

Solutions

1. Suppose that in January there is a magnitude 3.5 earthquake hitting the east coast of the United States. Six months later, a magnitude 5.5 earthquake hits the west coast. How many times more intense was the west coast quake compared to the east coast quake?

$$5.5 - 3.5 = 2.00$$

$$\text{Relative Intensity} = 10^{\Delta \text{ magnitude}} = 10^2 = \boxed{100}$$

the west coast quake was 100 times more intense than the east coast quake.

2. How many times more intense is a 4.0 magnitude earthquake compared to a 1.0 magnitude earthquake?

$$4 - 1.00 = 3.00$$

$$\text{Relative Intensity} = 10^3 = \boxed{1000 \text{ times}}$$

3. A speaker is playing music at 80 decibels. A second speaker playing the same music at the same decibel reading is placed beside the first. What is the decibel reading of the pair of speakers?

$$\text{Relative Intensity of one speaker} = 1.26^{80}$$

$$\text{Relative Intensity of a pair of speakers} = 2 \times 1.26^{80}$$

$$\text{Decibels} = 10 \log(\text{Relative Intensity}) = 10 \log(2 \times 1.26^{80}) \approx \boxed{83.31}$$

4. If the per capita growth rate of the world population continues to be what it was in the year 2000, the world population  $t$  years after July 1, 2000, will be  $6.085 \times 1.0121^t$  billion. According to this formula, when will the world population reach 12 billion?

$$12 = 6.085 \times 1.0121^t \Rightarrow \frac{12}{6.085} = 1.0121^t \Rightarrow t = \frac{\log\left(\frac{12}{6.085}\right)}{\log(1.0121)} \approx \frac{0.287}{0.0053} \approx \boxed{56.46 \text{ years}}$$

5. The acidity of a solution is determined by the concentration  $H$  of hydrogen ions. The formula is  $\text{pH} = -\log H$ . The accompanying exponential formula is  $H = 0.1^{\text{pH}}$ . Lower pH values indicate a more acidic solution. Normal rain has a pH of 5.7. Suppose acid rain has a pH of 3.3. How many times as acidic as normal rain is this?

$$H = 0.1^{5.7}$$

$$H = 0.1^{3.3}$$

$$\Rightarrow \frac{0.1^{3.3}}{0.1^{5.7}} \approx \boxed{251.2 \text{ times}}$$

After  
July 1, 2000

6. What is the solution to  $3.5 = 1.05^t$ ?

$$t = \log_{1.05} 3.5 = \frac{\log 3.5}{\log 1.05} \approx 25.68 \approx \boxed{25.7}$$

7. You have \$1000 and wish to buy a computer. You find an investment that increases by 5% each month, and you put your \$1000 into the account. When will the amount enable you to purchase a computer costing \$2000?

$$2000 = 1000(1 + 0.05)^t \Rightarrow 2 = 1.05^t$$

$$\Rightarrow t = \log_{1.05} 2 = \frac{\log 2}{\log 1.05} \approx 14.21 \text{ Months}$$

8. Suppose that a certain jet engine up close produces sound at 165 decibels. What is the decibel reading of a pair of nearby jet engines?

$$\text{Relative Intensity of one jet engine} = 1.26^{165}$$

$$\text{Relative Intensity of a pair of jet engines} = 2 \times 1.26^{165}$$

$$\text{Decibels} = 10 \log (\text{Relative Intensity}) =$$

$$= 10 \log (2 \times 1.26^{165}) \approx 168.62$$

9. From 1929 to the early 1930s, the prices of consumer goods actually decreased. Economists call this phenomenon *deflation*. The rate of deflation during this period was around 7% per year. Suppose this rate of deflation persisted over a period of 10 years. What would be the cost after 10 years of an item that costs \$2000 initially?

$$2000(1 - 0.07)^{10} = 2000(0.93)^{10} = 967.96$$

10. The energy released by an earthquake is related to the magnitude by an exponential function:  $\text{Energy} = 25,000 \times 31.6^{\text{Magnitude}}$ . The unit of energy in the above equation is a *joule*. One joule is approximately the energy expressed in lifting  $\frac{3}{4}$  of a pound 1 foot. The earthquake that devastated a certain country on January 12, 2010 had a magnitude of 6.0 and killed hundreds of thousands of people. How much energy was released by this earthquake?

$$\text{Energy} = 25000 \times 31.6^{6.0} = 2.489 \times 10^{13} \text{ Joules}$$