

MA 110 TEST REVIEW WORKSHEET

Name Solutions

1. The weekly demand for mouthwash in a chain of drug stores is 1,160 bottles at a price of \$3.79 per bottle. If the price is lowered to \$3.59, the weekly demand increases to 1,320. Assume that the relationship between demand  $D$  and price per  $p$  is linear.

- A. Write a linear equation that expresses  $D$  in terms of  $p$ .

$D$  in terms of  $p$  means the equation should have the form  $D = mp + b$ . The ordered pairs should be in the form  $(p, D)$ . The ordered pairs are  $(3.79, 1160)$  and  $(3.59, 1320)$ . The slope  $m = (1320 - 1160)/(3.59 - 3.79) = -800$ .

$D = -800p + b$  is known so far. Substituting  $D = 1160$  and  $p = 3.79$  (from the 1<sup>st</sup> ordered pair) gives  $1160 = -800(3.79) + b$  and solving gives  $b = 4192$

$$D = -800p + 4192$$

- B. What should the price of a bottle of mouthwash be so that the demand is 2000 bottles?

$$2000 = -800p + 4192 \rightarrow -800p = -2192 \rightarrow p = -2192/-800 = \$2.74$$

- C. How many bottles would the stores sell each week if the price were lowered to \$3.29?

$$D = -800(3.29) + 4192 = 1560 \text{ bottles}$$

- D. Suppose that the Supply function for the mouthwash is  $S = 500p$ . Is there a surplus or shortage when the price is \$3.29? Explain.

$S = 500(3.29) = 1645$  bottles. The supply is greater than the demand so there is a surplus.

- E. Find the equilibrium price.

$$\text{Solve } S = D. \quad 500p = -800p + 4192 \rightarrow 1300p = 4192 \rightarrow p = 4192/1300 = \$3.22$$

- F. For what prices is there a shortage? Explain.

For prices less than the equilibrium price,  $p < \$3.22$ . The graph of  $S$  (supply) is lower than the graph of  $D$  (demand) for  $p < \$3.22$ .

2. A charter company buys a new machine for \$4500 and assumes that it will have a trade in value of \$500 after 8 years. Supposing the machine depreciates linearly,
- A. Find a linear model  $V$  for the depreciated value of the machine  $t$  years after it was purchased. **Clearly show your ordered pairs and a few other steps!!**

Form of equation should be  $V = mt + b$ .

$$(0, 4500) \quad (8, 500) \quad \text{slope} = (4500 - 500)/(0 - 8) = -500 \rightarrow V = -500t + 4500$$

- B. INTERPRET the slope of your linear function. Write your answer in sentence form.

Each year the value of the machine decreases by \$500.

- C. What is the depreciated value of the machine after 3 years?

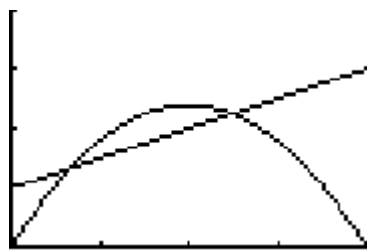
$$V = -500(3) + 4500 = \$3000$$

- D. When will the depreciated value be \$2000? **Answer to nearest whole year.**

$$-500t + 4500 = 2000 \rightarrow -500t = -2500 \rightarrow t = 5 \text{ years}$$

4. The research department in a company that manufactures dish washers established the following functions: Revenue:  $R(x) = -3x^2 + 120x$  and Cost:  $C(x) = 500 + 25x$ ,  $0 \leq x \leq 40$ , is in hundreds,  $p(x)$  is in dollars, and  $R(x)$  and  $C(x)$  are in hundreds of dollars.

- A. Find the production level(s) of dish washers (to the nearest hundred) at which the company has break-even point(s). You should find this answer graphically. **Give units with your answer!!**



$$x = 6.7 \text{ hundred or } 670 \text{ and } x = 25 \text{ hundred or } 2500$$

Rounding to the nearest hundred, the answers are 700 and 25 hundred dish washers.

- B. If the company sells 30 hundred dish washers will they make a profit or experience a loss? Explain.

Loss, 30 falls outside the break-even points.

$$\text{Also, } R(30) = -3(30)^2 + 120(30) = 900 \text{ and } C(30) = 500 + 25(30) = 1250 \\ \text{so Cost} > \text{Revenue}$$

4. The research department in a company that manufactures cell phone established the following price-demand function:  $p(x) = 300 - 15x$  where  $x$ ,  $0 \leq x \leq 20$ , is in hundreds and  $p(x)$  is in dollars.

A. Write the revenue function  $R(x)$  where  $x$  is the number of cell phones sold (in hundreds) and  $R(x)$  is the revenue in hundreds of dollars.

$$R(x) = x(300 - 15x) = -15x^2 + 300x$$

B. Determine the number of cell phones that must be sold to maximize the revenue. **Give units with your answer!!** You may find this answer algebraically or graphically. Show your steps or a simple sketch of your graph to support your answer.

$$x = -b/2a = -300/(2*-15) = 10 \text{ hundred or } 1000 \text{ cell phones}$$

C. What is the price per cell phone when revenue is maximized? Show which equation you use and your substitution.

$$P(10) = 300 - 15(10) = \$150 \text{ per phone}$$

D. What is the maximum revenue? **Give units with your answer!!** Show your substitution or a simple sketch of your supporting graph.

$$R(10) = -15(10)^2 + 300(10) = 1500 \text{ hundred dollars or } \$ 150,000$$

5. What will the pay-off amount be for a 16 month loan of \$2,500 at an annual simple interest rate of 9.5%?  **$I = Prt$  and  $A = P(1 + rt)$**

$$A = 2500(1 + .095(16/12)) = \$2816.67$$

6. What interest rate compounded quarterly is needed to have a \$1,000 investment grow to \$5,000 in five years?

<b>N = 20</b>
<b>I% =</b>
<b>PV = -1000</b>
<b>PMT = 0</b>
<b>FV = 5000</b>
<b>P/Y = 4</b>

Answer: 33.52%

7. If you deposit \$450 a month into your child's college fund for 18 years at 4.7% compounded monthly, how much will you accumulate?

<b>N = 216</b>
<b>I% = 4.7</b>
<b>PV = 0</b>
<b>PMT = -450</b>
<b>FV =</b>
<b>P/Y = 12</b>

Answer: \$152,401.72

How much interest does this account earn in the 18<sup>th</sup> (last) year? Hint: You have to use the TVM solver and then simple arithmetic. Show arithmetic here:

<b>N = 216 - 12 = 204</b>
<b>I% = 4.7</b>
<b>PV = 0</b>
<b>PMT = -450</b>
<b>FV = 140,152.91</b>
<b>P/Y = 12</b>

At the beginning of the 18<sup>th</sup> year, there have been 204 payments into this account. The amount in the account at the beginning of the 18<sup>th</sup> year is \$140,152.91. The total increase in the account during the 18<sup>th</sup> year is  $152,401.72 - 140,152.91 = 12,248.81$ . Of that amount,  $12 * 450 = 5400$  is deposited by the parents. The total interest earned in the 18<sup>th</sup> year is the difference:  $12,248.81 - 5400 = \$6848.81$ .

Answer: \$6848.81

8. You purchase a home set up a 30-year mortgage \$320,000 with a loan company that charges 7.5% compounded monthly. What will your monthly mortgage payments be?

<b>N = 360</b>
<b>I% = 7.5</b>
<b>PV = 320000</b>
<b>PMT =</b>
<b>FV = 0</b>
<b>P/Y = 12</b>

Answer: \$2237.49

How much interest will you pay over the entire length of the loan?

$$360(2237.49) - 320,000 = \$485,496.40$$

9. How much should you deposit now to accumulate \$45,000 in 10 years in an account that earns 4.3% compounded **quarterly**? If you use the TVM solver fill in the table, otherwise, you can list the formula and show your substitutions.

<b>N = 40</b>
<b>I% = 4.3</b>
<b>PV =</b>
<b>PMT = 0</b>
<b>FV = 45,000</b>
<b>P/Y = 4</b>

Answer: \$29,340.16

10. How many **years** will it take for \$8,000 to grow to \$15,000 in an account that earns 3.5% compounded **quarterly**?

<b>N =</b>
<b>I% = 3.5</b>
<b>PV = -8000</b>
<b>PMT = 0</b>
<b>FV = 15,000</b>
<b>P/Y = 4</b>

Answer:  $N \sim 72, 72/4 = 18$  years.

11. How much will you need to save each month for 35 years to accumulate \$2,300,000 in your retirement account if you can earn 6.8% compounded monthly.

<b>N = 35*12 = 420</b>
<b>I% = 6.8</b>
<b>PV = 0</b>
<b>PMT =</b>
<b>FV = 2,300,000</b>
<b>P/Y = 12</b>

Answer:      **\$1339.15**