

Dr. Katiraie Math 115A Notes section 3.2

Key Concept: An Exponential _____ is a function that changes at a _____ percentage rate.

Example 3-8: If a population triples each hour, does this represent constant percentage growth?

If so, what is the percentage increase each hour?

Is the population size an exponential function of time?

Now, You Try Yourself:

If a population quadruples each hour, does this represent constant percentage growth?

If so, what is the percentage?

Is the population size an exponential function of time?

Exponential Formulas

The formula for an exponential function y of t is:

**An exponential function y of t is characterized by the following property:
When t increases by 1, to find the new value of y , we multiply the current value by the base.**

Example 3-9

- ▶ **Example:** The value of a certain investment grows according to the rule:

$$\text{Next year's balance} = 1.07 \times \text{Current balance}$$

1. Find the percentage increase each year, and explain why the balance is an exponential function of time.
2. Assume that the original investment is \$800. Find an exponential formula that gives the balance in terms of time.
3. What is the balance after 10 years?

Now, You Try Yourself 3.9:

An investment grows according to the rule:

$$\text{Next Month's Balance} = 1.03 \times \text{Current Balance}$$

Find the percentage increase each month.

If the initial investment is \$450, find an exponential formula for the balance as a function of time.

What is the balance after two years?

Example 3-10.

- ▶ **Example:** Consider the investment from the previous example where the balance B after t years is given by:

$$B = 800 \times 1.07^t \text{ dollars}$$

What is the growth of the balance over the first 10 years?
Compare this with the growth from year 40 to year 50.

- ▶ **Solution:** The balance after 10 years was \$1573.72.
Growth over first 10 years = \$1573.72 - 800 = \$773.72.

To calculate the growth from year 40 to year 50:

$$\text{Balance after 40 years} = 800 \times 1.07^{40} = \$11,979.57$$

$$\text{Balance after 50 years} = 800 \times 1.07^{50} = \$23,565.62$$

That is an increase of \$23,565.62 - \$11,979.57 = \$11,586.05.
That is almost 15 times the growth over the first 10 years.

Try it yourself 3-10:

After t months, an investment has a balance of $B = 400 \times 1.05^t$ dollars

Compare the growth rate over the first 20 months with the growth from months 50 to month 70.

Definition: Exponential Growth

1. A quantity grows exponentially when it increases by a _____ percentage over a given period.
2. If r is the percentage growth per period then the base of the exponential function is $1 + r$. For exponential growth, the base is always greater _____

$$\text{Amount} = \text{Initial value} \times (1 + r)^t$$

Here, t is the number of _____

3. Typically, exponential growth starts _____ and then increases _____.

Example 3-11

▶ **Example:** U.S. health-care expenditures in 2010 reached 2.47 trillion dollars. In the near term this is expected to grow by 6.5% each year. Assuming that this growth rate continues, find a formula that gives health-care expenditures as a function of time. If this trend continues, what will health-care expenditures be in 2030?

▶ **Solution:** Let H be the expenditures t years after 2010.

$$H = \text{Initial value} \times (1 + r)^t = 2.47 \times (1 + 0.065)^t.$$

To predict health-care expenditures in 2030, use $t = 20$ in the formula for H :

$$\text{Expenditures in 2030} = 2.47 \times (1 + 0.065)^{20} \text{ trillion dollars.}$$

The above result is about 8.7 trillion dollars.

Try it yourself 3-11:

U.S defense spending was about 719.2 billion dollars in 2010. Since that time, it has grown by about 9.5% each year. Assume that this growth rate continues, and find a formula that gives defense spending as a function of time. What prediction does this formula give for defense spending in 2015?

► **Exponential Decay**

1. A quantity decays exponentially when it decreases by a constant percentage over a given period.
2. If r is the percentage decay per period, then the base of the exponential function is $1 - r$: For exponential decay, the base is always less than 1.

Here, t is the number of periods.

3. Typically, exponential decay is rapid first but eventually slows.

$$\text{Amount} = \text{Initial value} \times (1 - r)^t$$

For exponential decay, the base is always less than one
Exponential decay is fast at first and then eventually _____

Example 3.12 on p. 156

After antibiotics are administered, the concentration in the bloodstream declines over time. Suppose that 70 milligrams (mg) of amoxicillin are injected and that the amount of the drug in the bloodstream declines by 49% each hour.

Find an exponential formula that gives the amount of amoxicillin in the bloodstream as a function of time since the injection. Another injection will be required when the level declines to 10 mg. Will another injection be required before five hours?

Try it yourself 3-12:

A certain population is initially 4000 and declines by 10% per year. Find an exponential formula that gives the population as a function of time.

What will be the population after 10 years?

Radioactive decay and half life

Key concept- The half-life of a _____ substance is the time it takes for half of the substance to decay

After h half-lives, the amount of a radioactive substance remaining is given by

the exponential formula: $Amount\ remaining = Initial\ Amount * \left(\frac{1}{2}\right)^h$

We can find the amount remaining after t years by first expressing t in terms of half-lives and then using the formula above.

Example: Pu-239 has a half life of 24,000 years. We start with 100 gm. So, after 24000 years we would get _____ gm.

$$Amount\ remaining = Initial\ Amount * \left(\frac{1}{2}\right)^h$$

How much material is left after 36,000 years?

Try it yourself 3-14:

A certain radioactive substance has a half-life of 450 years. If there are initially 20 grams, find a formula for the exponential function that gives the amount remaining after h half lives.

How much remains after 720 years? Round your answer to one decimal place.