110 Dr. Katiraie Solutions to Practice Quiz 8 Sections 4.1 -- 4.4
Name Solutions

1) FINDING THE INTERSECTION POINT OF TWO LINES BY THE ELIMINATION METHOD:
Example: Solve the following system of equations.

$3x + 9y = 45$
$2x + y = 10$

STEP 1: Decide whether to eliminate the x or y variable. Usually you make this choice by seeing which variable's coefficients has the smaller LCM. The coefficients of x are 2 and 3 LCM(2, 3) = 6. The coefficients of y are 1 and 9 LCM(1, 9) = 9. So we will eliminate the x variable.

Multiply the top equation by 2 and the bottom equation by –3, so that one coefficient of x is +6 and the other is –6. You must make one coefficient positive and one negative for the variable you want to eliminate.

$2(3x + 9y) = 2(45) \rightarrow 6x + 18y = 90$
$-3(2x + y) = -3(10) \rightarrow -6x - 3y = -30$

STEP 2: ***ELIMINATE*** one variable (in this case x), by adding the two equations.

$15y = 60$

STEP 3: Solve for the remaining variable (in this case y).

Y = 60/15 = 4

STEP 4: ***SUBSTITUTE*** into either original equation to find other coordinate of intersection point.

$3x + 9(4) = 45$ or $2x + 4 = 10$
$3x + 36 = 45$ $2x = 6$
$3x = 9$ $x = 3$
$x = 3$

STEP 5: Write final answer.

The intersection point is (3, 4).

2) FINDING THE INTERSECTION POINT OF TWO LINES BY THE SUBSTITUTION METHOD:
Solve the following system of equations by Substitution Method

$3x + 9y = 45$
$2x + y = 10$

STEP 1: Solve each equation for y.

$3x + 9y = 45$
$9y = -3x + 45$
$y = -\frac{1}{3}x + 5$

$2x + y = 10$
$y = -2x + 10$

STEP 2: ***SUBSTITUTE*** the solution for y of the first equation into y for the second equation.

$-\frac{1}{3}x + 5 = -2x + 10$

STEP 3: Solve for x.

$3(-\frac{1}{3}x + 5) = 3(-2x + 10)$ *Clear the fraction*
$-x + 15 = -6x + 30$ *Combine like terms*
$5x = 15$ *Isolate x*
$x = 3$ *Solution*

STEP 4: ***SUBSTITUTE*** to find other coordinate of intersection point.

$y = -2x + 10 = -2(3) + 10 = 4$

STEP 5: Write final answer.

The intersection point is (3, 4).

3) Use these matrices to answer the following questions.

$$A = \begin{bmatrix} 4 & -3 & 7 \\ 5 & 0 & -8 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 5 \\ 0 & -8 \end{bmatrix} \quad C = \begin{bmatrix} 4 & -3 & 7 \end{bmatrix} \quad D = \begin{bmatrix} 5 & -2 & 9 \\ 3 & 0 & -6 \\ 4 & -1 & -2 \end{bmatrix}$$

$$E = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \qquad F = \begin{bmatrix} 4 & -3 \\ 5 & 0 \\ 9 & 2 \\ 7 & -8 \end{bmatrix} \qquad G = \begin{bmatrix} w & x \\ y & z \end{bmatrix} \qquad H = \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix}$$

A. List the size of each of the following matrices:

Solution:

Please recall that the size of a matrix is always the number of rows X number of columns

A = 2 X 3 B = 2 X 2 C = 1 X 3 D = 3 X 3

E = 3 X 2 F = 4 X 2 G = 2 X 2 H = 3 X 1

B. Do not compute – just answer question!! Are the following products possible to compute? If so, write yes in the blank. If not, explain why not – be brief – but specific!

AD = (2 X 3) (3 X 3) = (2 X 3) YES EF = (3 X 2)(4 X 2) = Not Possible

FD = (4 X 2)(3 X 3) = Not Possible FG = (4 X 2)(2 X 2) = Yes, this is Possible

C. Find the product BG

Solution:

The size of BG = (2 X 2)(2 X 2) = (2 X 2) which is possible

$$B = \begin{bmatrix} -3 & 5 \\ 0 & -8 \end{bmatrix} \quad G = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$$

$$\text{Then } BG = \begin{bmatrix} -3 & 5 \\ 0 & -8 \end{bmatrix} * \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} -3w+5y & -3x+5z \\ 0w-8y & 0x-8z \end{bmatrix} = \begin{bmatrix} -3w+5y & -3x+5z \\ 0-8y & 0-8z \end{bmatrix}$$

D. Find the result of 3 times matrix B, namely: 3B

Solution:

$$3B = 3\begin{bmatrix} -3 & 5 \\ 0 & -8 \end{bmatrix} = \begin{bmatrix} 3(-3) & 3*5 \\ 3(0) & 3*(-8) \end{bmatrix} = \begin{bmatrix} -9 & 15 \\ 0 & -24 \end{bmatrix}$$

E. Find matrix B added to matrix G, namely B + G

Solution:

$$B + G = \begin{bmatrix} -3 & 5 \\ 0 & -8 \end{bmatrix} + \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} -3+w & 5+x \\ 0+y & -8+z \end{bmatrix}$$

F. Find the inverse of matrix B namely, B^-1

Solution:

First go to Matrix menu of your calculator, and input matrix B
Then go to Matrix menu of your calculator and pick matrix B, then use the x^-1 button of your calculator, and then use the MATH Frac part of your calculator, and get

$$B^{-1} = \begin{bmatrix} -\frac{1}{3} & \frac{-5}{24} \\ 0 & \frac{-1}{8} \end{bmatrix}$$

G. Find the result of matrix G being subtracted from matrix B, namely B - G

Solution:

$$B - G = \begin{bmatrix} -3 & 5 \\ 0 & -8 \end{bmatrix} - \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} -3-w & 5-x \\ 0-y & -8-z \end{bmatrix}$$

4) A grain dealer sold to one customer 5 bushels of wheat, 2 of corn, and 3 of rye, for $ 31.00. To another customer he sold 2 bushels of wheat, 3 of corn, and 5 of rye, for $ 27.60. To a third customer he sold 3 bushels of wheat, 5 of corn, and 2 of rye for $ 32.70. What was the price per bushel for each of the different grains?

Solution:

Set up matrix equations for this problem and use inverses to solve.

Let x represent the price per bushel for wheat,
y the price per bushel for corn, and
z the price per bushel for rye.

Write the matrix algebra system for this problem:

$$\begin{bmatrix} 5 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 31.00 \\ 27.60 \\ 32.70 \end{bmatrix}$$

Use inverses to solve the system

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 31.00 \\ 27.60 \\ 32.70 \end{bmatrix} = \begin{bmatrix} 3.61 \\ 3.61 \\ 1.91 \end{bmatrix}$$

Write out the solution to the problem.

The wheat sells for $3.61 per bushel, the corn sells for $3.61 per bushel, and the rye sells for $1.91 per bushel.