

MATH 120 Section 3.3 Future Value of an Annuity; Sinking Funds
Future Value of an Annuity

An annuity is a stream of periodic payments. If payments are made at the end of each time interval, then the annuity is called an ordinary annuity. The future value of an annuity is the sum of all payments plus all interest earned

$$FV = PMT \frac{\left(1 + \frac{r}{m}\right)^{mt} - 1}{\left(\frac{r}{m}\right)}$$

Sinking Fund: When an account is set up to accumulate funds for a future obligation, the account is Sinking fund.

$$PMT = FV \frac{\left(\frac{r}{m}\right)}{\left(1 + \frac{r}{m}\right)^{mt} - 1}$$

1. Recently, More Money 4U offered an annuity that pays 7.5% compounded monthly. If \$250 is deposited into this annuity every month, how much is in the account after 15 years? How much of this is interest?

| | |
|-------|------------------------|
| N = | $15 \times 12 = 180$ |
| I% = | 7.5 |
| PV = | 0 |
| PMT = | -250 |
| FV = | 0 Alpha solve 82778.07 |
| P/Y = | 12 |
| C/Y = | 12 |

\$ 82778.07 in the account after 15 years.

$$\text{Interest} = 82778.07 - 45000 = \underline{\underline{\$ 37778.07}}$$

2. In order to accumulate enough money for a down payment on a house, a couple deposits \$1389 per month into an account paying 8% compounded monthly. If payments are made at the end of each period, how much money will be in the account in 8 years? How much interest was earned?

| | |
|-------|---------------------------|
| N = | $8 \times 12 = 96$ |
| I% = | 8 |
| PV = | 0 |
| PMT = | -1389 |
| FV = | 0 Alpha solve \$185943.46 |
| P/Y = | 12 |
| C/Y = | 12 |

$$\text{Interest earned} = 185943.46 - 96 \times 1389 =$$
$$= \underline{\underline{\$52599.46}}$$

3. Acme Annuities recently offered an annuity that pays 3.3% compounded monthly. What equal monthly deposit should be made into this annuity in order to have \$500,000 in 40 years? How much of this amount is interest?

| | |
|-------|----------------------|
| N = | $40 \times 12 = 480$ |
| I% = | 3.3 |
| PV = | 0 |
| PMT = | -502.44 |
| FV = | 500000 |
| P/Y = | 12 |
| C/Y = | 12 |

$$\text{Interest earned} = 500000 - 502.44 \times 480 = \underline{\underline{\$258828.80}}$$

4. A company estimates that it will need \$199,000 in 5 years to replace a computer. If it establishes a sinking fund by making fixed monthly payments into an account paying 3.1% compounded monthly, how much should each payment be?

| | |
|-------|--------------------------|
| N = | $5 \times 12 = 60$ |
| I% = | 3.1 |
| PV = | 0 |
| PMT = | 0 Alpha solve -3070.54 |
| FV = | 199000 |
| P/Y = | 12 |
| C/Y = | 12 |

5. Bob makes his first \$2500 deposit into an IRA earning 6.4% compounded annually on his 23rd birthday and his last \$2500 deposit on his 48th birthday (26 equal deposits in all). With no additional deposits, the money in the IRA continues to earn 6.4% interest compounded annually until Bob retires on his 64th birthday. How much is in the IRA when Bob retires?

| | |
|---------------------------------|---------------------------------|
| N = 26 | N = 64 - 48 = 16 |
| I% = 6.4 | I% = 6.4 |
| PV = 0 | PV = 156931.31 |
| PMT = -2500 | PMT = 0 |
| FV = 0 Alpha solve 156931.31 | FV = 0 Alpha solve 423424.17 |
| P/Y = 1 Compounded annually | P/Y = 1 |
| C/Y = 1 | C/Y = 1 |

8 423424.17