MATH 120 Section 3.4
Annuity Present Value and Amortization

The **present value** (PV) of an ordinary annuity is the amount of money you would need to invest today to receive payments (PMT) in the future. Example: How much do you need to invest today (PV) in order to receive payments when you retire (PMT)?

\[
PV = PMT \frac{1 - (1 + \frac{r}{m})^{-mt}}{\frac{r}{m}}
\]

_Amortizing a debt means that the debt (PV) is paid after a given length of time by equal periodic payments (PMT) that include compound interest. Examples: Paying off car payments and home mortgages._

\[
PMT = PV \frac{\frac{r}{m}}{1 - (1 + \frac{r}{m})^{-mt}}
\]
1. E-Loan, an online lending service, recently offered 42-month auto loans at 6.6% compounded monthly to applicants with good credit ratings. If you have a good credit rating and can afford monthly payments of $225, how much can you borrow from E-Loan? What is the total interest you will pay for this loan?

<table>
<thead>
<tr>
<th>N</th>
<th>42</th>
</tr>
</thead>
<tbody>
<tr>
<td>I%</td>
<td>6.6</td>
</tr>
<tr>
<td>PV</td>
<td>$8417.37</td>
</tr>
<tr>
<td>PMT</td>
<td>$225</td>
</tr>
<tr>
<td>FV</td>
<td>0</td>
</tr>
<tr>
<td>P/Y</td>
<td>12</td>
</tr>
<tr>
<td>C/Y</td>
<td>12</td>
</tr>
</tbody>
</table>

Interest paid = \( 225 \times 42 = \$8417.37 \)

\[
\text{PMT} = \frac{(\frac{I}{100})}{(1 + \frac{1}{n}) - 1} \times FV 
\]

\[
\text{PMT} = \frac{(\frac{6.6}{100})}{(1 + \frac{1}{12}) - 1} \times 8417.37 
\]

\[
\text{PMT} = \frac{0.066}{0.105833333} \times 8417.37 
\]

\[
\text{PMT} = 578.00 
\]
2. If you buy a computer directly from the manufacturer for $2100 and agree to repay it in 24 equal installments at 1.3% interest per month (because this is written as a monthly interest rate, this is \( i \) and not \( r \)) on the unpaid balance, how much are your monthly payments? How much total interest will be paid?

<table>
<thead>
<tr>
<th>( N )</th>
<th>( 24 \text{ monthly} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i% )</td>
<td>1.3</td>
</tr>
<tr>
<td>( PV )</td>
<td>2100</td>
</tr>
<tr>
<td>( PMT )</td>
<td>0, Alpha solve (-102.42)</td>
</tr>
<tr>
<td>( FV )</td>
<td>0</td>
</tr>
<tr>
<td>( P/Y )</td>
<td>1</td>
</tr>
<tr>
<td>( C/Y )</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
N = \frac{r}{m}
\]

Interest paid = 102.42 \times 24 - 2100 = 2458.08 - 2100 = $\boxed{358.08}$
3. You want to purchase an automobile for $30,877. The dealer offers you 0% financing for 60 months or a $6778 rebate. You can obtain 6% financing for 60 months at the local bank. Which option should you choose?

\[
\begin{array}{|c|}
\hline
N = 60 \\
FV = 0 \\
P/Y = 12 \\
C/Y = 12 \\
PMT = 0 \quad \text{Alpha Solve} = 514.62 \\
PV = 30877 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|}
\hline
N = 60 \\
I\% = 6 \\
PMT = 0 \quad \text{Alpha Solve} = 514.62 \\
FV = 0 \\
P/Y = 12 \\
C/Y = 12 \\
PV = 30877 - 6778 = 24099 \\
\hline
\end{array}
\]

Rebate is a better option.
4. A sailboat costs $23,636. You pay 15% down and amortize the rest with equal monthly payments over a 8-year period. If you must pay 6.9% compounded monthly, what is your monthly payment?

<table>
<thead>
<tr>
<th>N</th>
<th>96 = 8*12</th>
</tr>
</thead>
<tbody>
<tr>
<td>I%</td>
<td>6.9</td>
</tr>
<tr>
<td>PV</td>
<td>$23636 - 0.15 \times 23636$</td>
</tr>
<tr>
<td>PMT</td>
<td>0 Alpha solve $\boxed{-272.91}$</td>
</tr>
<tr>
<td>FV</td>
<td>0</td>
</tr>
<tr>
<td>P/Y</td>
<td>12</td>
</tr>
<tr>
<td>C/Y</td>
<td>12</td>
</tr>
</tbody>
</table>
5. A home in Rockville costs $575,000. You pay 20% down and finance the rest at 4.7% for 30 years. What is the mortgage? How much interest will you pay during the life of the loan?

\[
\begin{array}{|l|}
\hline
N = & 30 \times 12 = 360 \\
I\% = & 4.7 \\
PV = & 575000 - 0.20(575000) = 460000 \\
PMT = & -$2385.73 \\
FV = & 0 \\
P/Y = & 12 \\
C/Y = & 12 \\
\hline
\end{array}
\]

\[2385.73 \times 360 - 460000 = \$398862.80\]