Dr. Katiraie Math 115A Notes Section 3.3
Logarithmic phenomena: compressed scales

Homework- Read section 3.3 Some hints for reading. Read the key concepts closely, look closely at the purple-lined boxes, and then read and try to understand the example chronologically. You can now begin your homework on-line for section 3-3.

Logarithmic function-is the kind of function that **Reverses** the effect of an exponential function

Open your books and look at p. 166 on the top

Exponential (Section 3.2) now Logarithmic (3.3)
Population vs. time Time vs. population

$100 = 10^2$ Log 100 = 2

t-chart—to reverse the function you switch the x axis and y axis and then graph.
This is what you are seeing on p. 166

Key Concept- The common logarithm of a positive number x, written **Log x**
Is the **Exponent** of 10 that gives x.

$$\boxed{\log x = t \text{ if and only if } 10^t = x}$$

1. log 10 = 1 because 10^ 1 = 10
2. log 100 = 2 because 10^ 2 = 100
3. log 1000= 3 because 10^ 3 = 1000
4. log 1/10 = -1 because 10^ -1 = 1 / 10

Example- What is the logarithm of

A) 1 million = 10^6 Therefore, Log 1000000 = 6
B) one thousandths = 0.001 = 10^-3 Therefore, log0.001 = -3
C) 5? 5 is about 10^0.699 Therefore, log 5 ~ 0.699

Now, you try 3.15: What is logarithm of 1 billion?

1 billion = 10^9 Therefore, log 1000000000 = 9

Real World Examples of Logarithms

I. ***Key Concept***- The relative intensity of an earthquake is a measurement of ground movement. The **Magnitude** of an earthquake is the _**Logarithm** of relative intensity

$$\text{Magnitude} = \log(\text{Relative intensity}),$$

$$\text{Relative intensity} = 10^{\text{Magnitude}}$$

Look at the chart on p. 169 Richter magnitude

Example. If an earthquake has a relative intensity of 6700, what is its magnitude?

- **Solution:** Magnitude = log(Relative intensity) = log(6700) ≈ 3.8

Meaning-an increase of 1 unit on the Richter scale corresponds to increasing the relative intensity by a factor of 10
-an increase of t units in magnitude corresponds to increasing relative intensity by a factor of __**10^t**______

- **Meaning of magnitude changes**

1. An increase of 1 unit on the Richter scale corresponds to increasing the relative intensity by a factor of 10.
2. An increase of t units in magnitude corresponds to increasing the relative intensity by a factor of 10^t.

- **Example:** In 1994 an earthquake measuring 6.7 on the Richter scale occurred in Northridge, CA. In 1958 an earthquake measuring 8.7 occurred in the Kuril Islands. How did the intensity of the Northridge quake compare with that of the Kuril Islands quake?

- **Solution:** The Kuril Islands quake was $8.7 - 6.7 = 2$ points higher. Increasing magnitude by 2 points means that relative intensity increases by 10^2. The Kuril Islands quake was 100 times as intense as the Northridge quake.

II. Decibel as a measure of sound

Key Concept- The decibel rating of sound is <u>**10**</u> times the logarithm of its <u>**Relative**</u> intensity.

Formulas

Decibels = 10 log(Relative Intensity)

The corresponding exponential equation is :

Relative Intensity = 10^ (0.1* Decibels)

Or in the approximation form:

Relative Intensity = 1.26 ^ Decibels

Decibels are given in whole numbers

Look at the sound chart in your book on p. 171

Summary

- An increase of one decibel multiplies relative intensity by 1.26
- An increase of t decibels multiplies relative intensity by 1.26^t

Example—Look in your book at p. 172 Example 3.18

Summary: Interpreting decibel Changes:Vaccum Cleaner to Bulldozer

Vacuum cleaner 80 decibels

Bull dozer 85 decibels

Compare: According to the above formula, an increase of t decibels multiplies relative intensity by 1.26^t. Therefore, increasing the number of decibels by five multiplies the intensity by 1.26^5 or about 3.2

Thus, the sound from the bulldozer is about 3.2 times as intense as that of a vacuum cleaner.

Example 3-19: A stereo plays 60 decibel. What decibel would we expect with a second speaker?

We are doubling the intensity of the sound not the loudness.
Relative Intensity = 1.26 ^ Decibels= 1.26 ^ 60
With a second speaker added, the new relative intensity is doubled
New Relative Intensity = 2 * 1.26^60
Therefore, the decibel reading of the pair of speakers is given by
Decibels = 10 log(Relative Intensity)= 10 log(2 * 1.26^60)= 63 decibels

- **Properties of Logarithms**

1. Logarithm rule 1: $\log(\mathrm{A}^t) = t\log(\mathrm{A})$
2. Logarithm rule 2: $\log(\mathrm{AB}) = \log(\mathrm{A}) + \log(\mathrm{B})$
3. Logarithm rule 3: $\log\left(\frac{\mathrm{A}}{\mathrm{B}}\right) = \log(\mathrm{A}) - \log(\mathrm{B})$

- **Example:** Suppose we have a population that is initially 500 and grows at a rate of 0.5% per month. How long will it take for the population to reach 800?

- **Solution:** The monthly percentage growth rate, $r = 0.005$.

The population size *N* after *t* months is:

$$N = \text{Initial value} \times (1+r)^t = 500 \times 1.005^t$$

To find out when $N = 800$, solve the equation:

$$800 = 500 \times 1.005^t$$

Divide both sides by 500:

$$1.6 = 1.005^t$$

Apply the logarithm function to both sides and use rule 1:

$$\log 1.6 = \log(1.005^t) = t\log(1.005)$$

Dividing by log 1.005 gives:

$$t = \frac{\log 1.6}{\log 1.005} = 94.2 \text{ months}$$

The population reaches 800 in about 7 years and 10 months.

$$t = \frac{\log A}{\log B}$$

- **Solving exponential equations**

The solution for *t* of the exponential equation $A = B^t$ is:

- **Example:** An investment is initially $5000 and grows by 10% each year. How long will it take the account balance to reach $20,000?
- **Solution:** The balance *B* after *t* years:

$$B = \text{Initial value} \times (1 + r)^t = 5000 \times 1.1^t$$

To find when $B = \$20{,}000$,

solve $20{,}000 = 5000 \times 1.1^t$, or $4 = 1.1^t$ with $A = 4$ and $B = 1.1$ of exponential equation $A = B^t$.

$$t = \frac{\log A}{\log B} = \frac{\log 4}{\log 1.1} = 14.5 \text{ years}$$

<u>Doubling time and More</u>

- **Doubling Time and more**

Suppose a quantity grows as an exponential function with a given base. The time *t* required to multiply the initial value by *K* is:

$$\text{Time required to multiply by } K \text{ is } t = \frac{\log K}{\log(\text{Base})}$$

The special case *K* = 2 gives the **doubling time:**

$$\text{Doubling time} = \frac{\log 2}{\log\,(\text{Base})}$$

- **Example:** Suppose an investment is growing by 7% each year. How long does it take the investment to double in value?

- **Solution:** The percentage growth is a constant, 7%, so the balance is an exponential function.
- The base $= 1 + r = 1.07$:

$$\text{Doubling time} = \frac{\log 2}{\log(\text{Base})} = \frac{\log 2}{\log(1.07)} = 10.2 \text{ years}$$

Ex. P. 176 in book look at Ex. 3.22

- **Example:** Recall that carbon-14 has a half-life of 5770 years. Suppose the charcoal from an ancient campfire is found to contain only one-third of the carbon-14 of a living tree. How long ago did the tree that was the source of the charcoal die?

- **Solution:** Use $K=1/3$, the base $=$ half-life $= 1/2$:

$$\text{Time to multiply by } 1/3 \text{ is } t = \frac{\log K}{\log(\text{Base})} = \frac{\log(1/3)}{\log(1/2)} = 1.58.$$

Each half-life is 5770 years, the tree died $1.58 \times 5770 = 9116.6$ years ago.

Chapter 3 Linear and Exponential Changes:

Chapter Summary

- **Lines and linear growth:** What does a constant rate mean?
  - Understand linear functions and consequences of a constant growth rate.

  Recognizing and solve linear functions
  Calculate the growth rate or slope
  Interpolating and using the slope
  Approximate the linear data with trend lines

- **Exponential growth and decay:** Constant percentage rates
  - Understand exponential functions and consequences of constant percentage change.

  The nature of exponential growth
  Formula for exponential functions
  The rapidity of exponential growth
  Relating percentage growth and base
  Exponential decay
  Radioactive decay and half-life

- **Logarithmic phenomena**: Compressed scales
  - Understand the use if logarithms in compressed scales and solving exponential equations.

  The Richter scale and interpolating change on the Richter scale
  The decibel as a measure of sound
  Solving exponential equations
  Doubling time and more