

MATH 120 Section 3.2 Compound, Continuous Interest and APY

Compound Interest: Earning Interest on Interest

With simple interest, the principal earns interest once a year (compounded once a year). In reality, interest is compounded more than once a year.

Problem

1. You put \$100 into a savings account @ 5% interest compounded monthly. Complete the table. Round to the nearest penny.

$$A = 100 \left(1 + 0.05 \times \frac{1}{12}\right) = 100.4166667$$

$$A = 100.4166667 \left(1 + 0.05 \times \frac{1}{12}\right) = 100.84$$

Month	Amount
0	100
1	100.4166667
2	100.84
3	101.26
4	101.68
5	102.10
6	102.53

Compound Interest Formula

When an account earns interest on interest, this is called compound interest and the formula is (in this textbook): $A = P(1 + i)^n$, where $i = \frac{r}{m}$ and $n = mt$ or you may write:

$$A = P \left(1 + \frac{r}{m}\right)^{mt}$$

- A: Amt after time [future value] t: time in years
- P: Principal [present value] n: total number of compoundings
- r: annual interest rate m: number of compoundings per year

Compounded	Annually	Semiannually	Quarterly	Monthly	Daily
m	1	2	4	12	365

Problem

2. You put \$100 into a savings account @ 5% interest compounded monthly. Use the compound interest formula to determine the amount in the account after

a) 6 months

b) after 6 years.

a) $N = 6$
 $I\% = 5$
 $PV = -100$
 $PMT = 0$
 $FV = 102.53$
 $P/Y = 12$
 $C/Y = 12$

b) $N = 72$
 $I\% = 5$
 $PV = -100$
 $PMT = 0$
 $FV = 134.90$
 $P/Y = 12$
 $C/Y = 12$

Using the TVM Solver for Compound Interest

Instead of the Compound Interest Formula, you may use the TVM Solver on your calculator. To access this feature, press APPS, Finance, TVM Solver and enter the values you are given. Use ALPHA ENTER (SOLVE) next to the unknown to calculate.

N=	$m \cdot t$, total number of compoundings
I%=	annual interest rate (don't change to a decimal)
PV=	present value (principal P)
PMT=	payment amount (0 if there is no payment)
FV=	future value (amount after time: A)
P/Y=	number of payments per year (m)
C/Y=	number of compoundings per year (m) same as P/Y

Problems

3. Do Example 2b using the TVM Solver.

4. If \$900 is invested at 13% compounded a) annually, b) quarterly what is the amount after 10 years? How much interest is earned?

$N = 10$
 $I\% = 13$
 $PV = -900$
 $PMT = 0$
 $FV = 0$ Alpha solve $\$3055.11$
 $P/Y = 1$
 $C/Y = 1$

$N = 10 \times 4 = 40$
 $I\% = 13$
 $PV = -900$
 $PMT = 0$
 $FV = 0$ Alpha solve $\$3234.78$
 $P/Y = 4$
 $C/Y = 4$

$I = A - P$
 $I = 3055.11 - 900 = \$2155.11$
 $I = 3234.78 - 900 = \$2334.78$

5. An investment company pays 10% compounded semiannually. You want to have \$26,000 in the future. How much should you deposit now to have that amount 5 years from now?

$N = 5 \times 2 = 10$
 $I\% = 10$
 $PV = 0$ Alpha solve $= -15961.74$
 $PMT = 0$
 $FV = 26000$
 $P/Y = 2$
 $C/Y = 2$

Deposit \$15961.74 Now

Continuous Compounding

Problem

7. You invest \$100 in an account that earns 5% compounded interest. What is the amount in the account after 6 years? Complete the table. Round to the nearest penny.

Compounded	C/Y	FV
Annually	1	\$134.01
Semiannually	2	\$134.49
Quarterly	4	\$134.74
Monthly	12	\$134.90
Daily	365	\$134.98
Hourly	8760	\$134.99
Every Minute	525600	\$134.99
Continuously	31536000	\$134.99

365×24 (pointing to Daily)
 $365 \times 24 \times 60$ (pointing to Every Minute)

Continuous Compound Interest Formula

When an account compounds interest continuously, the compound interest formula becomes:

$$A = Pe^{rt}$$

$$A = 100e^{(0.05 \times 6)}$$

$$A = \$134.99$$

A = future value, P = principal, $e \approx 2.718281828459\dots$, r = rate, t = time in years

Problem

$$A = Pe^{rt}$$

8. You invest \$100 into an account that earns 5% compounded continuously. Use the continuous interest formula to determine the amount in the account after

a) 6 months

$$A = Pe^{rt}$$

$$A = 100e^{(0.05 \times \frac{6}{12})} = \boxed{102.53}$$

b) 6 years

$$A = 100e^{(0.05 \times 6)} = \boxed{134.99}$$

APY: Annual Percentage Yield

Annual Percentage Yield (APY) is the simple interest rate that will produce the same amount A in 1 year. Calculating APY allows you to compare different interest rates with different compounding, in order to decide which is the best rate for your investment.

APY Formulas

Compound Interest: $APY = \left(1 + \frac{r}{m}\right)^m - 1$

Continuous Interest: $APY = e^r - 1$

9. Find the APYs for the following banks which offer certificates of deposit (CDs). Which bank has the better rate for your investment. Round the percent to the nearest 3 decimal places.

$$APY = \left(1 + \frac{0.0493}{12}\right)^{12} - 1$$

$$APY = \left(1 + \frac{0.0497}{4}\right)^4 - 1$$

$$APY = e^{0.0494} - 1$$

Bank	Rate	Compounded	APY
Advanta	4.93%	monthly	5.043%
Charter One	4.97%	quarterly	5.063%
Liberty	4.94%	continuously	5.064%

Liberty has the Best Rate for Investment

Using the TVM Solver for APY

You may use the TVM solver to calculate APY for compound interest. To access this feature, press APPS, Finance, scroll down to C: Eff(. You must enter the interest rate (don't convert to a decimal), and the number of compoundings per year:

$$\text{Eff}(I\%, m)$$

Problem

10. Do problem 9 using the TVM solver.

$$\text{Eff}(4.93, 12) = 5.043 \text{ Percent}$$

$$\text{Eff}(4.97, 4) = 5.063 \text{ Percent}$$

$$\text{Eff}(4.94, 365 \times 24 \times 60) = 5.064 \text{ Percent}$$