Quantitative Literacy:
Thinking Between the Lines

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Chapter 4: Personal Finance

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Lesson Plan

- Saving money: The power of compounding
- Borrowing: How much car can you afford?
- Savings for the long term: Build that nest egg
- Credit cards: Paying off consumer debt
- Inflation, taxes, and stocks: Managing your money

Learning Objectives:

- Use the simple interest and compound interest formulas
- Compute Annual Percentage Yield (APY)
- Understand and calculate the Present value and the Future value
- Compute the exact doubling time
- Estimate the doubling time using the Rule of 72

- **Principal:** The initial balance of an account.
- **Simple interest:** calculated by the interest rate to the principal only, not to interest earned.

Simple Interest Formula

Simple interest earned = Principal × Yearly interest rate × Time

- **Example:** We invest \$2000 in an account that pays simple interest of 4% each year. Find the interest earned after five years.

- **Solution:** The interest rate of 4% written as a decimal is 0.04.

Simple interest earned

= Principal × Yearly interest rate × Time in years

= \$2000 × 0.04 /year × 5 years

= \$400

4.1 Saving money: The power of compounding

- **APR (Annual Percentage Rate)** – multiply the period interest rate by the number of periods in a year.

APR Formula

$$\text{Period interest rate} = \frac{\text{APR}}{\text{Number of periods in a year}}$$

- **Compound interest** – interest paid on **both** the principal and on the interest that the account has earned.

Compound Interest Formula

$$\text{Balance after } t \text{ periods} = \text{Principal} \times (1 + r)^t$$

- **Example:** Suppose we invest $10,000 in a five-year certificate of deposit (CD) that pays an APR of 6%. What is the value of the mature CD if interest is:

1. compounded **annually**?
2. compounded **quarterly**?
3. compounded **monthly**?
4. compounded **daily**?

Solution:

1. Compounded **annually:** the rate is the same as the APR:

$$r = 6\% = 0.06 \text{ and } t = 5\ years.$$

$$\begin{aligned}\text{Balance after 5 years} &= \text{Principal} \times (1+r)^t \\ &= \$10000 \times (1+0.06)^5 \\ &= \$10{,}000 \times 1.06^5 = \$13{,}382.26\end{aligned}$$

Solution:

2. Compounded **quarterly:**

$$r = \text{Quarterly rate} = \frac{APR}{4} = \frac{0.06}{4} = 0.015.$$

5 years is 5 × 4 quarters, so $t = 20$ in compound interest formula:

$$\text{Balance after 5 years} = \text{Principal} \times (1 + r)^t$$
$$= \$10{,}000 \times (1 + 0.015)^{20}$$
$$= \$13{,}468.55$$

Solution:

3. Compounded **monthly:**

$$r = \text{Monthly rate} = \frac{APR}{12} = \frac{0.06}{12} = 0.005$$

5 years is 5 × 12 months, so $t = 60$:

$$\text{Balance after 5 years} = \text{Principal} \times (1 + r)^t$$
$$= \$10{,}000 \times (1 + 0.005)^{60}$$
$$= \$13{,}488.50$$

Solution:

4. Compounded **daily:**

$$r = \text{daily rate} = \frac{APR}{365} = \frac{0.06}{365} = 0.00016.$$

5 years is 5 × 365 = 1825 days.

$$\text{Balance after 5 years} = \text{Principal} \times (1 + r)^t$$
$$= \$10{,}000 \times 1.00016^{1825}$$
$$= \$13{,}498.26$$

Solution:

We summarize the results.

Compounding period	Balance at maturity
Yearly	\$13,382.26
Quarterly	\$13,468.55
Monthly	\$13,488.50
Daily	\$13,498.26

- The annual percentage yield (**APY**) – the actual percentage return earned in a year.

$$\text{APY} = \left(1 + \frac{APR}{n}\right)^{n} - 1$$

Where *n* is the number of compounding periods per year.

- **Example:** You have an account that pays APR of 10%. If interest is compounded monthly, find the APY.

- **Solution:** APR = 10% = 0.10, n = 12 , so we use the APY formula:

$$\text{APY} = \left(1+\frac{APR}{n}\right)^{n} - 1 = \left(1+\frac{0.10}{12}\right)^{12} - 1 = 0.1047$$

Round the answer as a percentage to two decimal places; the APY is about 10.47%.

APY Balance Formula

$$\text{Balance after } t \text{ years} = \text{Principal} \times (1 + APY)^t$$

- **Example:** Suppose we earn 3.6% APY on a 10-year \$100,000 CD. Find the balance at maturity.
- **Solution:** APY = 3.6% = 0.036, t = 10, and so we use APY balance formula:

$$\begin{aligned}\text{Balance after 10 years} &= \$100{,}000 \times (1 + 0.036)^{10} \\ &= \$142{,}428.71\end{aligned}$$

4.1 Saving money: The power of compounding

- **Present value:** the amount we initially invest.

  Present value = Principal

- **Future value:** the value of that investment at some specific time in the future.

  Future value = balance after *t* periods

Compound Interest Formula

$$\text{Balance after } t \text{ periods} = \text{Principal} \times (1+r)^t$$

$$\text{Future value} = \text{Present value} \times (1+r)^t$$

- **Example:** Find the future value of an account after three years if the present value is \$900, the APR is 8%, and interest is compounded quarterly.

- **Solution:** $r = \frac{APR}{4} = \frac{.08}{4} = 0.02, \quad t = 3 \times 4 = 12$:

$$\text{Future value} = \text{Present value} \times (1 + r)^t$$
$$= \$900 \times (1 + 0.02)^{12}$$
$$= \$1141.42$$

- **Exact doubling time** for investments

$$\text{Number of periods to double} = \frac{\log 2}{\log(1+r)}$$

where r is the period interest rate as a decimal.

- **Approximate doubling time** using *the rule of 72*

$$\text{Estimate for doubling time} = \frac{72}{\text{APR}}$$

where APR is expressed as a **percentage**.

- **Example:** Suppose an account has an APR of 8% compounded quarterly. Estimate the doubling time using the rule of 72. Calculate the exact doubling time.
- **Solution:** The rule of 72 gives the estimate doubling time 9 years.

$$\boxed{\text{Estimate for doubling time} = \frac{72}{\text{APR}} = \frac{72}{8} = 9 \text{ years}}$$

To find the exact doubling time, $r = \frac{0.08}{4} = 0.02$, since the period is quarterly.

$$\boxed{\text{Number of periods to double} = \frac{\log 2}{\log(1 + 0.02)} = 35.0}$$

Thus, the actual doubling time is 35.0 quarters, or 8 years and 9 months.

Learning Objectives:

- Understand installment loans
- Calculate Monthly payment of a fixed loan
- Calculate amount borrowed
- Understand Amortization table and equity
- Understand mortgage options: fixed-rate mortgage vs. Adjustable-rate mortgage
- Compute the monthly payment for a mortgage

4.2 Borrowing: How much car can you afford?

- With an **installment loan** you borrow money for a fixed period of time, called the term of the loan, and you make regular payments (usually monthly) to pay off the loan plus interest accumulated during that time.
- The amount of payment depends on three things:

1. the amount of money we borrow (the principal)
2. the interest rate (or APR)
3. the term of the loan

4.2 Borrowing: How much car can you afford?

- Monthly payment formula

$$\text{Monthly payment} = \frac{\text{Amount borrowed } \times r(1+r)^t}{\left((1+r)^t - 1\right)}$$

where t is the term in months and $r = APR/12$ is the monthly rate as a decimal.

- **Example (College Loan):** You need to borrow $5,000 so you can attend college next fall. You get the loan at an APR of 6% to be paid off in monthly installments over three years. Calculate monthly payment.

- **Solution:** The interest rate (or APR): APR of 6%

  The monthly rate as a decimal is:

$$r = \text{Monthly rate} = \frac{APR}{12} = \frac{0.06}{12} = 0.005$$

We want to pay off the loan in three years, so we use a term of $t = 3 \times 12 = 36$ in the monthly payment formula:

$$\text{Monthly payment} = \frac{\text{Amount borrowed } \times r(1+r)^t}{\left((1+r)^t - 1\right)}$$

$$= \frac{\$5000 \times 0.005 \times 1.005^{36}}{(1.005^{36} - 1)} = \$152.11$$

4.2 Borrowing: How much car can you afford?

- Suppose you can afford a certain monthly payment, how much you can borrow to stay that budget.

<u>Companion monthly payment formula</u>

$$\text{Amount borrowed} = \frac{\text{Monthly payment} \times \left((1+r)^t - 1\right)}{(r \times (1+r)^t)}$$

- **Example (Buying a car):** We can afford to make payments of \$250 per month for three years. Our car dealer is offering us a loan at an APR 5%. For what price automobile should we be shopping?

Solution: The monthly rate as a decimal is:

$$r = \text{Monthly rate} = \frac{0.05}{12} = 0.0042$$

Three years is 36 months, so we use a term of t = 36 in the companion monthly payment formula:

$$\text{Amount borrowed} = \frac{\text{Monthly payment} \times \left((1+r)^t - 1\right)}{(r \times (1+r)^t)} = \frac{\$250 \times ((1+0.0042)^{36} - 1)}{(0.0042 \times (1+0.0042)^{36})} = \$8{,}341.43$$

We should shop for cars that cost $8,341.43 or less.

4.2 Borrowing: How much car can you afford?

- **Amortization table (schedule):** shows for each payment made the amount applied to interest, the amount applied to the balance owed, and the outstanding balance.

- If you borrow money to pay for an item, your **equity** in that item at a given time is the part of the principal you have paid.

- **Example:** Suppose you borrow $1,000 at 12% APR to buy a computer. We pay off the loan in 12 monthly payments. Make an amortization table showing payments over the first five months. What is your equity in the computer after five payments?

- **Solution:**

The monthly rate $r = \frac{12\%}{12} = 1\% = 0.01$ as a decimal.

The monthly payment formula with $t = 12$:

$$\text{Amount borrowed}$$
$$= \frac{\text{Monthly payment } \times \left((1+r)^t - 1\right)}{(r \times (1+r)^t)}$$
$$= \frac{\$1000 \times ((1+0.01)^{12} - 1)}{(1.01^{12} - 1)}$$
$$= \$88.85$$

4.2 Borrowing: How much car can you afford?

- Make our **1st payment:** the outstanding balance is $1000

Interest = 1% of $1000 = $10

$88.85 – $10 = $78.85 to interest, and the outstanding balance.

$$\text{Balance owed after 1 payment} = \$1000 - \$78.85 = \$921.15$$

- Make our **2nd payment:** the outstanding balance is $921.15

Interest = 1% of $921.15 = $9.21

$88.85 – $9.21 = $79.64 to interest, and the outstanding balance.

$$\text{Balance owed after 2 payments} = \$921.15 - \$79.64 = \$841.51$$

4.2 Borrowing: How much car can you afford?

- If we continue in this way, we get the following table:

- "Table on page 212 here"

- Equity after five payments = \$1000 – \$514.92 = \$405.08.

4.2 Borrowing: How much car can you afford?

- **Home Mortgage:** a loan for the purchase of a home.
- **Example (30-year mortgage):** You decide to take a 30-year mortgage for \$300,000 at an APR of 9%. Find the total interest paid.
- **Solution:** The monthly payment rate: $r = \frac{APR}{12} = \frac{0.09}{12} = 0.0075.$
  - The loan is for 30 years: $t = 30 \times 12 = 360$ months in the monthly payment formula:

$$\text{Monthly payment} = \frac{\text{Amount borrowed } \times r(1+r)^t}{\left((1+r)^t - 1\right)}$$
$$= \frac{\$300{,}000 \times 0.0075 \times 1.0075^{360}}{(1.0075^{360} - 1)} = \$2413.87$$

  - Total amount paid = $360 \times \$2413.87 = \$868{,}993.20$
  - Total interest paid = $\$868{,}993.20 – \$300{,}000 = \$568{,}993.20$

4.2 Borrowing: How much car can you afford?

- **Fixed-rate mortgage:** keeps the same interest rate over the life of the loan.
- **Adjustable-rate mortgage (ARM):** the interest may vary over the life of the loan.

- **Example (comparing monthly payment):** Fixed-rate mortgage and ARM.

We want to borrow \$200,000 for a 30-year home mortgage. We have found an APR of 6.6% for a fixed-rate mortgage and an APR of 6% for an ARM. Compare the initial monthly payments for these loans.

4.2 Borrowing: How much car can you afford?

- **Solution:** principal = \$200,000 and t = 360 months.
  - **Fixed-rate:** $r = \frac{0.066}{12} = 0.0055$. The monthly payment formula gives:

$$\text{Monthly payment} = \frac{\text{Amount borrowed } \times r(1+r)^t}{\left((1+r)^t - 1\right)}$$
$$= \frac{\$200{,}000 \times 0.0055 \times 1.0075^{360}}{(1.0055^{360} - 1)} = \$1277.32$$

  - **ARM:** $r = \frac{0.06}{12} = 0.005$. The monthly payment formula gives:

$$\text{Monthly payment} = \frac{\text{Amount borrowed } \times r(1+r)^t}{\left((1+r)^t - 1\right)}$$
$$= \frac{\$300{,}000 \times 0.005 \times 1.005^{360}}{(1.005^{360} - 1)} = \$1199.10$$

Learning Objectives:

- Calculate a balance after *t* deposits
- Calculate needed deposit to achieve a financial goal
- Determine the value of nest eggs (an annuity)
- Determine a monthly annuity yield
- Determine an annuity yield goal (the nest egg needed)

- **Example:** You deposit $100 to your savings account at the end of each month and suppose the account pays a monthly rate of 1% on the balance in the account. Find the balance at the end of three months.
- **Solution:**
  - At the end of 1st month:

    New balance = Deposit = $100
  - At the end of 2nd month:

    New balance = Previous balance + Interest + Deposit

    = $100 + $(1\% \times \$100 = \$1)$ + $100 = $201
  - At the end of 3rd month:

    New balance = Previous balance + Interest + Deposit

    = $201 + $(1\% \times \$201 = \$2.01)$ + $100 = $303.01

- The following table shows the growth of this account through 10 months.

- Table 4.2(page 224) here

- **Regular deposits balance:** regular deposits at the end of each period.

<u>Regular deposit formula</u>

$$\text{Balance after } t \text{ deposits} = \frac{\text{Deposit} \times ((1 + r)^t - 1)}{r}$$

- **Example:** Suppose we have a savings account earning 7% APR. We deposit \$20 to the account at the end of each month for five years. What is the account balance after five years?

- **Solution:** The monthly interest rate $r = \frac{APR}{12} = \frac{0.07}{12}$. The number of deposits $t = 5 \times 12 = 60$. The regular deposit formula gives:

$$\text{Balance after } t \text{ deposits} = \frac{\text{Deposit} \times ((1+r)^t - 1)}{r}$$

$$= \frac{\$20 \times \left(\left(1+\frac{0.07}{12}\right)^{60} - 1\right)}{\frac{0.07}{12}} = \$1431.86$$

- The future value is \$1431.86.

- Determining the savings needed

Deposit needed formula

$$\text{Needed Deposit} = \frac{\text{Goal } \times r}{((1+r)^t - 1)}$$

- **Example (Saving for college):** How much does your younger brother need to deposit each month into a savings account that pays 7.2% APR in order to have $10,000 when he starts college in five years?

- **Solution:** We want to achieve a goal of \$10,000 in five years.

  The monthly interest rate $r = \frac{APR}{12} = \frac{0.072}{12} = 0.006.$

  The number of deposits $t = 5 \times 12 = 60$. The deposit needed formula gives:

$$\text{Needed Deposit} = \frac{\text{Goal} \times r}{((1+r)^t - 1)}$$

$$= \frac{\$10000 \times 0.006}{((1+0.006)^{60} - 1)} = \$138.96.$$

He needs to deposit \$138.96 each month.

- **Example (Saving for retirement):** Suppose that you'd like to retire in 40 years and you want to have a future value of \$500,000 in a savings account, and suppose that your employer makes regular monthly deposits into your retirement account. If your expect an APR of 9% for your account, how much do you need your employer to deposit each month?

- **Solution:** Goal = $\$500{,}000$, $t = 40 \times 12 = 480$

  r = Monthly rate = $\frac{APR}{12} = \frac{0.09}{12} = 0.0075$

$$\text{Needed Deposit} = \frac{\text{Goal} \times r}{((1+r)^t - 1)}$$

$$= \frac{\$500{,}000 \times 0.0075}{((1+0.0075)^{480} - 1)} = \$106.81$$

- **Example (cont.):** Assume the interest rate is constant over the period in question. Over a period of 40 years interest rates can vary widely. Assume a constant APR of 6% for your retirement account. How much do you need your employer to deposit each month under this assumption?

- **Solution:** r = Monthly rate = $\frac{APR}{12} = \frac{0.06}{12} = 0.005$

$$\text{Needed Deposit} = \frac{\text{Goal} \times r}{((1+r)^t - 1)}$$

$$= \frac{\$500{,}000 \times 0.005}{((1+0.005)^{480} - 1)} = \$251.07$$

Note that the decrease in the interest rate from 9% to 6% requires that the monthly deposit more than doubles.

- **Nest egg:** the balance of your retirement account at the time of retirement.
- **Monthly yield:** the amount you can withdraw from your retirement account each month.
- An **Annuity:** an arrangement that withdraws both principal and interest from your nest egg.

Annuity Yield Formula

$$\text{Monthly annuity yield} = \frac{\text{Nest egg} \times r \times (1+r)^t}{((1+r)^t - 1)}$$

- **Example:** Suppose we have a nest egg of \$800,000 with an APR of 6% compounded monthly. Find the monthly yield for a 20-year annuity.

- **Solution:** $r = \frac{APR}{12} = \frac{0.06}{12} = 0.005, \quad t = 20 \times 12 = 240$ months.

$$\text{Monthly annuity yield}$$
$$= \frac{\text{Nest egg } \times r \times (1+r)^t}{((1+r)^t - 1)}$$
$$= \frac{\$800{,}000 \times 0.005 \times (1+0.005)^{240}}{((1+0.005)^{240} - 1)}$$
$$= \$5731.45$$

Annuity Yield Goal

$$\text{Nest egg needed} = \frac{\text{Annuity yield goal} \times ((1+r)^t - 1)}{(r \times (1+r)^t)}$$

- **Example:** Suppose our retirement account pays 5% APR compounded monthly. What size nest egg do we need in order to retire with 20-year annuity that yields $4000 per month?
- **Solution:** $r = \frac{APR}{12} = \frac{0.05}{12}, \quad t = 20 \times 12 = 240 \text{ months}$

$$\text{Nest egg needed} = \frac{\text{Annuity yield goal} \times ((1+r)^t - 1)}{(r \times (1+r)^t)}$$
$$= \frac{\$4000 \times ((1+0.05/12)^{240} - 1)}{(0.05/12 \times (1+0.05/12)^{240})} = \$606{,}101.25$$

Learning Objectives:

- Understand credit cards
- Determine an amount subject to finance charges
- Determine the minimum payment balance formula to find a balance after *t* minimum payments

- **Credit card basics**:

- **Amount subject to finance charges**

= Previous balance – Payment + Purchases

Where the finance charge is calculated by applying monthly interest rate ($r = \text{APR}/12$) to this amount.

- **New balance**

= Amount subject to finance charges + Finance charge

- **Example:** Suppose your Visa card calculates finance charges using an APR of 22.8%. Your previous statement showed a balance of \$500, in response to which you made a payment of \$200. You then bought \$400 worth of clothes, which you charged to your Visa card. Find a new balance after one month.

- **Solution:**
- Amount subject to finance charges

  = Previous balance – Payment + Purchases

  = \$500 – \$200 + \$400 = \$700
- Finance charge = $\frac{APR}{12} \times \$700 = \frac{0.228}{12} \times \$700 = \$13.30$
- New Balance

  = Amount subject to finance charges + Finance charge

  = \$700 + \$13.30 = \$713.30

	Previous balance	Payments	Purchases	Finance charge	New balance
Month1	\$500	\$200	\$400	1.9% of \$700 = \$13.30	\$713.30

4.4 Credit cards: Paying off consumer debt

- **Example:** We have a card with an APR of 24%.

The minimum payment is 5% of the balance. Suppose we have a balance of $400 on the card.

We decide to stop charging and to pay it off by making the minimum payment each month.

Calculate the new balance after we have made our first minimum payment, and then calculate the minimum payment due for the next month.

- **Solution:**

- 1^{st} minimum payment = 5% of balance = 0.05 × \$400 = \$20
- Amount subject to finance charges

  = Previous balance – Payment + Purchases

  = \$400 – \$20 + \$0 = \$380
- Finance charge $= \frac{APR}{12} \times \$380 = \frac{0.24}{12} \times \$380 = 0.02 \times \$380 = \$7.60$
- New Balance

  = Amount subject to finance charges + Finance charge

  = \$380 + \$7.60 = \$387.60
- The next minimum payment will be 5% of \$387.60.

  Minimum payment = 5% of balance = 0.05 × \$387.60 = \$19.38

- **Minimum payment balance**

Minimum payment balance formula

$$\text{Balance after } t \text{ minimum payments} = \text{Initial balance } \times [(1+r)(1-m)]^t$$

Where *r* is the monthly rate and *m* is the minimum monthly payment as a percent of the balance.

- **Example:** We have a card with an APR of 20% and a minimum payment that is 4% of the balance. We have a balance of $250 on the card, and we stop charging and pay off that balance by making the minimum payment each month.

Find the balance after two years of payments.

- **Solution:**

- The monthly interest rate: $r = \frac{20\%}{12} = \frac{0.2}{12}$
- The minimum payment = 4% of new balance: $m = 0.04$
- The initial balance = \$250
- The number of payments: $t = 2 \times 12 = 24$ months

$$\begin{aligned}\text{Balance after } t \text{ minimum payments} \\ &= \text{Initial balance } \times [(1+r)(1-m)]^t \\ &= \$250 \times \left[\left(1+\frac{0.2}{12}\right)(1-0.04)\right]^{24} \\ &= \$139.55\end{aligned}$$

- **Example:** Suppose you have a balance \$10,000 on your Visa card, which has an APR of 24%. The card requires a minimum payment of 5% of the balance. You stop charging and begin making only the minimum payment until your balance is below \$100.

1. Find a formula that gives your balance after *t* monthly payments.
2. Find your balance after five years of payments.
3. Determine how long it will take to get your balance under \$100.
4. Suppose that instead of the minimum payment, you want to make a fixed monthly payment so that your debt is clear in two years. How much do you pay each month?

- **Solution:** 1. The minimum payment as a decimal:

  $m = 0.05$.

- The monthly rate: $r = 0.24/12 = 0.02$
- The initial balance = $10,000

$$\begin{aligned}\text{Balance after } t \text{ minimum payments} \\ &= \text{Initial balance } \times [(1+r)(1-m)]^t \\ &= \$10{,}000 \times [(1+0.02)(1-0.05)]^t \\ &= \$10{,}000 \times 0.969^t\end{aligned}$$

2. Now five years: $t = 5 \times 12 = 60$ months

Balance after 60 months = $\$10{,}000 \times 0.969^{60} = \$1511.56$

- After five years, we still owe over $1500.

3. Determine how long it takes to get the balance down to $100.

- **Method 1 (Using a logarithm):** Solve for t the equation

$$\$100 = \$10{,}000 \times 0.969^t$$

Divide each side of the equation by $10,000:

$$\frac{\$100}{\$10{,}000} = \frac{\$10{,}000}{\$10{,}000} \times 0.969^t$$

$$0.01 = 0.969^t$$

Solve exponential equation using logarithm:

$$A = B^t \text{ is } t = \frac{logA}{logB}$$

Use this formula: $t = \frac{log0.01}{log0.969} = 146.2$ months.

Hence, the balance will be under $100 after 147 monthly payments.

3. Determine how long it takes to get the balance down to $100.

- **Method 2 (Trial and error):** If you want to avoid logarithms, you can solve this problem using trial and error with a calculator. The information in part 2 indicates that it will take some time for the balance to drop below $100.

Try five years or 120 months,

Balance after 120 months = $\$10{,}000 \times 0.969^{120} = \$228.48$.

So we should try large number of months. If you continue in this way, we find the same answer as that obtained for Method 1: the balance drops below $100 at payment 147.

4. Consider your debt as an installment loan:

- Amount borrowed = \$10,000
- Monthly interest rate r = APR/12 = 24%/12 = 0.02
- Pay off the loan over 24 years: t = 24
- Use the monthly payment formula from section 4.2:

$$\text{Monthly payment} = \frac{\text{Amount borrowed } \times r(1+r)^t}{\left((1+r)^t - 1\right)}$$

$$= \frac{\$10{,}000 \times 0.02 \times 1.02^{24}}{(1.02^{24} - 1)} = \$528.71$$

- So, a payment of \$528.71 each month will clear the debt in two years.

Learning Objectives:

- Understand Consumer Price Index (CPI), inflation, rate of inflation, and deflation
- Determine the buying power formula and the inflation formula
- Understand and calculate taxes and stock price
- Determine the Dow Jones Industrial Average (DJIA) changes

- **Consumer Price Index (CPI):** a measure of the average price paid by urban consumers for a "market basket" of consumer goods and services.
- **Inflation:** an increase in prices.
- **The rate of inflation:** measured by the percentage change in the CPI over time.
- **Deflation:** when prices decrease, the percentage change is negative.

$$\text{Percentage change} = \frac{\text{Changes in CPI}}{\text{Previous CPI}} \times 100\%$$

- **Example:** Suppose the CPI increases this year from 200 to 205. What is the rate of inflation for this year?

- **Solution:**

1. The change in CPI = 205 – 200 = 5.
2. The percentage change $= \dfrac{\text{Changes in CPI}}{\text{Previous CPI}} \times 100\%$

$$= \frac{5}{200} \times 100\% = 2.5\%$$

Thus, the rate of inflation is 2.5%.

Buying Power Formula

$$\text{Percent decrease in buying power} = \frac{100\,i}{100 + i}$$

Where *i* is the inflation rate expressed as a percent (not a decimal).

- **Example:** Suppose the rate of inflation this year is 5%. What is the percentage decrease in the buying power of a dollar?
- **Solution**: $i = 5\%$;

$$\text{Percent decrease in buying power} = \frac{100\,i}{100 + i} = \frac{(100 \times 5)}{(100 + 5)}$$

This is about 4.8%.

Inflation Formula

$$\text{Percent rate of inflation} = \frac{100\,B}{100 - B}$$

Where B is the decrease in buying power expressed as a percent (not a decimal).

- **Example:** Suppose the buying power of a dollar decreased by 2.5% this year. What is the rate of inflation this year?
- **Solution:** $B = 2.5\%$;

$$\text{Percent rate of inflation} = \frac{100\,B}{100 - B} = \frac{(100 \times 2.5)}{(100 - 2.5)}$$

This is about 2.6%.

- **Example (calculating the tax: a single person):** In the year 2000, Alex was single and had a taxable income of \$70,000. How much tax did she owe?
- **Solution**:

Table 4.5 on page 254 here

- According to Table 4.5, Alex owed \$14,381.50 plus 31% of the excess taxable income over \$63,550. The total tax is:

$$\$14{,}381.50 + 0.31 \times (\$70{,}000 - \$63{,}550) = \$16{,}381.00$$

- **Example:** In the year 2000, Betty and Carol were single, and each had a total income of \$75,000. Betty took a deduction of \$10,000 but had no tax credits.

  Carol took a deduction of \$9,000 and had an education tax credit of \$1,000. Compare the taxes owed by Betty and Carol.
- **Solution:**

1. **Betty:** the taxable income = \$75,000 – \$10,000 = \$65,000.

By Table 4.5, Betty owes \$14,381.50 plus 31% of the excess taxable income over \$63,550. The total tax is:

$$\$14{,}381.50 + 0.31 \times (\$65{,}000 - \$63{,}550) = \$14{,}831.00$$

Betty has no tax credits, so the tax she owes is \$14,831.00.

2. **Carol:** the taxable income = $75,000 – $9,000 = $66,000.

By Table 4.5, Carol owes $14,381.50 plus 31% of the excess taxable income over $63,550. The total tax is:

$$\$14{,}381.50 + 0.31 \times (\$66{,}000 - \$63{,}550) = \$15{,}141.00$$

Carol has a tax credit of $1,000, so the tax she owes is:

$$\$15{,}141.00 - \$1{,}000 = \$14{,}141.00$$

Betty owes:

$$\$14{,}831.00 - \$14{,}141.00 = \$690.00$$

more tax than Carol.

For every $1 move in any Dow company's stock price, the Dow Jones Industrial Average (DJIA) changes by about 7.56 points.

- **Example (Finding changes in the Dow):** Suppose the stock of Walt Disney increases in value by $3 per share. If all other Dow stock prices remain unchanged, how does this affect the DJIA?
- **Solution:** Each $1 increases causes the average to increase by about 7.56 points. So, $3 increase would cause an increase of about $3 \times 7.56 = 22.68$ points in the Dow.

Chapter 4 Personal Finance: **Chapter Summary**

- **Savings:** simple interest or compound interest
  - Formulas: simple interest earned
    period interest rate
    balance after *t* periods
    APY
    Present value or Future value
    Number of periods to double
- **Borrowing:** an installment loan
  - Formulas: Monthly payment
    Amount borrowed
  - Fixed-rate mortgage vs. ARM

Chapter 4 Personal Finance: **Chapter Summary**

- Saving for the long term: Build the nest egg (Annuity)
  - Formulas: Balance after *t* deposits
    Needed deposit
    Monthly annuity yield
    Nest egg needed
- Credit cards
  - Formulas: Amount subject to finance charges
    Balance after *t* minimum payments
- Inflation, taxes, and stocks
  - Understand CPI, taxes, DJIA