# Quantitative Literacy: Thinking Between the Lines

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## Chapter 3:

## Linear and Exponential Change:

### Comparing Growth Rates

# Chapter 3: Linear and Exponential Changes

## Lesson Plan

- Lines and linear growth: What does a constant rate mean?
- Exponential growth and decay: Constant percentage rates
- Logarithmic phenomena: Compressed scales

Chapter 3 Linear and Exponential Changes

3.1 Lines and linear growth: What does a constant rate mean?

# Learning Objectives:

- Understand linear functions and consequences of a constant growth rate.
- Interpret linear functions.
- Calculate and interpret the slope.
- Understand linear data and trend lines for linear approximations.

## Chapter 3 Linear and Exponential Changes

### 3.1 Lines and linear growth: What does a constant rate mean?

- A **linear function** is a function with a constant growth rate.
- A **graph** of a linear function is a straight line.
- **Example (Determining linear or not):** Find the growth rate of the function. Make a graph of the function. Is the function linear?

  *For my daughter's wedding reception, I pay $500 rent for the building plus $15 for each guest. This describes the total cost of the reception as a function of the number of guests.*

- **Solution:** The growth rate is the extra cost incurred for each additional guest, that is $15. So, the growth rate is **constant**.

The additional cost means each additional guest.

The total cost of the reception is a **linear** function of the number of guests.

## 3.1 Lines and linear growth: What does a constant rate mean?

- **Example (Determining linear or not):** Find the growth rate of the function and give its practical meaning. Make a graph of the function. Is the function linear?

  *My salary is initially \$30,000, and I get a 10% salary raise each year for several years. This describes my salary as a function of time.*

- **Solution:** The growth rate:

$$1^{\text{st}}\text{ year increased} = 10\%\text{ of }\$30{,}000 = \$3{,}000$$

$$\therefore\quad 1^{\text{st}}\text{ year salary} = \$33{,}000$$

$$2^{\text{nd}}\text{ year increased} = 10\%\text{ of }\$33{,}000 = \$3{,}300$$

$$\therefore\quad 2^{\text{nd}}\text{ year salary} = \$36{,}300$$

The growth rate is **not the same each year**. So, the graph is **not** a straight line. Thus, my salary is **not** a linear function of time in years.

# Chapter 3 Linear and Exponential Changes

## 3.1 Lines and linear growth: What does a constant rate mean?

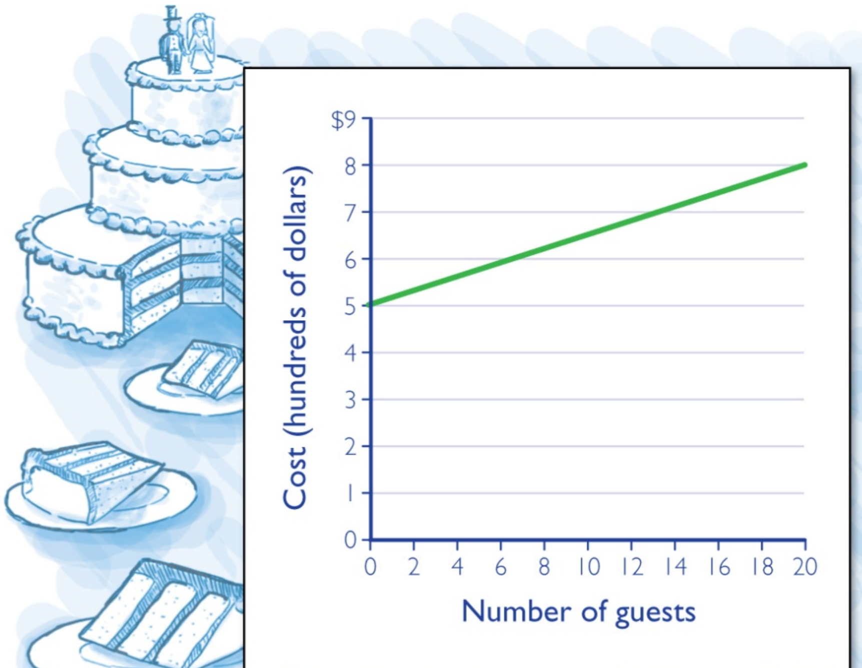


**FIGURE 3.5** Cost is a linear function of number of wedding guests.

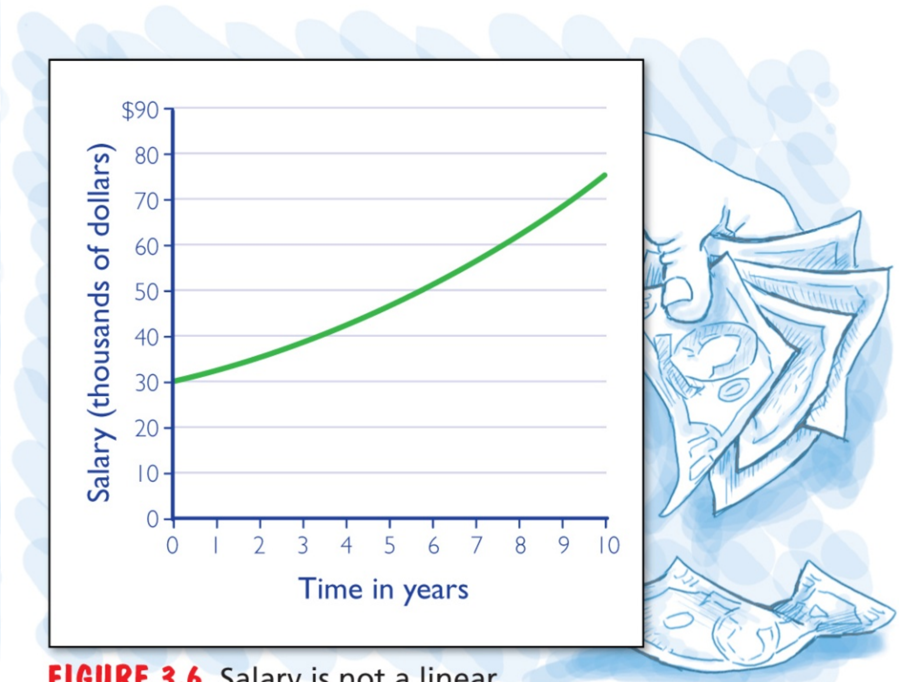


**FIGURE 3.6** Salary is not a linear function of time.

## 3.1 Lines and linear growth: What does a constant rate mean?

**Formula for Linear Function**

$$y = \text{Growth rate } \times x + \text{Initial value}$$

If *m* is the growth rate or slope and *b* is the initial value,

$$y = mx + b.$$

- **Example:** Let $L$ denote the length in meters of the winning long jump in the early years of the modern Olympic Games. Suppose $L$ is a function of the number $n$ of Olympic Games since 1990, an approximate linear formula is $L = 0.14n + 7.20$.

Identify the initial values and growth rate, and explain in practical terms their meaning.

- **Solution:** The initial value is 7.20 meters. The growth rate is 0.14 meter per Olympic Game. It means that the length of the winning long jump increased by 0.14 meters from one game to the next.

## 3.1 Lines and linear growth: What does a constant rate mean?

- **Example:** A rocket starting from an orbit 30,000 kilometers (km) above the surface of Earth blasts off and flies at a constant speed of 1000 km per hour away from Earth.

1. Explain why the function giving the rocket's distance from Earth in terms of time is linear.
2. Identify the initial value and growth rate.
3. Find a linear formula for the distance.

The Saturn V carried the first men to the moon in 1969.

- **Solution:**

1. We first choose letters to represent the function and variable. Let $d$ be the distance in km from Earth after $t$ hours.

   $$\text{The growth rate} = \text{velocity} = 1000\ \text{km/hour} = \text{a constant}$$

   Thus, $d$ is a linear function of $t$.

2. $$\begin{aligned}\text{The Initial value} &= 30{,}000\ \text{km} \\ &= \text{the height above Earth at blastoff}\end{aligned}$$

3. $$\begin{aligned} d &= \text{Growth rate} \times t + \text{Initial value} \\ &= 1000\,t + 30{,}000\end{aligned}$$

## Chapter 3 Linear and Exponential Changes

## 3.1 Lines and linear growth: What does a constant rate mean?

- **Interpreting and using the Slope**

❑ The slope of a linear function is using:

$$\boxed{\text{Slope} = \text{Growth rate} = \frac{\text{Change in function}}{\text{Change in independent variable}}}$$

❑ Write the equation of a linear function as: $y = mx + b$, the formula for the slope becomes

$$\boxed{m = \text{Slope} = \frac{\text{Change in } y}{\text{Change in } x}}$$

❑ Each 1-unit increase in *x* corresponds to a change of *m* units in *y.*

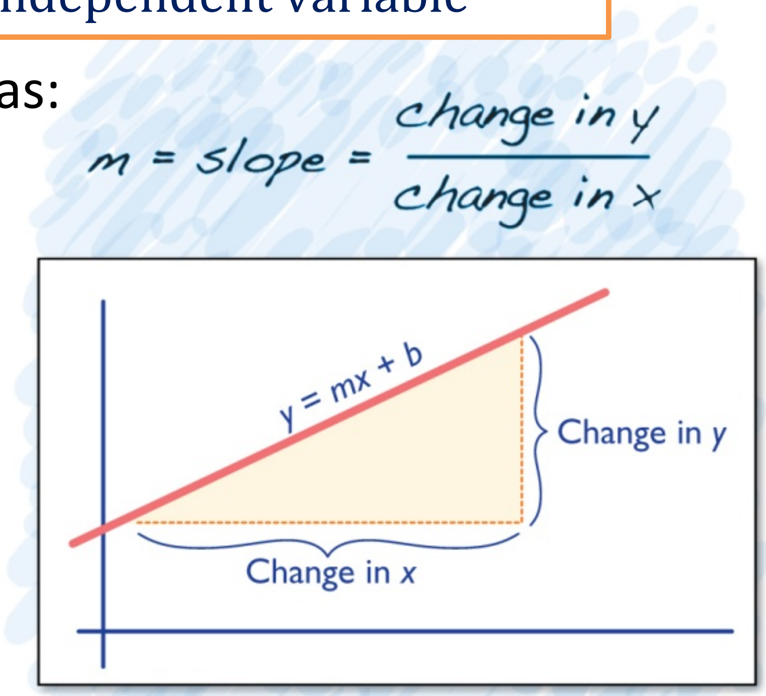


**FIGURE 3.9** How slope is calculated.

## 3.1 Lines and linear growth: What does a constant rate mean?

- Given a set of data points, the **regression line** (or **trend line**) is a line that comes as close as possible to fitting those data.
- **Example:** The following table shows the running speed of various animals vs. their length. Show the scatterplot and find the formula for the trend line. Explain in practical terms the meaning of the slope.

| Animal | Length (inches) | Speed (feet per second) |
|---|---|---|
| Deer mouse | 3.5 | 8.2 |
| Chipmunk | 6.3 | 15.7 |
| Desert crested lizard | 9.4 | 24.0 |
| Grey squirrel | 9.8 | 24.9 |
| Red fox | 24.0 | 65.6 |
| Cheetah | 47.0 | 95.1 |

## 3.1 Lines and linear growth: What does a constant rate mean?

- **Solution:**

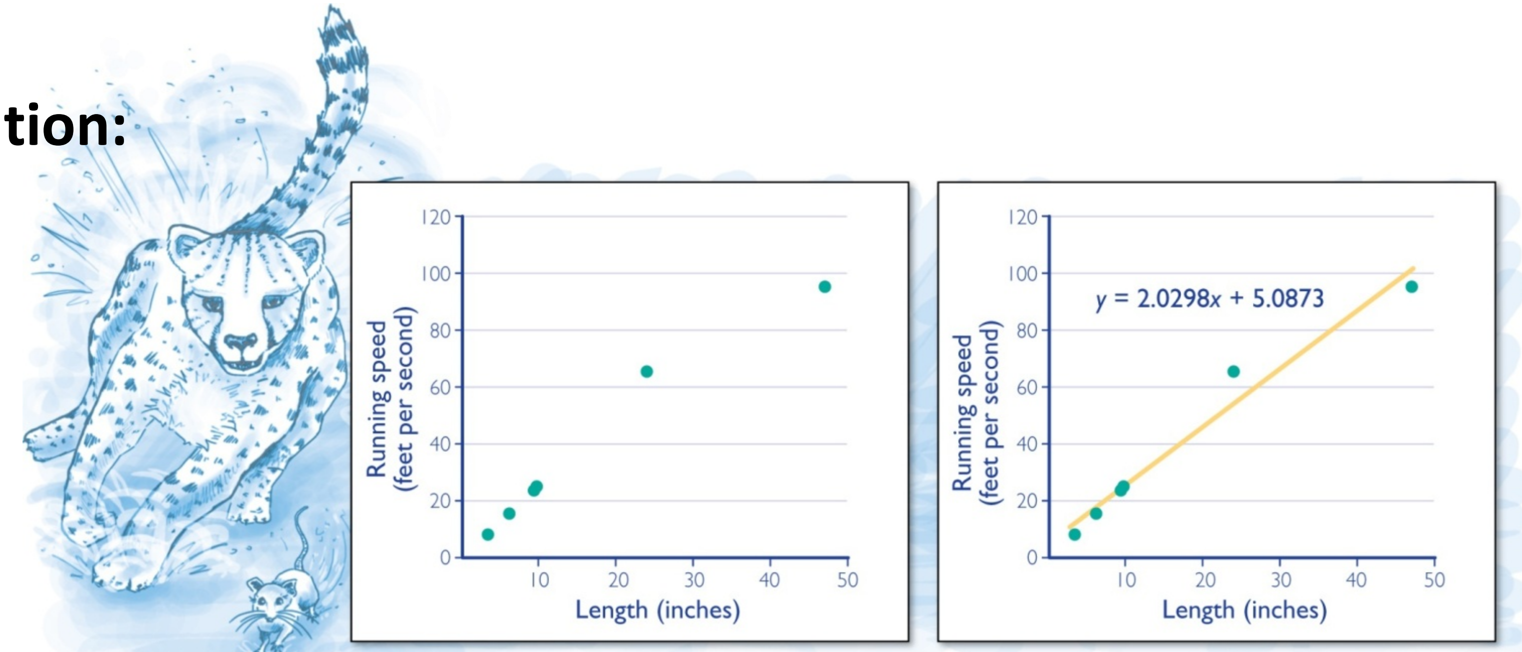


**FIGURE 3.14** Scatterplot of running speed versus length.

**FIGURE 3.15** Trend line added.

The points do not fall on a straight line, so the data in the table are not exactly linear. In Figure 3.15, we have added the trend line produced by the spreadsheet program Excel.

- **Solution:** The equation of the trend line is:

$$y = 2.03x + 5.09$$

This means that running speed *S* in feet per second can be closely estimated by:

$$S = 2.03\, L + 5.09,$$

where *L* is the length measured in inches.

The slope of the trend line is 2.03 feet per second per inch.

This value for the slope means that an animal that is 1 inch longer than another would be expected to run about 2.03 feet per second faster.