Quantitative Literacy: Thinking Between the Lines

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Chapter 2:

Analysis of Growth

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Lesson Plan

- Measurements of growth: How fast is it changing?
- Graphs: Picturing growth
- Misleading graphs: Should I believe my eyes?

Learning Objectives:

- Understand intuitive notion of functions
- Read data table and calculate the percentage change
- Calculate the average growth rate
- Estimate by interpolation and extrapolation from a function value

Chapter 2 Analysis of Growth

2.1 Measurements of growth: How fast is it changing?

- When one quantity (or variable) depends on another, the latter is referred to as the **independent variable** and the former is referred to as the **dependent variable**.

- A **function** describes *how* the dependent variable *depends on* the independent variable.

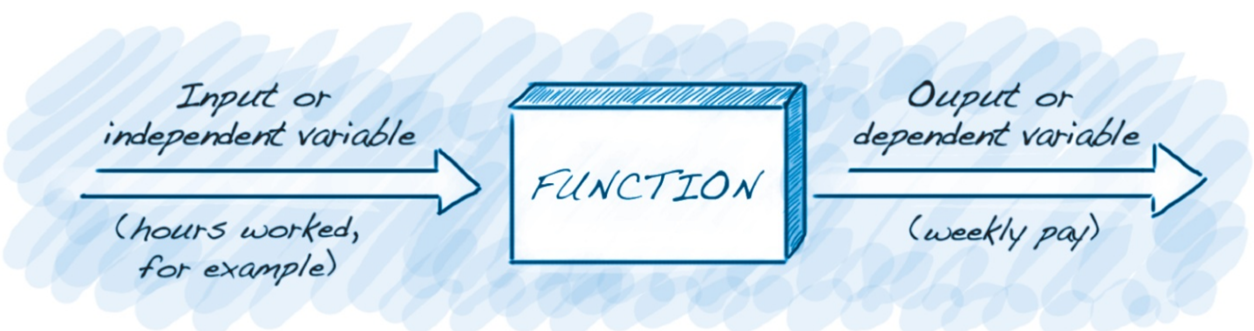
Chapter 2 Analysis of Growth

2.1 Measurements of growth: How fast is it changing?

- The **Independent variable =** the **input** value
- The **dependent variable =** the **output** value of a function.

- **Example:** If you work for an hourly wage, your pay for the work depends on the number of hours you work.

FIGURE 2.2 Visual representation of a function.



2.1 Measurements of growth: How fast is it changing?

- **Example:** The following table shows the most medals won by any country in a given year of the Olympic Winter Games:

Year	Most medals won by a country	Country
1988	29	Soviet Union
1992	26	Germany
1994	26	Norway
1998	29	Germany
2002	36	Germany
2006	29	Germany
2010	37	United States

What would you label as the independent and the dependent variables for this table?

- **Solution:**

The year = the Independent variable

The number of medals won = the dependent variable

- **Note:** the date cannot be a dependent variable because there were 3 different years in which 29 medals were won. The year does **not** depend only on the number of medals won.

- Thus, the year is **not a function** of the number of medals won.

2.1 Measurements of growth: How fast is it changing?

- The **percentage change** (or **relative** change) in a function is the percentage increase in the function from one value of the independent variable to another.

$$\text{Percentage change} = \frac{\text{Change in function}}{\text{Previous function value}} \times 100\%$$

- **Example:** Calculate the percentage change in the U.S. population from 1790 to 1800. Round your answer in the nearest whole number.

2.1 Measurements of growth: How fast is it changing?

- **Solution:**

TABLE 2.1 Population of the United States

Year	Population (millions)	Year	Population (millions)
1790	3.93	1900	76.21
1800	5.31	1910	92.23
1810	7.24	1920	106.02
1820	9.64	1930	123.20
1830	12.87	1940	132.16
1840	17.07	1950	151.33
1850	23.19	1960	179.32
1860	31.44	1970	203.30
1870	38.56	1980	226.54
1880	50.19	1990	248.71
1890	62.98	2000	281.42
		2010	308.75

$$\text{Percentage change} = \frac{\text{Change in function}}{\text{Previous function value}} \times 100\%$$

$$= \frac{\text{Change from 1790 to 1800}}{\text{Population in 1790}} \times 100\%$$

$$= \frac{5.31\text{million } - 3.93 \text{ million}}{3.93 \text{ million}} \times 100\% = 35\%$$

2.1 Measurements of growth: How fast is it changing?

- **Example:** The following table shows the world population (in billions) on the given data:

Date	1950	1960	1970	1980	1990	2000
Population (billions)	2.56	3.04	3.71	4.45	5.26	6.08

1. Identify the independent variable and the function, and make a bar graph that displays the data.

2. Make a table and bar showing percentage changes between decades.

- **Solution:** 1. The independent variable = the date.

  The function = the world population on that date.

2. The percentage increase from 1950 to 1960 is

$$\text{Percentage change} = \frac{\text{Change in function}}{\text{Previous function value}} \times 100\%$$
$$= \frac{3.04 - 2.56}{2.56} \times 100\% = 19\%$$

2.1 Measurements of growth: How fast is it changing?

- **Solution:** The table is given below, and the bar graph is in Figure 2.6.

Decade	1950-1960	1960-1970	1970-1980	1980-1990	1990-2000
% change	19%	22%	20%	18%	16%

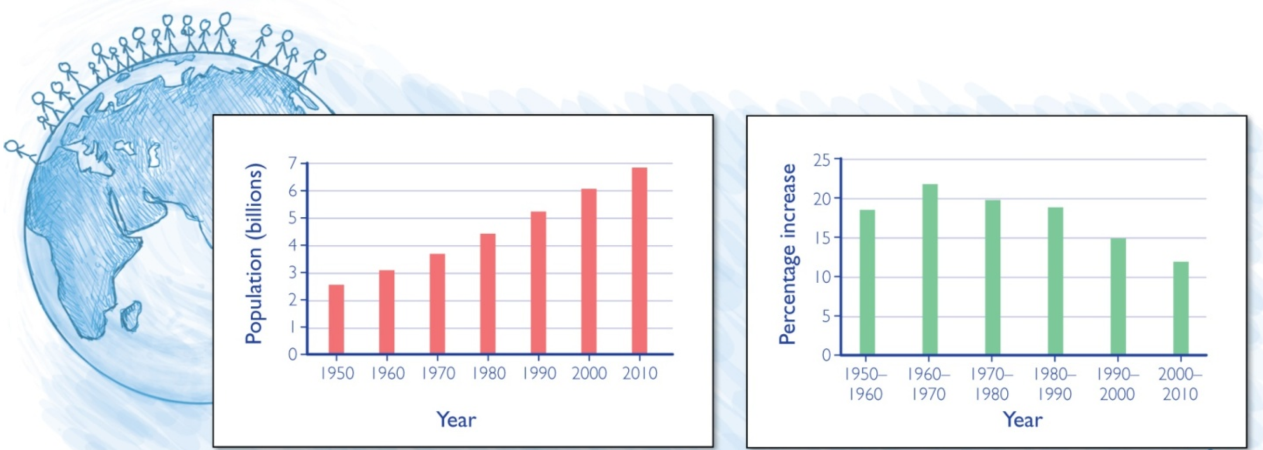


FIGURE 2.5 World population.

FIGURE 2.6 Percentage growth of world population.

2.1 Measurements of growth: How fast is it changing?

- The **average growth rate** of a function over an interval is the change in the function divided by the change in the independent variable.

$$\boxed{\text{Average growth rate} = \frac{\text{Change in function}}{\text{Change in independent variable}}}$$

- **Example:** The population of Russia declined from about 146 million in 2000 to about 143 million in 2007. Calculate the average growth rate over this period and explain its meaning.

- **Solution:** The change in population is negative,

143 – 146 = –3 million. The change in time is 7 years.

$$\text{Average growth rate} = \frac{\text{Change in function}}{\text{Change in independent variable}}$$
$$= -\frac{3}{7} = -0.429 \text{ million per year}$$

- This means that over this interval the population declined, on average, by about 429,000 per year.

2.1 Measurements of growth: How fast is it changing?

- **Example:** Assume that the independent variable is the year and the function gives the tuition cost, in dollars, at your university.

Give the units of the average growth rate and explain in practical terms what that rate means.

2.1 Measurements of growth: How fast is it changing?

- **Solution:** The change in the independent variable is the elapsed time measured in years.

- The change in the function is the tuition increase measured in dollars.

- The units of the average growth rate are dollars per year.

- The average growth rate means how much we expect the tuition to increase each year.

2.1 Measurements of growth: How fast is it changing?

- **Interpolation** is the process of estimating unknown values between known data points using the average growth rate.

- **Example:** In the fall of 2005, 37.7% of college freshmen in the United States believed that marijuana should be legalized.

In the fall of 2008, that figure was 41.3%. Use these figures to estimate the percentage in the fall of 2007.

The actual figure for 2007 was 38.2%.

What does this say about how the growth rate in the percentage varied over time?

- **Solution:** The change in the independent variable from 2005 to 2008 is three years.

❑ The change in the dependent variable over that period is $41.3 - 37.3 = 3.6$ percentage points.

$$\frac{\text{Change in function}}{\text{Change in independent variable}} = \frac{3.6}{3} = 1.2$$

❑ Hence, the average growth rate from the fall of 2005 to the fall of 2008 was 1.2 percentage points per year.

- **Solution:**

- The percentage in 2007 was about:

$\text{Percentage in } 2005 + 2 \text{ years of increase} = 37.7 + 2 \times 1.2 = 40.1\%$

- Our estimate of 40.1% for the fall of 2007 is higher than the actual figure of 38.2%. The figure grew more quickly from 2007 to 2008.

2.1 Measurements of growth: How fast is it changing?

- **Example:** Severe Acute Respiratory Syndrome (SARS) is a viral respiratory disease. There was a serious outbreak initially in China from November 2002 to July 2003. There were 8096 known cases and 774 deaths worldwide.

1. Calculate the average growth rate of cases from March 26 to March 31.

2. Use your answer from part 1 to estimate the cumulative number of SARS cases by March 28. The actual cumulative number on March 28 was 1485. What does this say about how the growth rate varied over time?

2.1 Measurements of growth: How fast is it changing?

- **Solution:** The table from the World Health Organization shows the cumulative number of SARS cases:

Date	March 26	March 31	April 5	April 10	April 15
Number of cases	1323	1622	2416	2781	3235

1. The independent variable is the data.

The function is the cumulative number of SARS cases reported.

$$\frac{\text{Change in reported cases}}{\text{Elapsed time}} = \frac{1622 - 1323}{5} = 59.8$$

The average growth rate from March 26 to March 31 is 59.8 new cases per day.

- **Solution:** 2. There were 1323 cases on March 26, and we expect to see about 59.8 new cases on March 27 and again on March 28.

$$\text{Estimated cases on March 28}$$
$$= \text{Cases on March 26} + 2 \times \text{Average new cases per day}$$
$$= 1323 + 2 \times 59.8$$
$$= 1442.6.$$

Thus, estimated cases on March 28 is about 1443 cases.

Our estimate using interpolation is relatively good but somewhat lower than the actual value of 1485.

2.1 Measurements of growth: How fast is it changing?

- **Extrapolation** is the process of estimating unknown values beyond known data points using the average growth rate.

- **Example:** The following table shows the average age, in years, of first-time mothers in the given year.

Year	1970	1980	1990	2000
Average age	21.4	22.7	24.2	24.9

1. Estimate the average age of first-time mothers in 1997.
2. Predict the average age of first-time mothers in 2005.
3. Predict the average age of first-time mothers in the year 3000. Explain why the resulting figure is not to be trusted.

- **Solution:**

1. We estimate the average age in 1997 **by interpolating**. The average growth rate between 1990 and 2000 is:

$$\boxed{\text{Average growth rate} = \frac{24.9 - 24.2}{10} = 0.07}$$

- So, the average age of first-time mothers increased at a rate of 0.07 year per year over this decade.
- The average age in 1997 is estimated to be

  $24.7 + 7 \times 0.07 = 24.69$ years, or about 24.7 years.

- **Solution:**

2. We estimate the average age **by extrapolating**.

- The average growth rate between 1990 and 2000, that we found in part 1 to be 0.07 year per year.

- Thus, we estimate an increase of 0.07 year over each of the five years from 2000 to 2005:

$\text{Estimate average age in } 2005 = 24.9 + 5 \times 0.07 = 25.25$ years

Or about 25.3 years.

- **Solution:**

3. This is exactly like part 2.

- The average growth rate from 1990 to 2000 is 0.07 year per year.
- The year 3000 is 1000 years from the year 2000.This growth rate gives a prediction for the year 3000 of:

$$24.9 + 1000 \times 0.07 = 94.9 \text{ years}$$

- Our projection for the average age in the year 3000 of first-time mothers is 95 years.
- The number is silly and clearly illustrates the danger of extrapolating too far beyond the limits of the given data.