3.1 Notes for Lines and Linear Growth: What does a constant rate mean?

Key concept: A function is called _**Linear**_ if it has a _**Constant** growth rate

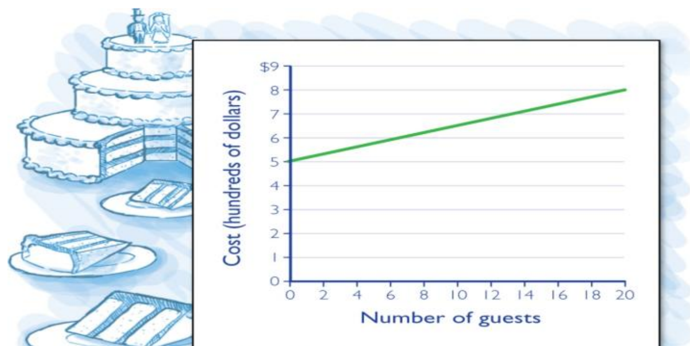
You take notes and put in your own words.

What is positive growth rate? Functions that are increasing
What is negative growth rate? Functions that are decreasing

Example 3.1 In each a function is described. Find the growth rate of the function and give its practical meaning. Make a graph of the function. Is the function linear?

a. For my daughter's wedding reception, I pay $500 rent for the buildings plus $15 for each guest. This describes the total cost of the reception as a function of the number of guest.

- **Solution:** The growth rate is the extra cost incurred for each additional guest, that is $15. So, the growth rate is **constant**.
  The additional cost means each additional guest.
  The total cost of the reception is a **linear** function of the number of guests.



Cost (x) = 15x + 500

FIGURE 3.5 Cost is a linear function of number of wedding guests.

b. My salary is initially $30,000, and I get a 10% salary raise each year for several years. This describes my salary as a function of time.

Solution: The growth rate:

1st year increased = 10% of $30,000 = $3,000

∴ 1st year salary = $33,000

2nd year increased = 10% of $33,000 = $3,300

∴ 2nd year salary = $36,300

The growth rate is **not the same each year**. So, the graph is **not** a straight line. Thus, my salary is **not** a linear function of time in years.

FIGURE 3.6 Salary is not a linear function of time.

Now try the following problem:
3.1 One of the following is a linear function, and one is not. In each case, determine the practical meaning of the growth rate, and then determine whether or not the given function is linear.
a. One inch is the same as 2.54 cm. Consider distance in cm to be a function of distance in inches.

x	y
0 inch	0 cm
1 inch	2.54 cm
2 inches	5.08

Formula **$y = 2.54x + 0$** **$y = mx + b$**

What does it mean? Every 1 inch is equivalent to 2.54 centimeter.

b. Consider the area of a square to be a function of the length of a side.

x	Y = Area
0 inches	0 squared inches
1 inch	1 squared inches
2 inches	4 squared inches
3 inches	9 squared inches

- **Example:** Let L denote the length in meters of the winning long jump in the early years of the modern Olympic Games. Suppose L is a function of the number n of Olympic Games since 1990, an approximate linear formula is $L = 0.14n + 7.20$. Identify the initial values and growth rate, and explain in practical terms their meaning.
- **Solution:** The initial value is 7.20 meters. The growth rate is 0.14 meter per Olympic Game. It means that the length of the winning long jump increased by 0.14 meters from one game to the next.

Now you try the following:
3.2)) Let H denote the height in meters of the winning pole vault in the early years of the modern Olympic Games. We think of H as a function of the number n of Olympic Games since 1900. An approximate linear formula is $H = 0.20n + 3.3$ Identify the initial value and growth rate, and explain in practical terms the meaning of each.

Solutions: The initial value of 3.3 meters is the (approximate) height of the winning pole vault in the 1900 Olympic Games. The growth rate is 0.20 meter per Olympic game means that the height of the winning pole vault increased by approximately 0.20 meter from one game to the next.

Example 3.3-Arocket starting from an orbit 30, 000 kilometers above the surface of Earth blasts off and flies at a constant speed of 1000 kilometers per hour away from the Earth. Explain why the function fiving the rocket's distance from Earth in terms of time is linear. Identify the initial value and growth rate, and find a linear formula for the distance.

- **Solution:**

1. We first choose letters to represent the function and variable. Let d be the distance in km from Earth after t hours.

   The growth rate = velocity = 1000 km/hour = a constant

   Thus, d is a linear function of t.

2. The Initial value = 30,000 km

   = the height above Earth at blastoff

3. d = Growth rate $\times\, t$ + Initial value

   $= 1000\, t + 30{,}000$

Now, you try the following Ex 3.3
3.3 Suppose that a stellar object is first detected at 1, 000, 000 kilometers from Earth and that it is traveling toward Earth at a speed of 2000 Kilometers per hour. Explain why the function giving the object's distance from the Earth in terms of time in linear. Identify the initial value and growth rate, and find a linear formula for distance.

Finding and interpreting the slope Slope = growth rate = -2000 kilometers per hour

Let d denote the distance from Earth in kilometers after t hours. Then d is a linear function of t because the growth rate (the velocity) is constant. The initial value is 1000000 kilometers, and the growth rate is -2000 kilometers per hour (negative because the object is traveling toward earth, so the distance is decreasing).
Therefore, the formula is

Example-Suppose we find snow is falling at a steady rate which means the depth of the snow on the ground is a linear function of the time since it started snowing. At some time during the snow fall, we find the snow is 8 inches deep. Four hours later we find that we now have 20 inches deep. What is the rate of snow?

I like to put my data in point from (0 , 8) (4 , 20)

Then, I use this to figure out the slope remember the slope formula
(x_1, y_1) (x_2, y_2) where $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20-8}{4-0} = \frac{12}{4} = 3 \text{ } inches/hour$

There are two ways to obtain the linear formula

a) By hand: y = 3x + 8
b) By plotting the above points in my calculator and use the regression function of your calculator.

Example_ Temperature conversion between Celsius and Fahrenheit. The temperature in degrees F is a linear function of the temperature in degrees C.

Freezing water at 0 degrees C Which is 32 Degrees F
Boiling temperature of water 100 degrees C which is 212 Degrees F

1. Find the slope $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{212 - 32}{100 - 0} = \frac{180}{100} = 1.8$

2. Find the formula y = 1.8x + 32

3. Graph it

1. Now use calculator to look at line

First we need to plot the points in the scatter plot mode in your calculator

Using your graphing calculator
1. Press 2nd Y= to access the STAT PLOT menu.
2. You can store up to three plots at a time. Choose Plot 1, and press ENTER.
3. Highlight On, press ENTER.
4. Highlight the first type plot shown, press ENTER.
5. For the XList, choose whichever list you used to store your first set of data (usually, this is L1), press ENTER.
6. For the YList, choose whichever list you used to store your second set of data (usually, this is L2), press ENTER.
7. Choose your mark, press ENTER. The first mark is the easiest to see.
8. Go to the Y = menu and clear or inactivate any existing functions.
9. Set the window automatically to see all of your data points by pressing ZOOM and then ZoomStat (#9). Alternatively, you can go to WINDOW and adjust the window settings yourself.

Next, we need to do the following:
Step 2 Enter second mode in your calculator
Step 3 In your calculator, Go to stat calc, and choose linearregression and enter.
Step 4 Graph the function you just found in your calculator.

Key concept-Given a set of data points, the **regression** line or **trend** line is a line that comes as close as possible to fitting those data.

- Given a set of data points, the **regression line** (or **trend line**) is a line that comes as close as possible to fitting those data.
- **Example:** The following table shows the running speed of various animals vs. their length. Show the scatterplot and find the formula for the trend line. Explain in practical terms the meaning of the slope.

Question: How can we find the speed of a 20 inch animal?

Animal	Length (inches)	Speed (feet per second)
Deer mouse	3.5	8.2
Chipmunk	6.3	15.7
Desert crested lizard	9.4	24.0
Grey squirrel	9.8	24.9
Red fox	24.0	65.6
Cheetah	47.0	95.1

- **Solution:**

The points do not fall on a straight line, so the data in the table are not exactly linear. In Figure 3.15, we have added the trend line produced by the spreadsheet program Excel.

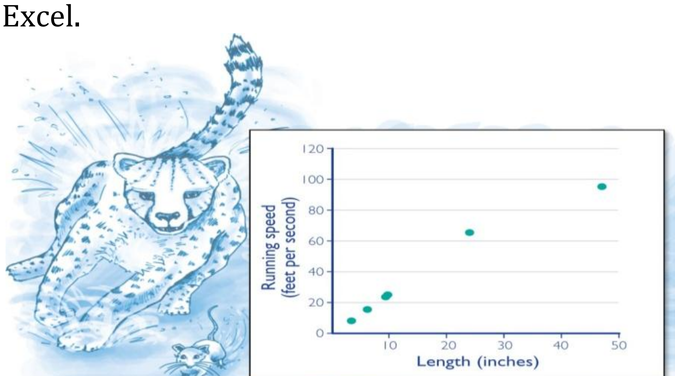


FIGURE 3.14 Scatterplot of running speed versus length.

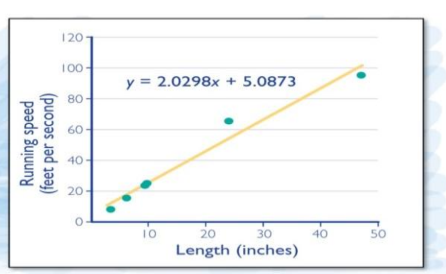


FIGURE 3.15 Trend line added.

Solution: The equation of the trend line is:

$$y = 2.03x + 5.09$$

This means that running speed S in feet per second can be closely estimated by:

$$S = 2.03\,L + 5.09,$$

where L is the length measured in inches.

The slope of the trend line is 2.03 feet per second per inch.

This value for the slope means that an animal that is 1 inch longer than another would be expected to run about 2.03 feet per second faster.

Now, you try the following:

3.4: Scientists believe that thousands of years ago the depth of ice in a glacier was increasing at a constant rate, so the depth in feet was a linear function of time in years. Using a core sample, they measured a depth of 25 feet at one time and a depth of 28 feet five years later. What is the slope of the linear function? Explain in practical terms the meaning of the slope?

Find the slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{28-25}{5-0} = \dfrac{3}{5} = 0.6$ foot per year

The depth of ice in a glacier was increasing at a constant rate of 0.6 foot each year.