MA 110:   SECTION 2.3 QUADRATIC FUNCTIONS
HOMEWORK: 9, 15, 19, 53, 57, 59, 61
A summary of quadratic functions is included on a separate document.

USING A QUADRATIC MODEL:
Referring to problems #58 & #60 on page 92, the marketing research department for the company that manufactures and sells "notebook" computers established the following price-demand-revenue functions:

$p(x) = 2,000 - 60x$ $1 \le x \le 25$ and $R(x) = xp = x(2000 - 60x) = -60x^2 + 2000x$

where $p(x)$ is the wholesale price at which x thousand computers can be sold, and $R(x)$ is the revenue (in thousands of dollars) from the sell of x thousand computers. Both functions have the physical domain $1 \le x \le 25$.

1.   Sketch a graph of $R(x)$.
       The domain of $R(x)$ is given in the problem $1 \le x \le 25$. This information tells you how to set the x window. You will have to figure out how to set the y window.

       $X_{min} = 0$ $Y_{min} = 0$

       $X_{max} = 25$ $X_{max} = 20,000$

       $X_{scl} = 5$ $Y_{scl} = 1000$

       To decide on the y window values, you have to figure out the range of $R(x)$. Since $R(x)$ models revenue only positive values of y make sense in our problem. Thus, the y_{min} is 0. To determine the y_{max}, choose a value of x about half way between the x_{min} and X_{max}. What value are you going to choose?

       12.5

       What is R of that value?
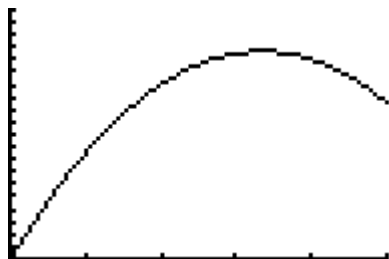
       15625

       What will you choose for your y_{max}.

       20,000

       The X_{scl} tells the calculator how often to make a tick mark.   For easy readability of the graph, you don't want too many or too few tick marks. What scale would you try?
   Answers above.

2. Find the output (the number of computers to the nearest hundred) that will produce the maximum revenue. You can determine this algebraically or graphically.

A. Algebraically: The x-coordinate of the vertex of a quadratic function is x = -b/2a. Compute x = -b/2a.

x = -2000/(2*-60) = 16.7

B. Graphically: Access the maximum feature by pressing the 2nd button then the TRACE button. Choose option 4: maximum.

The calculator will prompt you for a left bound. Use your (left/right) arrow keys to move the cursor to the left of the vertex. Press ENTER. The calculator will then prompt you for a right bound. Use the (left/right) arrow keys again to move the cursor to the right of the vertex. Press ENTER. The calculator will then prompt you for a guess. Use the (left/right) arrow keys to move the cursor in between your other marks. Press ENTER. (You must press ENTER a 3rd time.) The display at the bottom of your calculator screen will give the x and y coordinates of the vertex for R(x). The x-coordinate is your answer.

Maximum
X=16.666668 _Y=16666.667 .
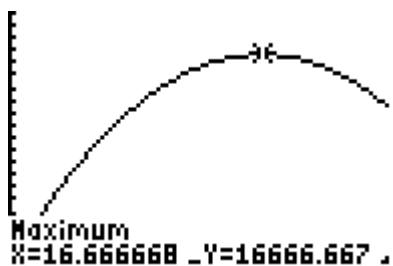
x = 16.7 thousand = 16,700 (nearest hundred)

3. What is the maximum revenue? This is the y-coordinate of the vertex of R(x).

y = -60(16.7)^2 + 2000(16.7) = 16666.67 thousand dollars = $ 16, 666, 667

4. What is the wholesale price of the notebook computer that produces the maximum revenue?

x = 16.667, find p(16.667) = 2000 – 60(16.7) = $1,000 ($999.98)

5. The cost function C(x) = 4000 + 500x (as given on page 93 #62) represents the cost (in thousands of dollars) to make x thousand computers.

Graph R(x) = -60x^2 + 2000x and C(x) = 4000 + 500x in the same window.
Your graph should look like the one below.
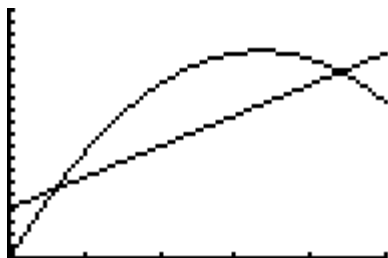
There are two break-even points. Recall a break-even point is when the Revenue equals the Cost. You can find the break-even points using the intersect function on your calculator. See instructions on the next page. An algebraic method is not recommended for finding the break even points.

Graphically: Access the intersect feature by pressing the 2nd button then the TRACE button. Choose option 5: intersect.

The calculator will prompt you for first curve. Use your (left/right) arrow keys to move the cursor close to one of the intersection points. Use the (up/down) arrow keys to move the cursor to the revenue function. Press ENTER. The calculator will then prompt you for second curve. Use the (up/down) arrow keys again to move the cursor to the cost function. Press ENTER. The calculator will then prompt you for a guess. Use the (right/left) arrow keys to move the cursor close to the intersection point. Press ENTER. (You must press ENTER a 3rd time.) The display at the bottom of your calculator screen will give the x and y coordinates of one of the break-even points. Repeat the process to find the other break-even point.

The x-coordinate is the number of items to produce and sell to break-even. The y-coordinate is the revenue (and cost since they are equal at the break even points) that is made by producing and selling that many items.

The break-even points are (3, 5518) & (22, 14,982).

6.   A profit will occur for the company if the number of items produced and sold is in between the break-even x-coordinates.

For each of the following output levels state whether the company will make a profit or a loss.

A.   2,000 notebook computers   loss

B.   4,000 notebook computers   profit

C.   15,000 notebook computers   profit

D.   25,000 notebook computers   loss

7.   The profit function
$$P(x) = R(x) - C(x) = -60x^2 + 2000x - [4000 + 500x] = -60x^2 + 1500x - 4000$$

Find the number of notebook computers the company should produce and sell to make the maximum profit.

The maximum profit occurs when 12.5 (thousand) notebook computers are manufactured and sold. The maximum profit is 5375 (thousand) or $5,375,000.

8.   COMMENTS:

A.   Note that the maximum revenue ($16,666, 167) was much greater than the maximum profit. Remember revenue is money in without deducting costs. The costs must be subtracted from the revenue to calculate the profit.

B.   The maximum revenue occurred when 16,667 notebook computers were made and sold, while the maximum profit occurred when only 12,500 notebook computers were made and sold. Thus, maximum revenue and maximum profit do not usually occur at the same production level.

C.   The price per notebook computer at which the maximum revenue occurred was p(16.667) = $1,000, while the price per notebook at which the maximum profit occurred was p(12.5) = $1250. Again, the maximum revenue and maximum profit do not occur at the same production levels. The production level (how many items you want to sell) affects the price.