MATH 120 Sections 4.2 - 4.3

Introduction to Matrices

A <u>matrix</u> is a rectangular array of numbers written within brackets. Each number in a matrix is called an <u>element</u>.

If a matrix has m rows and n columns, it is called an <u>m x n</u> matrix and is the <u>size</u> of the matrix. If the number of rows and the number of columns are the same, the matrix is called a <u>square</u> matrix.

The elements are organized according to the row and column they are in:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Problems

a) What is the size of the matrix? b) Is it a square matrix? Square Matrix

Matrix 1 $\begin{bmatrix} 1 & 5 \\ -7 & 0 \end{bmatrix}$

c) $a_{11} = ?$ d) $a_{21} = ?$ e) $a_{12} = ?$

1) $\begin{bmatrix} 1 & 5 \\ -7 & 0 \end{bmatrix}$ 2) $\begin{bmatrix} 5 & 2 \\ -1 & 8 \\ 0 & 1 \end{bmatrix}$ 3) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

1) $a_{11} = 1$ $a_{21} = -7$ $a_{12} = 5$

2) $a_{11} = 5$, $a_{21} = -1$ $a_{12} = 2$

3) $a_{11} = 3$ $a_{21} = -2$ $a_{12} = \text{DNE}$

Systems of equations can be written in what is called an <u>augmented matrix</u>.

The system $\begin{cases} 2x + y = 8 \\ x + 3y = 9 \end{cases}$ can be written as an <u>augmented matrix</u>: $\left[\begin{array}{cc|c} 2 & 1 & 8 \\ 1 & 3 & 9 \end{array}\right]$.

The <u>coefficient matrix</u> is $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$.

Examples

4) Write the system as an augmented matrix: $\begin{cases} 3x + 2y = 1 \\ x + 2y = 3 \end{cases}$. What is the coefficient matrix? $\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

5) Write the augmented matrix as a system of linear equations:

a) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 5 & 4 & 1 \end{array}\right]$ b) $\left[\begin{array}{cc|c} 1 & 0 & 10 \\ 0 & 1 & 5 \end{array}\right]$

a) $\begin{cases} 1x + 2y = 3 \\ 5x + 4y = 1 \end{cases}$

b) $1x = 10$, $1y = 5$

Recall solving a system by elimination:

- You may multiply one equation by a non-zero constant.
- You may add two equations together.

Solving a System by <u>Row Operations</u>. GOAL $\left[\begin{array}{cc|c} 1 & 0 & \# \\ 0 & 1 & \# \end{array}\right]$ or $\left[\begin{array}{ccc|c} 1 & 0 & 0 & \# \\ 0 & 1 & 0 & \# \\ 0 & 0 & 1 & \# \end{array}\right]$

- You may interchange two rows. $R_1 \leftrightarrow R_2$
- You may multiply a row by a non-zero constant. $2R_1 \rightarrow R_1$
- You may add two rows together. $2R_1 + R_2 \rightarrow R_2$

Problems

Solve the system by matrix methods. This method is also known as solving by augmented matrix methods or *Gaussian Elimination* or *Gauss-Jordon Elimination*, named after the mathematicians Carl Friedrich Gauss (1777-1855) and Wilhelm Jordan (1842 - 1899).

6. $\begin{cases} 2x + y = 8 \\ x + 3y = 9 \end{cases}$

$A = \begin{bmatrix} 2 & 1 & 8 \\ 1 & 3 & 9 \end{bmatrix}$

$\text{rref}[A] = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

$\boxed{x = 3 \quad y = 2}$

7. $\begin{cases} 3x_1 + 4x_2 = 1 \\ x_1 - 2x_2 = 7 \end{cases}$

$A = \begin{bmatrix} 3 & 4 & 1 \\ 1 & -2 & 7 \end{bmatrix}$

$\text{rref}[A] = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \end{bmatrix}$

$\boxed{x_1 = 3 \quad x_2 = -2}$

8. $\begin{cases} x + 5y - 12z = 1 \\ 2x + 4y - 10z = -2 \\ 3x + 9y - 21z = 0 \end{cases}$

$A = \begin{bmatrix} 1 & 5 & -12 & 1 \\ 2 & 4 & -10 & -2 \\ 3 & 9 & -21 & 0 \end{bmatrix}$

$\text{rref}[A] = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$\boxed{x = -2 \quad y = 3 \quad z = 1}$