MATH 120 Sections 4.2 - 4.3

Introduction to Matrices

A matrix is a rectangular array of numbers written within brackets. Each number in a matrix is called an element.

If a matrix has m rows and n columns, it is called an $m \times n$ matrix and is the size of the matrix. If the number of rows and the number of columns are the same, the matrix is called a square matrix.

The elements are organized according to the row and column they are in:

$$\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}$$

Problems

a) What is the size of the matrix? b) Is it a square matrix?

\[ \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \]

1) $a_{11} = 1$ 2) $a_{12} = 2$ 3) $a_{13} = 3$

\[ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \]

1) $a_{21} = -1$ 2) $a_{22} = 2$ 3) $a_{23} = 0$

\[ \begin{bmatrix} 1 & 2 \end{bmatrix} \]

1) $a_{11} = 5$ 2) $a_{12} = 2$

Square Matrix 1

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Matrix x 1

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Systems of equations can be written in what is called an augmented matrix.

The system $\begin{cases} 2x + y = 8 \\ x + 3y = 9 \end{cases}$ can be written as an augmented matrix:

$$\begin{bmatrix} 2 & 1 & 8 \\ 1 & 3 & 9 \end{bmatrix}$$

The coefficient matrix is $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$.

Examples

4) Write the system as an augmented matrix: $\begin{cases} 3x + 2y = 1 \\ x + 2y = 3 \end{cases}$. What is the coefficient matrix?

$$\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$$

5) Write the augmented matrix as a system of linear equations:

\[ \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 1 \end{bmatrix} \]

a) $x + 2y = 3$ b) $x + y = 10$

\[ \begin{bmatrix} 10 & 1 & 5 \\ 0 & 1 & 5 \end{bmatrix} \]

\[ \begin{cases} x + 2y = 3 \\ 5x + 4y = 1 \end{cases} \]

\[ \begin{cases} x = 10 \\ y = 5 \end{cases} \]
Recall solving a system by elimination:

- You may multiply one equation by a non-zero constant.
- You may add two equations together.

Solving a System by Row Operations. GOAL \[ \begin{bmatrix} 1 & 0 & | & \# \\ 0 & 1 & | & \# \end{bmatrix} \] or \[ \begin{bmatrix} 1 & 0 & | & \# \\ 0 & 0 & | & \# \end{bmatrix} \]

- You may interchange two rows. \( R_1 \leftrightarrow R_2 \)
- You may multiply a row by a non-zero constant. \( 2R_1 \rightarrow R_1 \)
- You may add two rows together. \( 2R_1 + R_2 \rightarrow R_2 \)

Problems

Solve the system by matrix methods. This method is also known as solving by augmented matrix methods or Gaussian Elimination or Gauss-Jordan Elimination, named after the mathematicians Carl Friedrich Gauss (1777-1855) and Wilhelm Jordan (1842 - 1899).

6. \[ \begin{cases} 2x + y = 8 \\ x + 3y = 9 \end{cases} \]
\[ A = \begin{bmatrix} 2 & 1 & 8 \\ 1 & 3 & 9 \end{bmatrix} \]
\[ \text{rref}[A] = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix} \]
\[ x = 3 \\ y = 2 \]

7. \[ \begin{cases} 3x_1 + 4x_2 = 1 \\ x_1 - 2x_2 = 7 \end{cases} \]
\[ A = \begin{bmatrix} 3 & 4 & 1 \\ 1 & -2 & 7 \end{bmatrix} \]
\[ \text{rref}[A] = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \end{bmatrix} \]
\[ x_1 = 3 \\ x_2 = -2 \]

8. \[ \begin{cases} x + 5y - 12z = 1 \\ 2x + 4y - 10z = -2 \\ 3x + 9y - 21z = 0 \end{cases} \]
\[ A = \begin{bmatrix} 1 & 5 & -12 & 1 \\ 2 & 4 & -10 & -2 \\ 3 & 9 & -21 & 0 \end{bmatrix} \]
\[ \text{rref}[A] = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix} \]
\[ x = -2 \\ y = 3 \\ z = 1 \]