3.3 Logarithmic phenomena: Compressed scales

Learning Objectives:

- Understand the use of logarithms in compressed scales.
- Understand the Richter scale and calculate its magnitude in terms of relative intensity.
- Understand and calculate decibel reading in terms of relative intensity.
- Solve exponential equations.
- Calculate the doubling time.

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- The **common logarithm** of a positive number *x*, written $\log x$, is the exponent of 10 that gives x:

$$\boxed{\log x = t \quad \text{if and only if} \quad 10^t = x}$$

- **Example:** Calculating logarithms

1. $\log 10 = 1$ because $10^1 = 10$.

2. $\log 100 = 2$ because $10^2 = 100$.

3. $\log 1000 = 3$ because $10^3 = 10000$.

4. $\log \frac{1}{10} = -1$ because $10^{-1} = \frac{1}{10}$.

- The **relative intensity** of an earthquake is a measurement of ground movement.
- The **magnitude** of an earthquake is the logarithm of relative

$$\text{Magnitude} = \log(\text{Relative intensity}),$$

$$\text{Relative intensity} = 10^{\text{Magnitude}}$$

- **Example:** If an earthquake has a relative intensity of 6700, what is its magnitude?
- **Solution:**

$$\text{Magnitude} = \log(\text{Relative intensity}) = \log(6700) \approx 3.8$$

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- **Meaning of magnitude changes**

1. An increase of 1 unit on the Richter scale corresponds to increasing the relative intensity by a factor of 10.
2. An increase of t units in magnitude corresponds to increasing the relative intensity by a factor of 10^t.

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- **Example:** In 1994 an earthquake measuring 6.7 on the Richter scale occurred in Northridge, CA. In 1958 an earthquake measuring 8.7 occurred in the Kuril Islands.

  How did the intensity of the Northridge quake compare with that of the Kuril Islands quake?

- **Solution:** The Kuril Islands quake was $8.7 - 6.7 = 2$ points higher. Increasing magnitude by 2 points means that relative intensity increases by 10^2. The Kuril Islands quake was 100 times as intense as the Northridge quake.

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- **Properties of Logarithms**

1. Logarithm rule 1: $\log(\mathrm{A}^t) = t\log(\mathrm{A})$
2. Logarithm rule 2: $\log(\mathrm{AB}) = \log(\mathrm{A}) + \log(\mathrm{B})$
3. Logarithm rule 3: $\log\left(\frac{\mathrm{A}}{\mathrm{B}}\right) = \log(\mathrm{A}) - \log(\mathrm{B})$

- **Example:** Suppose we have a population that is initially 500 and grows at a rate of 0.5% per month. How long will it take for the population to reach 800?

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- **Solution:** The monthly percentage growth rate, $r = 0.005$.

The population size *N* after *t* months is:

$$N = \text{Initial value} \times (1 + r)^t = 500 \times 1.005^t$$

To find out when $N = 800$, solve the equation:

$$800 = 500 \times 1.005^t$$

Divide both sides by 500:

$$1.6 = 1.005^t$$

Apply the logarithm function to both sides and use rule 1:

$$\log 1.6 = \log(1.005^t) = t \log(1.005)$$

Dividing by log 1.005 gives:

$$t = \frac{\log 1.6}{\log 1.005} = 94.2 \text{ months}$$

The population reaches 800 in about 7 years and 10 months.

- **Solving exponential equations**

The solution for t of the exponential equation $A = B^t$ is:

$$\boxed{t = \frac{\log \mathrm{A}}{\log \mathrm{B}}}$$

- **Example:** An investment is initially \$5000 and grows by 10% each year. How long will it take the account balance to reach \$20,000?
- **Solution:** The balance *B* after *t* years:

$$B = \text{Initial value} \times (1+r)^t = 5000 \times 1.1^t$$

To find when $B = \$20{,}000$,

solve $20{,}000 = 5000 \times 1.1^t$, or $4 = 1.1^t$ with $A = 4$ and $B = 1.1$ of exponential equation $A = B^t$.

$$t = \frac{\log \mathrm{A}}{\log \mathrm{B}} = \frac{\log 4}{\log 1.1} = 14.5 \text{ years}$$

- **Doubling Time and more**

Suppose a quantity grows as an exponential function with a given base. The time *t* required to multiply the initial value by *K* is:

$$\boxed{\text{Time required to multiply by } K \text{ is } t = \frac{\log K}{\log(\text{Base})}}$$

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$$\boxed{\text{Doubling time} = \frac{\log 2}{\log(\text{Base})}}$$

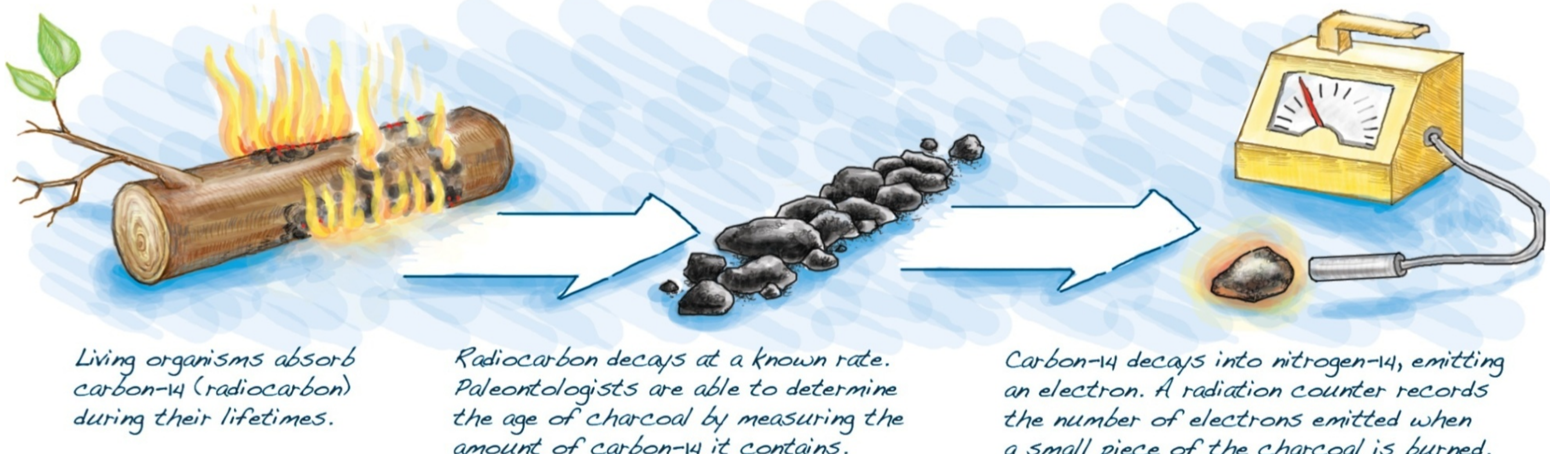
- **Example:** Suppose an investment is growing by 7% each year. How long does it take the investment to double in value?

- **Solution:** The percentage growth is a constant, 7%, so the balance is an exponential function.
- The base $= 1 + r = 1.07$:

$$\text{Doubling time} = \frac{\log 2}{\log(\text{Base})}$$

$$= \frac{\log 2}{\log(1.07)} = 10.2 \text{ years}$$

- **Example:** Recall that carbon-14 has a half-life of 5770 years. Suppose the charcoal from an ancient campfire is found to contain only one-third of the carbon-14 of a living tree. How long ago did the tree that was the source of the charcoal die?



- **Solution:** Use $K = 1/3$, the base = half-life = $1/2$:

$$\text{Time to multiply by } 1/3 \text{ is } t = \frac{\log K}{\log(\text{Base})} = \frac{\log(1/3)}{\log(1/2)} = 1.58.$$

Each half-life is 5770 years, the tree died $1.58 \times 5770 = 9116.6$ years ago.

Chapter Summary

- **Lines and linear growth:** What does a constant rate mean?
  - Understand linear functions and consequences of a constant growth rate.

    Recognizing and solve linear functions

    Calculate the growth rate or slope

    Interpolating and using the slope

    Approximate the linear data with trend lines

Chapter 3 Linear and Exponential Changes:

Chapter Summary

- **Exponential growth and decay:** Constant percentage rates
  - Understand exponential functions and consequences of constant percentage change.

    The nature of exponential growth

    Formula for exponential functions

    The rapidity of exponential growth

    Relating percentage growth and base

    Exponential decay

    Radioactive decay and half-life

Chapter Summary

- **Logarithmic phenomena:** Compressed scales
  - Understand the use if logarithms in compressed scales and solving exponential equations.

    The Richter scale and interpolating change on the Richter scale

    The decibel as a measure of sound

    Solving exponential equations

    Doubling time and more