

1. Suppose that in January there is a magnitude 2.5 earthquake hitting the east coast of the United States. Six months later, a magnitude 6.5 earthquake hits the west coast. How many times more intense was the west coast quake compared to the east coast quake?

$$6.5 - 2.5 = 4.00$$

$$\text{Relative Intensity} = 10^{\Delta \text{Magnitude}} = 10^4 = 10000$$

The west coast quake was 10000 more intense than the east coast quake.

2. How many times more intense is a 5.0 magnitude earthquake compared to a 1.0 magnitude earthquake?

$$5.0 - 1.0 = 4.0$$

$$\text{Relative Intensity} = 10^{\Delta \text{Magnitude}} = 10^4 = 10000 \text{ times}$$

3. A speaker is playing music at 60 decibels. A second speaker playing the same music at the same decibel reading is placed beside the first. What is the decibel reading of the pair of speakers?

$$\text{Relative Intensity of One speaker} = 1.26^{60}$$

$$\text{Relative Intensity of a pair of speakers} = 2 \times (1.26^{60})$$

$$\text{Decibel} = 10 \log(\text{Relative Intensity}) = 10 \log(2 \times 1.26^{60}) \approx 63$$

4. If the per capita growth rate of the world population continues to be what it was in the year 2000, the world population t years after July 1, 2000, will be 6.085×1.0121^t billion. According to this formula, when will the world population reach 8 billion?

$$8 = 6.085 \times 1.0121^t \Rightarrow \frac{8}{6.085} = 1.0121^t \Rightarrow t = \frac{\log\left(\frac{8}{6.085}\right)}{\log 1.0121}$$

$$t = 22.75 \text{ years after July 1, 2000}$$

5. The acidity of a solution is determined by the concentration H of hydrogen ions. The formula is $\text{pH} = -\log H$. The accompanying exponential formula is $H = 0.1^{\text{pH}}$. Lower pH values indicate a more acidic solution. Normal rain has a pH of 7.6. Suppose acid rain has a pH of 3.9. How many times as acidic as normal rain is this?

$$H = 0.1^{7.6}$$

$$= 2.5119 \times 10^{-8}$$

$$1.2589 \times 10^{-4}$$

$$\frac{1.2589 \times 10^{-4}}{2.5119 \times 10^{-8}} = 5011.85$$

$$\approx 5012 \text{ times}$$

6. What is the solution to $9.5 = 4.05^t$?

$$t = \log_{4.05} 9.5 = \frac{\log 9.5}{\log 4.05} = 1.6095 \approx 1.61$$

7. You have \$400 and wish to buy a computer. You find an investment that increases by 7% each month, and you put your \$400 into the account. When will the amount enable you to purchase a computer costing \$1200?

$$1200 = 400(1+0.07)^t \Rightarrow \frac{1200}{400} = 1.07^t$$

$$3 = 1.07^t \Rightarrow t = \frac{\log 3}{\log 1.07} \approx 16.24 \text{ months}$$

8. Suppose that a certain jet engine up close produces sound at 185 decibels. What is the decibel reading of a pair of nearby jet engines?

For one
jet engine

$$\text{Relative Intensity} = 1.26^{185} \text{ Decibels} = 1.26^{185}$$

$$\text{A pair of jet engines} \Rightarrow \text{Relative Intensity} = 2 \times 1.26^{185}$$

$$\text{Decibels} = 10 \log(\text{Relative Intensity})$$

$$= 10 \log(2 \times 1.26^{185}) \approx 188.7 \text{ Decibels}$$

9. From 1929 to the early 1930s, the prices of consumer goods actually decreased. Economists call this phenomenon *deflation*. The rate of deflation during this period was around 5% per year. Suppose this rate of deflation persisted over a period of 15 years. What would be the cost after 15 years of an item that costs \$2200 initially?

$$2200(1-0.05)^{15} = 2200(0.95)^{15} \approx 1019.24$$

10. The energy released by an earthquake is related to the magnitude by an exponential function: $\text{Energy} = 25,000 \times 31.6^{\text{Magnitude}}$. The unit of energy in the above equation is a *joule*. One joule is approximately the energy expressed in lifting $\frac{3}{4}$ of a pound 1 foot. The earthquake that devastated a certain country on January 12, 2010 had a magnitude of 7.5 and killed hundreds of thousands of people. How much energy was released by this earthquake?

$$\text{Energy} = 25000 \times 31.6^{7.5} = 4.42 \times 10^{15} \text{ Joules}$$