MA 110 TEST 3 REVIEW   Dr.   Katiraie

Name __Solutions_____

1. A poll is conducted among the customers in a diner. The people are asked to state whether they would prefer a chicken (C) or beef (B) entrée, and which of the following sides they would prefer: rice (R), potatoes (P), or steamed veggies (S).

    A. Write the sample space for this poll.

    S = {CR, CP, CS, BR, BP, BS}

    B. List the outcomes which belong to the event A = "the customer prefers chicken"
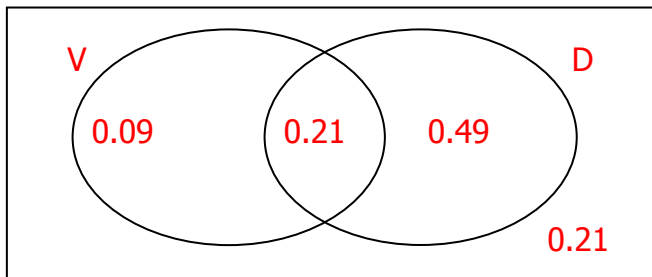
    A = {CR, CP, CS}

    C. List the outcomes which belong to the event B = "The customer prefers chicken and not rice"

    B = {CP, CS}

2.   In a certain town 70% of the people own dogs. Thirty percent of the people in the town are Vegetarians, and 21% of the vegetarians own dogs.

    A. Draw a two-circle Venn Diagram using the information given.

    B. What is the probability that a person is a vegetarian or owns a dog? 0.79= (0.09 + 0.21 + 0.49)

    C. What is the probability that a person is a vegetarian, but does not own a dog? 0.09

    D. What is the probability that a person is a vegetarian, given they own a dog?   0.21/0.7 = 0.3

    E. Are the events "person is a vegetarian" and "person owns a dog" independent? Use an appropriate calculation not intuition to answer.

    Yes, P(V) = .3 = P(V|D) = 0.21/0.7 = 0.3

3. A shipment of 70 hand-held digital planners, including eight that are defective, is sent to a large electronics store.

    A. If one planner is selected, what is the probability that it is defective?   8/70 = 0.114

    B. If five planners are selected, what is the probability that all five are defective?

    $$\frac{C_{8,5}}{C_{70,5}} = \frac{56}{12,103,014} \approx 0.0000046 \text{ or } \frac{8}{70} \bullet \frac{7}{69} \bullet \frac{6}{68} \bullet \frac{5}{67} \bullet \frac{4}{66} = \frac{6720}{1,452,361,680} \approx 0.0000046$$

    C. If five planners are selected, what is the probability that exactly three are defective?

    $$\frac{C_{8,3} \bullet C_{62,2}}{C_{70,5}} = \frac{56 \bullet 1891}{12,103,014} = \frac{105,896}{12,103,014} \sim 0.009$$

    D. If the original shipment of 70 hand-held digital planners, with 8 defective were representative of a larger batch of 4000 planners, how many planners would you expect to be defective in this larger batch of 4000?

    (8/70)(4000) ~ 457 planners

4. The odds of Americans living in the state in which they were born are 24 to 7. What is the probability that an American lives in the state in which he or she was born?

    24/(24 + 7) = 24/31 = 0.77

5. The probability that a person wins a certain game is 11/32, what are the odds that the person wins the game?

    11 to 21   (32 − 11 = 21)

6. College students (245 students total) were given three choices of pizza toppings and asked to choose one favorite. The following table shows the results.
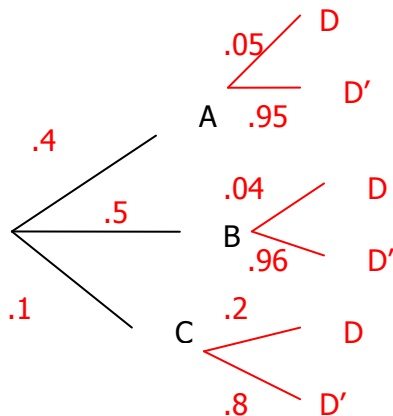
Toppings	Freshman F	Sophomore So	Junior J	Senior S
Cheese C	15	13	24	26
Meat M	27	26	13	15
Veggie V	18	15	27	26

What is the probability that the person selected

A. is a freshman **or** chooses veggie as their favorite topping: P(F ∪ V)

(15 + 27 + 18 + 15 + 27 + 26)/ 245 ➜ 128/245

B. chooses veggie as their favorite topping: P(V)

(18 + 15 + 27 + 26)/ 245 ➜ 86/245

C. is a freshman **and** chooses cheese as their favorite topping: P(F ∩ C)

15/ 245

D. chooses **neither** Cheese **nor** veggie as their favorite topping: P(C' ∩ V') or P((C ∪ V)')

(27 + 26 + 13 + 15)/ 245 ➜ 81/245

E. is a freshman **given that** the person chooses veggie as their favorite topping: P(F|V)

18/(18 + 15 + 27 + 26) ➜ 18/86

F. choose veggie as their favorite topping **given that** the person is a freshman: P(V|F)

18/ (15 + 27 + 18) ➜ 18/60

7. A store sells three brands of CD players. Forty percents of the CD players they sell are manufactured by Amex, 50% are made by Bates and the remainder are made by Circo. The store has found that 5% of the CD players made by Amex are defective, 4% of the Bates CD players are defective, and 20% of the Circo CD players are defective.

A. Draw a tree diagram with the given information. It is partially outlined for you.

```
              .05    D
                 /
          A   ———
      .4     .95  D'

          .04    D
       .5    /
          B ———
        .96    D'

    .1    .2    D
          /
       C ———
        .8    D'
```

B. If it is **known that** a CD player is supplied by the Circo company, what is probability that the CD Player is not defective?

P(D'|C) = .8

C. What is the probability that a CD player at the store comes from the Bates company?

P(B) = .5

D. What is the probability that a CD player is both made by Bates **and** is defective?

P(B ∩ D) = .5(.04) = .02

E. What is the probability that a CD player is defective?

P(D) = .4(.05) + .5(.04) + .1(.2) = .06

F. If it is **known that** a CD player is defective, what is the probability that it came from the Bates company?

$$P(B|D) = \frac{P(B \cap D)}{P(D)} = \frac{.02}{.06} = \frac{2}{6} = \frac{1}{3} = .33$$

8) FINDING THE INTERSECTION POINT OF TWO LINES BY THE ELIMINATION METHOD:
<u>Example:</u>   Solve the following system of equations.

 3x + 9y = 45
 2x + y = 10

<u>STEP 1:</u>   Decide whether to eliminate the x or y variable. Usually you make this choice by seeing which variable's coefficients has the smaller LCM. The coefficients of x are 2 and 3 LCM(2, 3) = 6. The coefficients of y are 1 and 9 LCM(1, 9) = 9.   So we will eliminate the x variable.

Multiply the top equation by 2 and the bottom equation by –3, so that one coefficient of x is +6 and the other is –6. You must make one coefficient positive and one negative for the variable you want to eliminate.

 2(3x + 9y) = 2(45) ➔   6x + 18y = 90
 -3(2x + y) = -3(10) ➔   -6x –   3y = -30

<u>STEP 2:</u>   **ELIMINATE** one variable (in this case x), by adding the two equations.

$$15y = 60$$

<u>STEP 3:</u>   Solve for the remaining variable (in this case y).

Y = 60/15 = 4

<u>STEP 4:</u>   **SUBSTITUTE**   into either original equation to find other coordinate of intersection point.

 3x + 9(4) = 45   or   2x + 4 = 10
 3x + 36 = 45   2x = 6
 3x = 9   x = 3
 x = 3

<u>STEP 5:</u>   Write final answer.

    The intersection point is (3, 4).

9) FINDING THE INTERSECTION POINT OF TWO LINES BY THE SUBSTITUTION METHOD:
Solve the following system of equations by Substitution Method

 3x + 9y = 45
 2x + y = 10

STEP 1:   Solve each equation for y.

$$3x + 9y = 45 \qquad\qquad 2x + y = 10$$
$$9y = -3x + 45 \qquad\qquad y = -2x + 10$$
$$y = -\frac{1}{3}x + 5$$

STEP 2:   **SUBSTITUTE** the solution for y of the first equation into y for the second equation.

$$-\frac{1}{3}x + 5 = -2x + 10$$

STEP 3:   Solve for x.

$$3(-\frac{1}{3}x + 5) = 3(-2x + 10) \qquad \textit{Clear the fraction}$$
$$-x + 15 = -6x + 30 \qquad \textit{Combine like terms}$$
$$5x = 15 \qquad \textit{Isolate x}$$
$$x = 3 \qquad \textit{Solution}$$

STEP 4:   **SUBSTITUTE** to find other coordinate of intersection point.

$$y = -2x + 10 = -2(3) + 10 = 4$$

STEP 5:   Write final answer.

The intersection point is (3, 4).

10) Use these matrices to answer the following questions.

$$A = \begin{bmatrix} 4 & -3 & 7 \\ 5 & 0 & -8 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 5 \\ 0 & -8 \end{bmatrix} \quad C = \begin{bmatrix} 4 & -3 & 7 \end{bmatrix} \quad D = \begin{bmatrix} 5 & -2 & 9 \\ 3 & 0 & -6 \\ 4 & -1 & -2 \end{bmatrix}$$

$$E = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \qquad F = \begin{bmatrix} 4 & -3 \\ 5 & 0 \\ 9 & 2 \\ 7 & -8 \end{bmatrix} \qquad G = \begin{bmatrix} w & x \\ y & z \end{bmatrix} \qquad H = \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix}$$

A.   List the size of each of the following matrices:

Solution:

Please recall that the size of a matrix is always the number of rows X number of columns

A = _2 X 3_   B = _2 X 2_   C = 1 X 3   D = 3 X 3

E = 3 X 2   F = 4 X 2   G = _2 X 2_   H = 3 X 1

B.   Do not compute – just answer question!! Are the following products possible to compute? If so, write yes in the blank. If not, explain why not – be brief – but specific!

AD = (2 X 3) (3 X 3) = (2 X 3) YES   EF = (3 X 2)(4 X 2) = Not Possible

FD = (4 X 2)(3 X 3) = Not Possible   FG = (4 X 2)(2 X 2) = Yes, this is Possible

C.   Find the product BG

Solution:

The size of BG = (2 X 2)(2 X 2) = (2 X 2) which is possible

$$B = \begin{bmatrix} -3 & 5 \\ 0 & -8 \end{bmatrix} \qquad G = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$$

$$\text{Then } BG = \begin{bmatrix} -3 & 5 \\ 0 & -8 \end{bmatrix} * \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} -3w + 5y & -3x + 5z \\ 0w - 8y & 0x - 8z \end{bmatrix} = \begin{bmatrix} -3w + 5y & -3x + 5z \\ 0 - 8y & 0 - 8z \end{bmatrix}$$

D.   Find the result of 3 times matrix B, namely: 3B

Solution:

$$3B = 3\begin{bmatrix} -3 & 5 \\ 0 & -8 \end{bmatrix} = \begin{bmatrix} 3(-3) & 3*5 \\ 3(0) & 3*(-8) \end{bmatrix} = \begin{bmatrix} -9 & 15 \\ 0 & -24 \end{bmatrix}$$

E.   Find matrix B added to matrix G, namely B + G

Solution:

$$B + G = \begin{bmatrix} -3 & 5 \\ 0 & -8 \end{bmatrix} + \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} -3+w & 5+x \\ 0+y & -8+z \end{bmatrix}$$

F.   Find the inverse of matrix B namely, B^-1

Solution:

First go to Matrix menu of your calculator, and input matrix B
Then go to Matrix menu of your calculator and pick matrix B, then use the x^-1 button of your calculator, and then use the MATH Frac part of your calculator, and get

$$B\text{^-1} = \begin{bmatrix} -\dfrac{1}{3} & \dfrac{-5}{24} \\ 0 & \dfrac{-1}{8} \end{bmatrix}$$

G.   Find the result of matrix G being subtracted from matrix B, namely B - G

Solution:

$$B - G = \begin{bmatrix} -3 & 5 \\ 0 & -8 \end{bmatrix} - \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} -3-w & 5-x \\ 0-y & -8-z \end{bmatrix}$$

11)   A grain dealer sold to one customer 5 bushels of wheat, 2 of corn, and 3 of rye, for $ 31.00. To another customer he sold 2 bushels of wheat, 3 of corn, and 5 of rye, for $ 27.60. To a third customer he sold 3 bushels of wheat, 5 of corn, and 2 of rye for $ 32.70. What was the price per bushel for each of the different grains?

Solution:

Set up matrix equations for this problem and use inverses to solve.

Let   x represent the price per bushel for wheat,
       y the price per bushel for corn, and
       z the price per bushel for rye.

Write the matrix algebra system for this problem:

$$\begin{bmatrix} 5 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 31.00 \\ 27.60 \\ 32.70 \end{bmatrix}$$


Use inverses to solve the system

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 31.00 \\ 27.60 \\ 32.70 \end{bmatrix} = \begin{bmatrix} 3.61 \\ 3.61 \\ 1.91 \end{bmatrix}$$

Write out the solution to the problem.

The wheat sells for $3.61 per bushel, the corn sells for $3.61 per bushel, and the rye sells for $1.91 per bushel.

12) The following probabilities have been determined from a recent survey. The probability that a person is favors the current president is 0.72. The probability that a person is male and favors the current president is 0.45. The probability that a person is female and does not favor the current president is 0.21.

A.   Draw a Venn diagram that displays the survey results.

Male   Favors
0.21
0.07   0.45
0.27
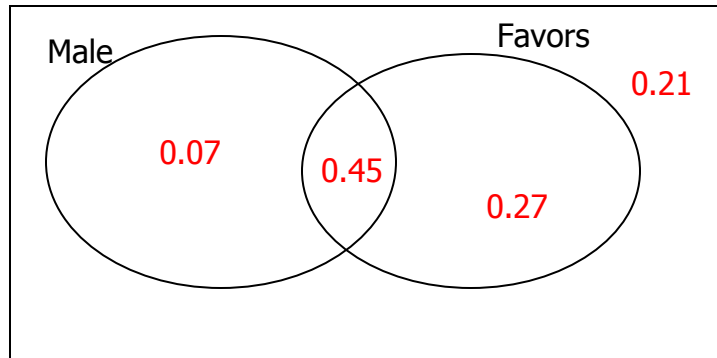
Use the diagram to determine the following:

B.   a person does not favor the current president

0.07 + 0.21 = 0.28

C.   a person is male or favors the current president

0.07 + 0.45 + 0.27 = 0.79

D.   a person is not male or does not favor the current president

0.27 + 0.21+ 0.07 = 0.55

E.   a person is male and does not favor the current president

0.07

13. Convert the following odds to probabilities:

   A. 56 to 95   B.   7 to 13

        56/(56 + 95) = 56/151   7/(7 + 13) = 7/20

   C. 40 to 11   D.   19 to 3

        40/(40 + 11) = 40/51   19/(19 + 3) = 19/22

14. Convert the following probabilities to odds:

   A. 3/5   B.   19/34

        3 to (5-3) ➔ 3 to 2   19 to (34-19) ➔ 19 to 15

   C. 0.123   D.   0.87

0.123 = 123/1000   123 to (123 - 1000) ➔ 123 to 877   0.87 = 87/100 87 to 13

15. If the odds of winning a game are 9 to 7, what is the probability of losing the game?

      The odds of losing are 7 to 9, so the probability of losing is 7/(9 + 7) = 7/16.

Or, The proability of winning is 7/(7 + 9) = 7/16. The probability of losing is 1 − 7/16 = 9/16.

16.   1800 people were surveyed. The following table contains data about their favorite colors.

	B BLUE	R RED	G GREEN	Y YELLOW	Pu PURPLE	Pi PINK	O ORANGE	TOTALS
WOMEN (**M'**)	80	170	80	20	250	350	30	980
MEN (**M**)	450	100	120	10	5	5	130	820
TOTALS	530	270	200	30	255	355	160	1800

Use the table to calculate the following probabilities.

A.   The probability that a person selected at random from this group is a man **and** their favorite color is green. $P(M \cap G)$

120/1800 = 1/15 ~ .067 or 6.7% (any answer is fine – the first is the preferred)

B.   The probability that a person selected is a woman **or** favorite color is red. $P(M' \cup R)$

(980 + 100)/1800 = 1080/1800 = 3/5 ~ .6 or 60%

C.   The probability that a person selected at random is a woman **given that** favorite color is purple. $P(M'|Pu)$

250/255 = 50/51 ~ .98 or 98%

D.   The probability that person selected at random has favorite color of red, **given that** the person is a man. $P(R|M)$

100/820 = 5/41 ~ .12 or 12%

E.   The probability that a person selected at random from this group is a man **and** their favorite color is not purple. $P(M \cap Pu')$

(450 + 100 + 120 + 10 + 5 + 130)/1800 = 815/1800 = 163/360 ~ 0.45 = 45%

Blood Type	Male	Female	TOTAL
O	80	370	450
A	150	250	400
B	50	50	100
AB	20	30	50
TOTAL	300	700	1,000

17.   Use the table to find the probability that a person is selected at random from the group
Note: Key words have bold type!!

A.   has blood type B?

100/1,000 = 0.1

B.   is female **and** has blood type B?

50/1,000 = 0.05

C.   has blood type B **given that** the person is female?

50/700 = 5/70

D.   is female **given that** the person has blood type B?

50/100=0.5

E.   the person is female **or** has blood type B?

(700 + 50) /1,000 = 750/1,000 = 0.75