MATH 120 Section 4.1 Systems of Linear Equations in Two Variables

A system of linear equations consists of two or more linear equations, which are solved simultaneously.
A solution to a system consists of the values of the variables which make the system true.

Three Methods for Solving Systems

I. Graphing
II. Substitution
   Solve one equation for a variable and substitute into the other equation.
III. Elimination (Sometimes called Addition)
   - Multiply one or both equations by a value or values, so that the coefficients of one variable in both equations are the same number but opposite signs.
   - Add the equations so that one variable is eliminated.

Problems

1. Solve the system of linear equations by the following methods:
   a) Graphing, b) Elimination, c) Substitution.
   \[ \begin{align*}
   2x + y &= 8 & (0,8) & (4,0) \\
   x + 3y &= 9 & (0,3) & (9,0) \\
   \end{align*} \]
   Solution is \((3,2)\).

2. Animals in an experiment are to be kept under a strict diet. Each animal should receive 60 grams of protein and 10 grams of fat. The laboratory technician is able to purchase two food mixes: Mix A has 20% protein and 6% fat. Mix B has 50% protein and 5% fat. Complete the chart. Write and solve a system of equations to determine how many grams of each mix should be used to obtain the right diet for one animal?

<table>
<thead>
<tr>
<th></th>
<th>Grams of Mix A</th>
<th>Grams of Mix B</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Protein</td>
<td>0.20</td>
<td>0.50</td>
<td>60</td>
</tr>
<tr>
<td>Fat</td>
<td>0.06</td>
<td>0.05</td>
<td>10</td>
</tr>
</tbody>
</table>

\[
A = \begin{bmatrix}
0.20 & 0.50 & 60 \\
0.06 & 0.05 & 10
\end{bmatrix}
\]

\[\text{REF}[A] = \begin{bmatrix}
1 & 0 & 100
\end{bmatrix}\]

In conclusion, they should use 100 grams of mix A and 80 grams of mix B.
4. A company produces lawn mowers. The company's daily fixed costs are $42,000 and variable costs are $1500 per lawn mower. The mowers are sold for $1800 each.

   a) Find the cost function \( C(x) = 1500x + 42000 \).

   b) Find the revenue function \( R(x) = 1800x \).

   c) Find the break-even point. Write your answer as an ordered pair.

   \[
   1800x = 1500x + 42000 \\
   -300x = 42000 \\
   x = \frac{42000}{300} = 140
   \]

   d) Graph the cost and revenue functions on the given coordinate system and show the break-even point.

   e) Write the meaning of the break-even point you found in part c using complete sentences with correct units. Include an interpretation of the regions between the lines that are to the left and to the right of the break-even point.

   If they sell more than 140 lawn mowers, they will make a profit.

5. A company markets exercise DVDs that sell for $34.95, including shipping and handling. The monthly fixed costs (advertising, rent, etc.) are $47,700 and the variable costs (materials, shipping, etc.) are $12.45 per DVD.

   a) Find the cost function \( C(x) = 12.45x + 47700 \).

   b) Find the revenue function \( R(x) = 34.95x \).

   c) Find the break-even point. Write your answer as an ordered pair.

   \[
   34.95x = 12.45x + 47700 \\
   22.5x = 47700 \\
   x = 2120
   \]

   d) Graph the cost and revenue functions on the given coordinate system and show the break-even point.

   e) Write the meaning of the break-even point you found in part c using complete sentences with correct units. Include an interpretation of the regions between the lines that are to the left and to the right of the break-even point.

   If they sell more than 2120 DVDs, they will make a profit.