Solutions

Dr. Katiraie Math 115A Notes for Section 3-4 Quadratics and Parabolas

The general form for solving quadratic equations is

$ax^2 + bx + c = 0$

Easy type factoring example: $x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x+3=0 \quad x+2=0$

Solutions $\boxed{x = -3}$ $\boxed{x = -2}$

Another way to obtain the solution for Quadratics is the quadratic formula

$ax^2+bx+c=0$

Formula: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Ex. Solve $x^2 - 3x - 2 = 0$

a = 1 b = -3 c = -2

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2} = \frac{3 \pm \sqrt{9+8}}{2}$$

Exact Solutions x = $\dfrac{3+\sqrt{17}}{2}$ x = $\dfrac{3-\sqrt{17}}{2}$

Approximate solutions x = 3.56 x = -0.56

Now you try.

Solve <u>by facoring</u>

1. **$x^2+4x-5 = 0$** $(x+5)(x-1)=0 \Rightarrow \boxed{x=-5}$ & $\boxed{x=1}$
2. **$x^2+6x-27 = 0$** $(x+9)(x-3)=0 \Rightarrow \boxed{x=-9}$ & $\boxed{x=3}$

Solve <u>by using the quadratic formula</u>

3. **$x^2+2x-2 = 0$** $a=1 \quad b=2 \quad c=-2$

$$x = \frac{-2 \pm \sqrt{2^2-4(1)(-2)}}{2} = \frac{-2 \pm \sqrt{12}}{2} \rightarrow \boxed{0.73}, \boxed{-2.73}$$

$a=1 \quad b=1 \quad c=-1$

4. $\mathbf{x^2 + x - 1 = 0}$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2}$$

$$x = \frac{-1 \pm \sqrt{1+4}}{2} \begin{cases} \rightarrow \boxed{0.618 \approx 0.62} \\ \rightarrow \boxed{-1.62} \end{cases}$$

I. Example of using the Quadratic Formula in real life: Throwing a Rock

If a rock is thrown downward with an initial speed of 32 feet per second, the distance D, in feet , that the rock travels in t seconds is given by

$$D = 16t^2 + 32t$$

Ex. If the rock is thrown from the top of a tower that is 128 feet tall, how long does it take for the rock to hit the ground?

Using the formula above we get $128 = 16t^2 + 32t$

Then we set the equation to zero so that we can factor this equation.

$$16t^2 + 32t - 128 = 0$$

Its easier to factor if we divide every term by 16. Then we obtain.

$$t^2 + 2t - 8 = 0$$

$$t = \frac{-2 \pm \sqrt{2^2 - 4(1)(-8)}}{2} \begin{cases} \rightarrow \frac{-2+\sqrt{36}}{2} = \boxed{2 \text{ seconds}} \\ \rightarrow \frac{-2-\sqrt{36}}{2} = -4 \text{ seconds} \end{cases}$$

Solving this we obtain t = 2 and t = -4 seconds

The answer is After 2 seconds the rock will hit the ground.

Now you try using this formula to throw a rock from a 48 feet tower, how long does it take for the rock to hit the ground?

$$48 = 16t^2 + 32t \Rightarrow 16t^2 + 32t - 48 = 0$$

$a=16 \quad b=32 \quad c=-48$

$$\frac{-32 \pm \sqrt{32^2 - 4(16)(-48)}}{32} \begin{cases} \rightarrow 1 \\ \rightarrow -3 \end{cases}$$

$\boxed{t = \text{1 second}}$

II. Example using the Quadratic Formula for stable population levels. For a certain population the growth rate G, in thousands of individuals per year depends on the size N, in thousands, of the population. The relation is

$$G = 1 + N - .2\,N^2$$

-the population level is stable so the growth rate is 0. At what level is the population stable?

$$-0.2N^2 + N + 1 = 0$$

a = −0.2 b = 1 c = 1

$$N = \frac{-1 \pm \sqrt{1^2 - 4(-0.2)(1)}}{2(-0.2)}$$

$$\frac{-1 + \sqrt{1 + 0.8}}{-0.4} = -0.854$$

$$\frac{-1 - \sqrt{1 + 0.8}}{-0.4} = \boxed{5.854}$$

Solution N = −0.85 N = 5.85

But the solution is 5.854 thousand because ~~5854 people~~ (Population can not be Negative)

You try to find the level at which the population is stable using

G= 2 + 2N - .3N²

$$-0.3N^2 + 2N + 2 = 0$$

$a = -0.3$ $b = 2$ $c = 2$

$$N = \frac{-2 \pm \sqrt{2^2 - 4(-0.3)(2)}}{2(0.3)} =$$

~~−0.883~~

$\boxed{7.550 \text{ thousand}}$

OR 7,550 individuals.

Dr. Katiraie-Math 115A Part 2 Section3.4 Parabolas Notes

Quadratics have the shape of a Parabola

i. opens upward if a is positive

Example $y = 3x^2 + 6x + 5$

$$x = \frac{-6}{2(3)} = -1$$

Formula $x = \frac{-b}{2a} = -1$ Vertex-- $(-1, 2)$ point when parabola opens upward.

ii. opens downward if a is Negative

$$y = -4x^2 + 8x + 6$$

$$x = -\frac{b}{2a} = \frac{-8}{2(-4)} = 1$$

Formula $x = \frac{-b}{2a}$ Vertex--- $(1, 10)$ point when parabola opens downward.

Vertex Formula $\left(x = \frac{-b}{2a},\ f\left(\frac{-b}{2a}\right) \right)$

x value $x = \frac{-b}{2a} = \frac{-8}{2(-4)} = 1$

y value $= -4(1)^2 + 8(1) + 6$
$= -4 + 14 = 10$

Example one—Find the vertex of $y = 3x^2 - 24x + 10$

$$x = \frac{-b}{2a} = \frac{-(-24)}{2(3)} = \frac{24}{6} = 4$$

$$y = 3(4)^2 - 24(4) + 10 = -38$$

Vertex is $(4, -38)$

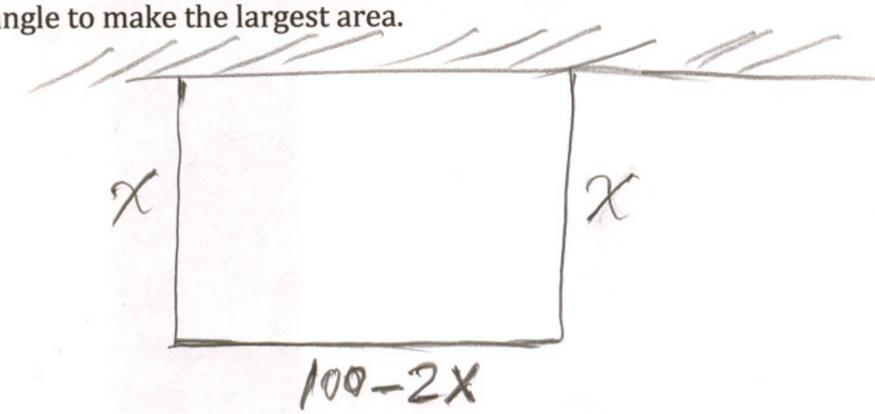
$x = \frac{-b}{2a}$

****Now you try to find the vertex of y = 2x² -28x +12

$x = \frac{-(-28)}{2(2)} = \frac{28}{4} = 7$, $y = -86$ $\boxed{(7, -86)}$

Example using vertex—A rectangular pen can be constructed using the side of a barn as one boundary and 100 ft. of fence to make the other three sides. Find the length and width of the rectangle to make the largest area.

Draw a picture-



Work to solve problem.

Area = length * width

$= (100-2X)X = 100X - 2X^2$

$A(X) = -2X^2 + 100X$

$x = \frac{-100}{2(-2)} = \frac{100}{4} = 25$

x = width = $\boxed{25 \text{ ft}}$ y = length= 100 – 2x = $\boxed{100 - 2(25) = 50 \text{ ft}}$

****Now you try the same problem but using 200 feet for fencing.

$X(200-2X) = 200X - 2X^2 = -2X^2 + 200X$

width $X = \frac{-200}{2(-2)} = \boxed{50 \text{ feet}}$ length $= 200 - 2(50) = \boxed{100 \text{ ft}}$