MATH 120 Sections 4.2 - 4.3

Introduction to Matrices

A matrix is a rectangular array of numbers written within brackets. Each number in a matrix is called an element.

If a matrix has m rows and n columns, it is called an \( m \times n \) matrix and is the size of the matrix. If the number of rows and the number of columns are the same, the matrix is called a square matrix.

The elements are organized according to the row and column they are in:

\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\]

Problems

a) What is the size of the matrix? b) Is it a square matrix?

c) \( a_{11} = ? \) d) \( a_{21} = ? \) e) \( a_{12} = ? \)

1) \[
\begin{bmatrix}
1 \\
-7 \\
0
\end{bmatrix}
\]
2) \[
\begin{bmatrix}
5 & 2 \\
-1 & 8 \\
0 & 1
\end{bmatrix}
\]
3) \[
\begin{bmatrix}
3 \\
-2 \\
1
\end{bmatrix}
\]

Systems of equations can be written in what is called an augmented matrix.

The system \[
\begin{align*}
2x + y &= 8 \\
x + 3y &= 9
\end{align*}
\] can be written as an augmented matrix: \[
\begin{bmatrix}
2 & 1 & 8 \\
1 & 3 & 9
\end{bmatrix}
\]

The coefficient matrix is \[
\begin{bmatrix}
2 \\
1
\end{bmatrix}
\]

Examples

4) Write the system as an augmented matrix: \[
\begin{align*}
3x + 2y &= 1 \\
x + 2y &= 3
\end{align*}
\] What is the coefficient matrix?

5) Write the augmented matrix as a system of linear equations:

a) \[
\begin{bmatrix}
1 & 2 & 3 \\
5 & 4 & 1
\end{bmatrix}
\]
b) \[
\begin{bmatrix}
1 & 0 & 10 \\
0 & 1 & 5
\end{bmatrix}
\]
Recall solving a system by elimination:

- You may multiply one equation by a non-zero constant.
- You may add two equations together.

Solving a System by Row Operations. \[ \text{GOAL} \begin{bmatrix} 1 & 0 & \# \\ 0 & 1 & \# \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 & 0 & \# \\ 0 & 1 & 0 & \# \\ 0 & 0 & 1 & \# \end{bmatrix} \]

- You may interchange two rows. \( R_1 \leftrightarrow R_2 \)
- You may multiply a row by a non-zero constant. \( 2R_1 \rightarrow R_1 \)
- You may add two rows together. \( 2R_1 + R_2 \rightarrow R_2 \)

Problems

Solve the system by matrix methods. This method is also known as solving by augmented matrix methods or Gaussian Elimination or Gauss-Jordan Elimination, named after the mathematicians Carl Friedrich Gauss (1777-1855) and Wilhelm Jordan (1842 - 1899).

6. \[ \begin{align*}
2x + y &= 8 \\
 x + 3y &= 9
\end{align*} \]

7. \[ \begin{align*}
3x_1 + 4x_2 &= 1 \\
x_1 - 2x_2 &= 7
\end{align*} \]

8. \[ \begin{align*}
x + 5y - 12z &= 1 \\
2x + 4y - 10z &= -2 \\
3x + 9y - 21z &= 0
\end{align*} \]