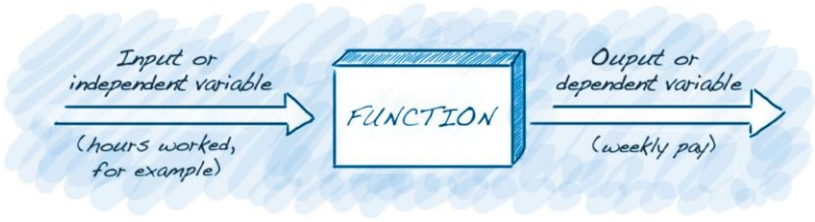
Dr. Katiraie Solutions to Notes 2.1 for Math 115A

Draw picture for function

FIGURE 2.2 Visual representation of a function.



I. Percentage Change

The percentage change or __Relative Change in a function is the percentage Increase in the function from one value of the _Independent variable to another.

Formula

$$\text{Percentage change} = \frac{\text{Change in function}}{\text{Previous function value}} \times 100\%$$

Example:

TABLE 2.1 Population of the United States

Year	Population (millions)	Year	Population (millions)
1790	3.93	1900	76.21
1800	5.31	1910	92.23
1810	7.24	1920	106.02
1820	9.64	1930	123.20
1830	12.87	1940	132.16
1840	17.07	1950	151.33
1850	23.19	1960	179.32
1860	31.44	1970	203.30
1870	38.56	1980	226.54
1880	50.19	1990	248.71
1890	62.98	2000	281.42
		2010	308.75

Find the percentage change in the U.S. population from 1790 to 1800

$$\text{Percentage change} = \frac{\text{Change in function}}{\text{Previous function value}} \times 100\%$$

$$= \frac{\text{Change from 1790 to 1800}}{\text{Population in 1790}} \times 100\%$$

$$= \frac{5.31\text{million} - 3.93\text{ million}}{3.93\text{ million}} \times 100\% = 35\%$$

II. Average growth rate

The average growth rate of a function over an Interval is the change in the function Divided by the change in the Independent variable.

Formula

$$\text{Average growth rate} = \frac{\text{Change in function}}{\text{Change in independent variable}}$$

Example: The population of Russia declined from about 146 million in 2000 to about 143 million in 2007. Calculate the average growth rate over this period and explain its meaning.

- **: The change in population is negative,**
  143 – 146 = –3 million. The change in time is 7 years.

$$\text{Average growth rate} = \frac{\text{Change in function}}{\text{Change in independent variable}} = -\frac{3}{7} = -0.429 \text{ million per year}$$

- **This means that over this interval the population declined, on average, by about 429,000 per year.**

III. Interpolation

Interpolation is the Process of estimating Unknown values between known data points using average Growth rate.

- **Example: In the fall of 2005, 37.7% of college freshmen in the United States believed that marijuana should be legalized. In the fall of 2008, that figure was 41.3%. Use these figures to estimate the percentage in the fall of 2007. The actual figure for 2007 was 38.2%.**
  - **Solution:** The change in the independent variable from 2005 to 2008 is three years.
  - The change in the dependent variable over that period is $41.3 - 37.3 = 3.6$ percentage points.

$$\frac{\text{Change in function}}{\text{Change in independent variable}} = \frac{3.6}{3} = 1.2$$

  - Hence, the average growth rate from the fall of 2005 to the fall of 2008 was 1.2 percentage points per year.

- **What does this say about how the growth rate in the percentage varied over time?**

- **Solution:**

❑ The percentage in 2007 was about:

$$\text{Percentage in 2005} + 2 \text{ years of increase} = 37.7 + 2 \times 1.2 = 40.1\%$$

❑ Our estimate of 40.1% for the fall of 2007 is higher than the actual figure of 38.2%. The figure grew more quickly from 2007 to 2008.

IV. Extrapolation

Extrapolation is the process of estimating unknown values beyond know data points using the average growth rate.

Example

- **The following table shows the average age, in years, of first-time mothers in the given year.**

Year	1970	1980	1990	2000
Average age	21.4	22.7	24.2	24.9

1. **Estimate the average age of first-time mothers in 1997.**
2. **Predict the average age of first-time mothers in 2005.**
3. **Predict the average age of first-time mothers in the year 3000. Explain why the resulting figure is not to be trusted.**

- **Solution:**

1. We estimate the average age in 1997 **by interpolating**.
The average growth rate between 1990 and 2000 is:

- So, the average age of first-time mothers increased at a rate of 0.07 year per year over this decade.
- The average age in 1997 is estimated to be
  $24.7 + 7 \times 0.07 = 24.69$ years, or about 24.7 years.

$$\text{Average growth rate} = \frac{24.9 - 24.2}{10} = 0.07$$

- **Solution:**

2. We estimate the average age **by extrapolating**.

- The average growth rate between 1990 and 2000, that we found in part 1 to be 0.07 year per year.
- Thus, we estimate an increase of 0.07 year over each of the five years from 2000 to 2005:

$$\text{Estimate average age in } 2005 = 24.9 + 5 \times 0.07 = 25.25 \text{ years}$$

Or about 25.3 years.

- **Solution:**

3. This is exactly like part 2.

- The average growth rate from 1990 to 2000 is 0.07 year per year.
- The year 3000 is 1000 years from the year 2000.This growth rate gives a prediction for the year 3000 of:
  $$24.9 + 1000 \times 0.07 = 94.9 \text{ years}$$
- Our projection for the average age in the year 3000 of first-time mothers is 95 years.
- The number is silly and clearly illustrates the danger of extrapolating too far beyond the limits of the given data.