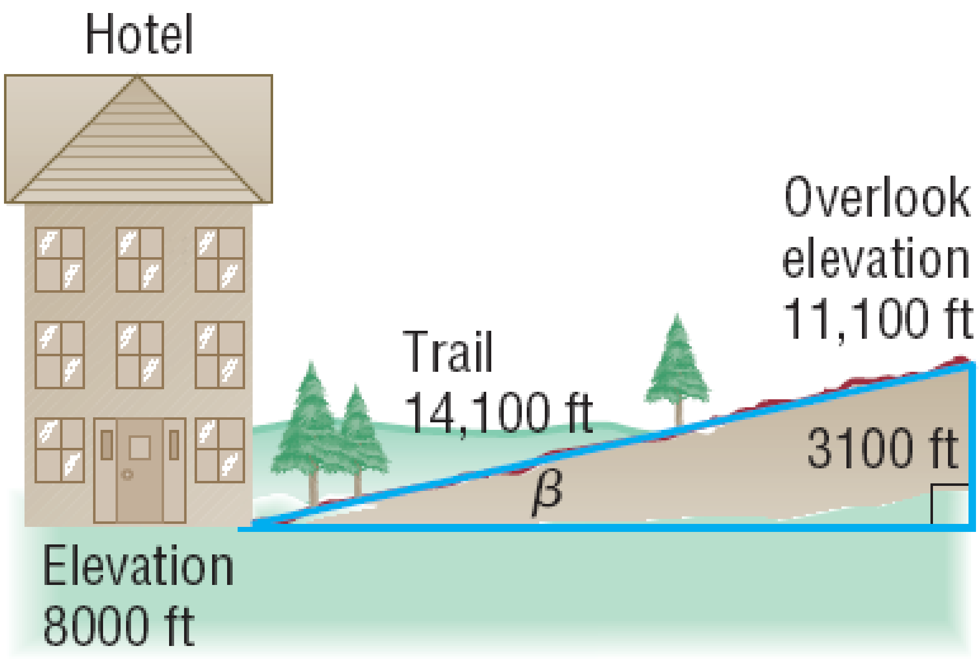
A surveyor can measure the width of a river by setting up a transit* at a point C on one side of the river and taking a sighting of a point A on the other side. Refer to Figure 10. After turning through an angle of 90° at C, the surveyor walks a distance of 200 meters to point B. Using the transit at B, the angle β is measured and found to be 20°. What is the width of the river rounded to the nearest meter?

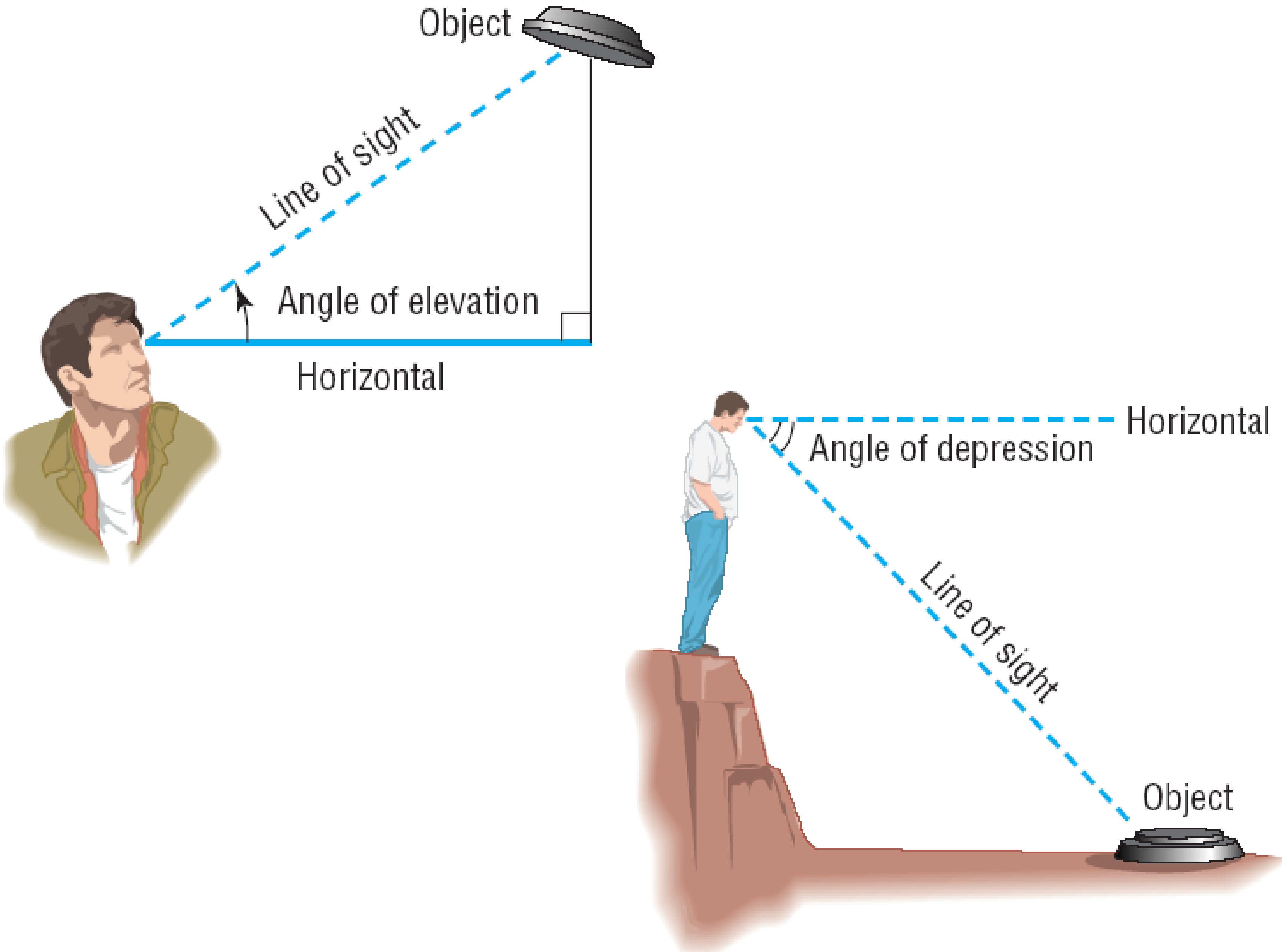


EXAMPLE

Finding the Inclination of a Mountain Trail

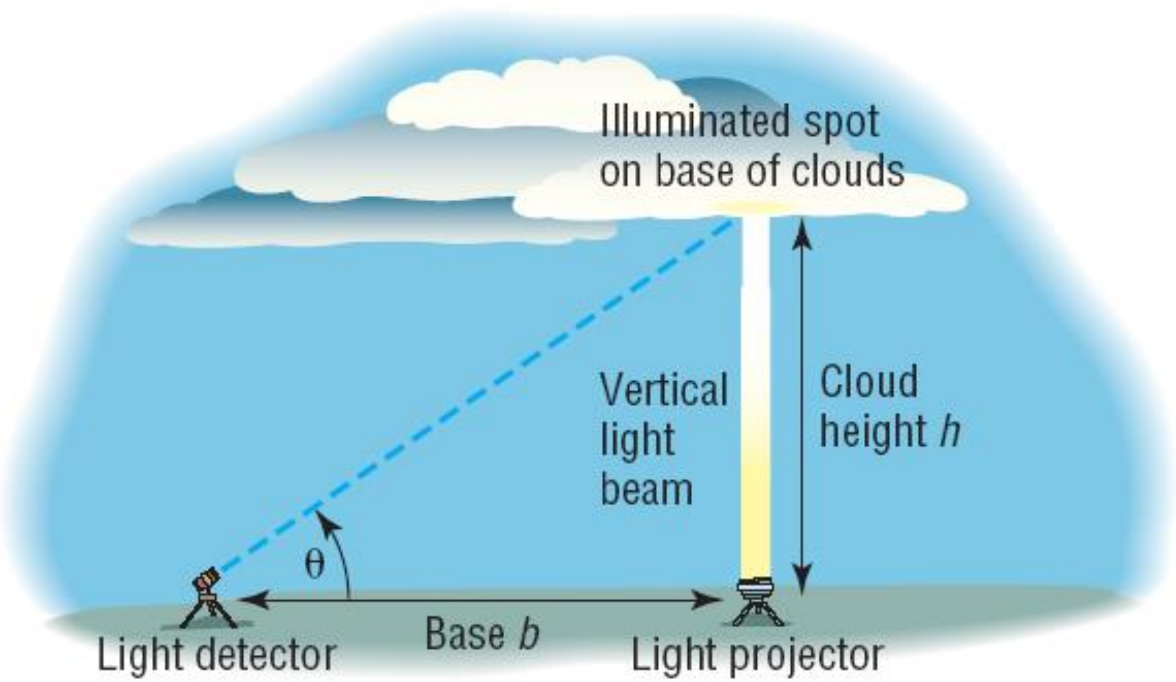
A straight trail leads from the Alpine Hotel, elevation 8000 feet, to a scenic overlook, elevation 11,100 feet. The length of the trail is 14,100 feet. What is the inclination (grade) of the trail? That is, what is the angle β in Figure 11?

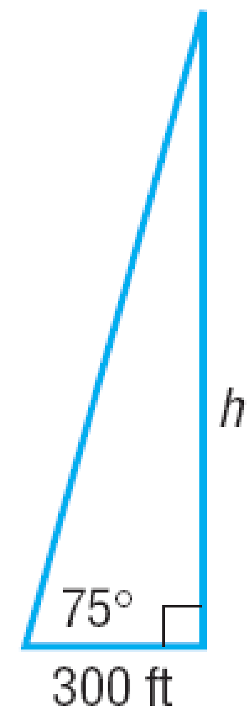



Object
Line of sight
Angle of elevation
Horizontal
Horizontal
Angle of depression
Line of sight
Object

EXAMPLE Finding the Height of a Cloud

Meteorologists find the height of a cloud using an instrument called a **ceilometer.** A ceilometer consists of a **light projector** that directs a vertical light beam up to the cloud base and a **light detector** that scans the cloud to detect the light beam. See Figure 13(a). On December 1, 2004, at Midway Airport in Chicago, a ceilometer with a base of 300 feet was employed to find the height of the cloud cover. If the angle of elevation of the light detector is 75°, what is the height of the cloud cover? Round the answer to the nearest foot.

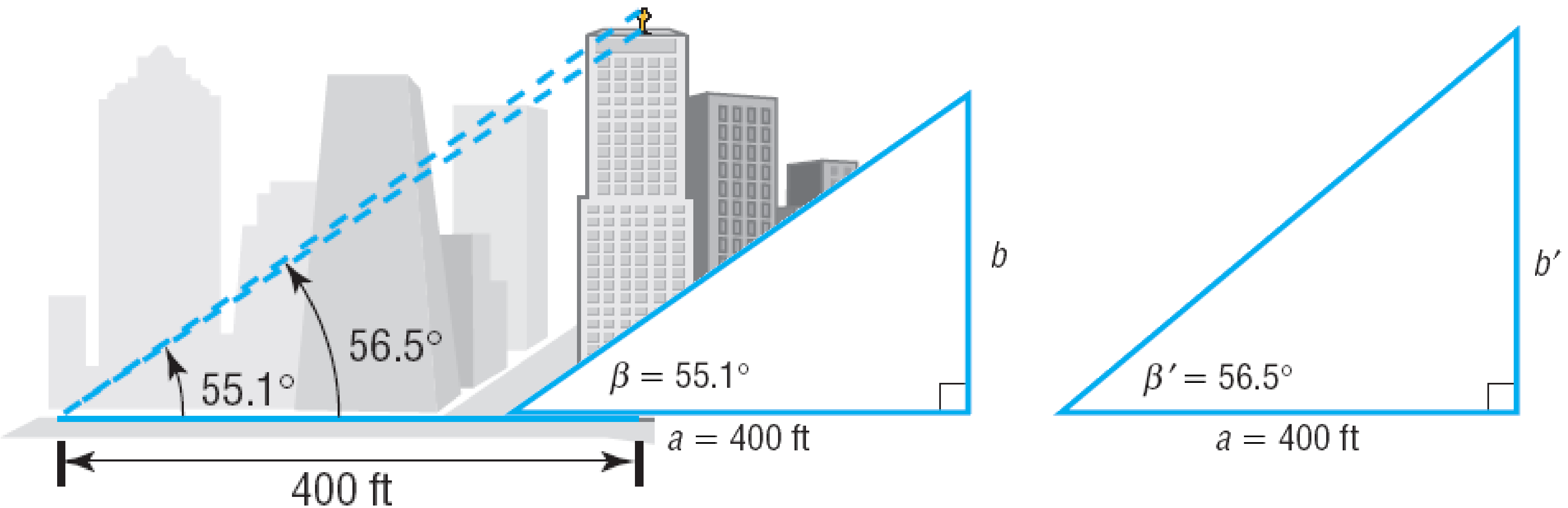




EXAMPLE

Finding the Height of a Statue on a Building

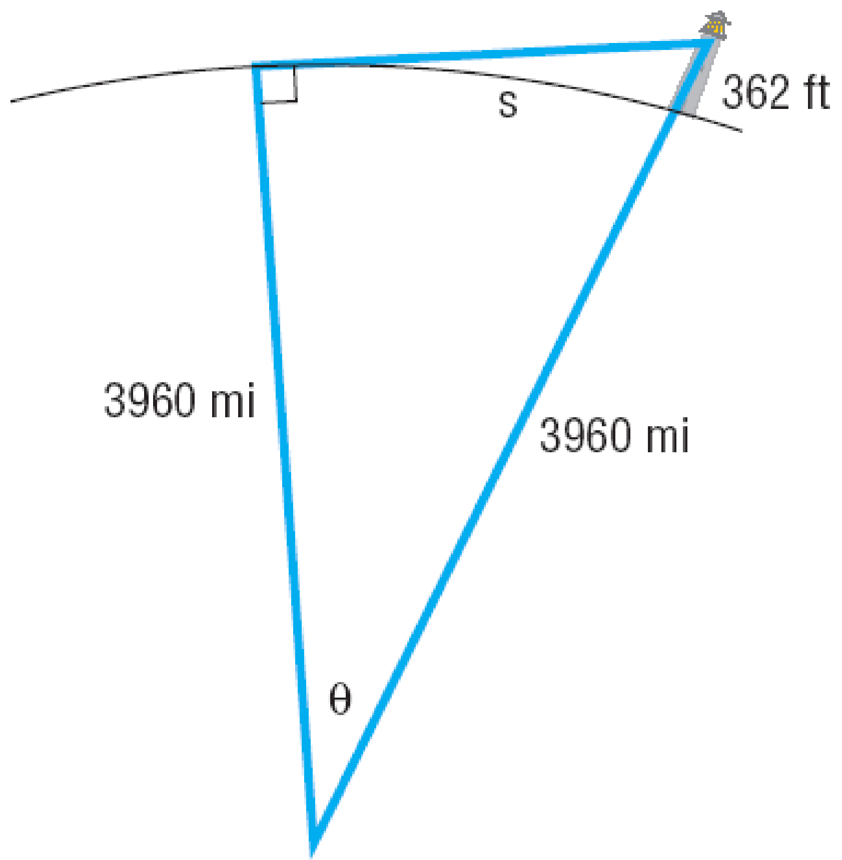
Adorning the top of the Board of Trade building in Chicago is a statue of Ceres, the Roman goddess of wheat. From street level, two observations are taken 400 feet from the center of the building. The angle of elevation to the base of the statue is found to be 55.1°; the angle of elevation to the top of the statue is 56.5°. See Figure 14(a). What is the height of the statue? Round the answer to the nearest foot.

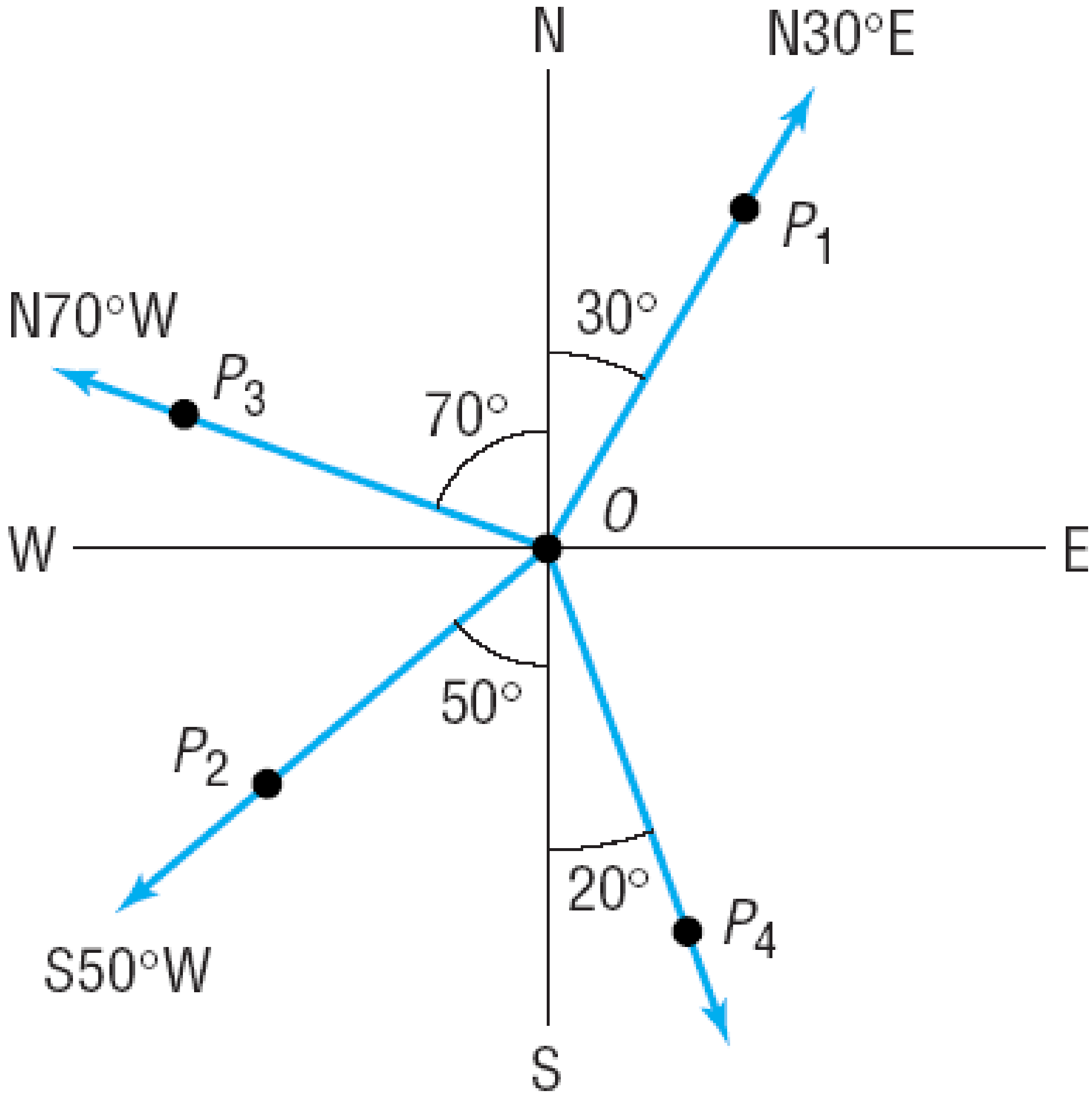


EXAMPLE

The Gibb's Hill Lighthouse, Southampton, Bermuda

In operation since 1846, the Gibb's Hill Lighthouse stands 117 feet high on a hill 245 feet high, so its beam of light is 362 feet above sea level. A brochure states that the light can be seen on the horizon about 26 miles distant. Verify the accuracy of this statement.



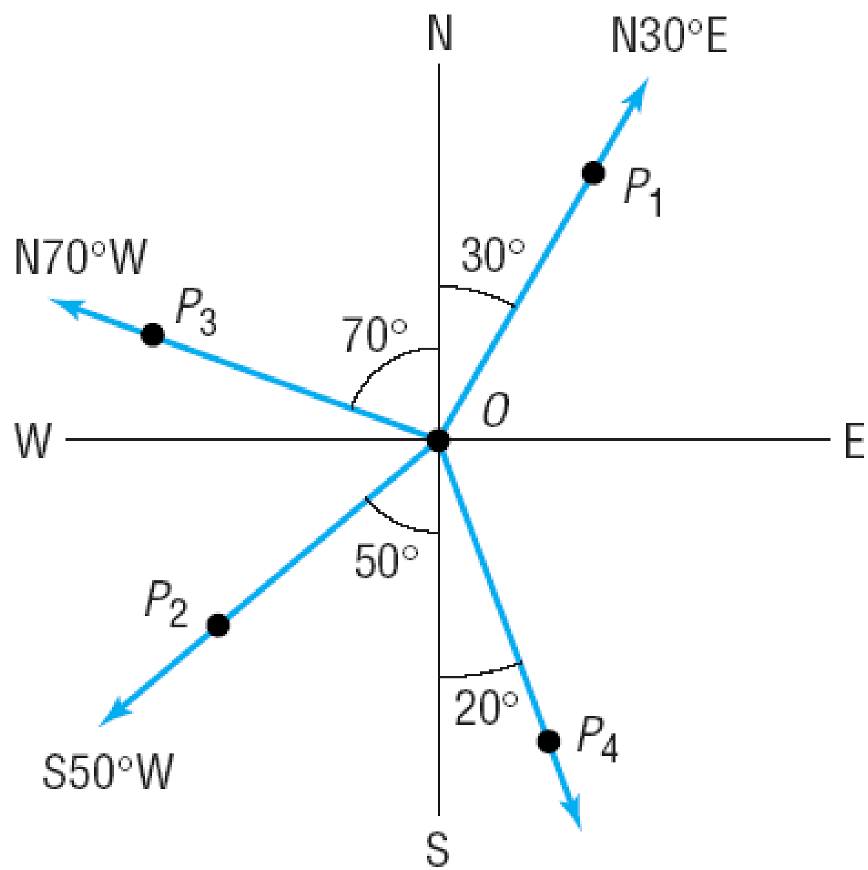

N
N30°E
P_1
30°
N70°W
P_3
70°
O
W
E
50°
P_2
20°
P_4
S50°W
S

EXAMPLE

Finding the Bearing of an Object

In Figure 16, what is the bearing from O to an object at P_4?

Figure 16



EXAMPLE Finding the Bearing of an Airplane

A Boeing 777 aircraft takes off from O'Hare Airport on runway 2 LEFT, which has a bearing of N20°E.* After flying for 1 mile, the pilot of the aircraft requests permission to turn 90° and head toward the northwest. The request is granted. After the plane goes 2 miles in this direction, what bearing should the control tower use to locate the aircraft?