SECTIONS 9.1 AND 9.2

1) A new grocery store has just opened. From past observations of new stores it has been determined that 70% of people who go to the new grocery store will go again next week, while 20% of the people who went to the old grocery store will go to the new store next week.

A. Set up the transition matrix for this situation. ***Label your rows and columns*** with new and old.

$$P = \begin{matrix} \\ \text{New} \\ \text{Old} \end{matrix} \begin{matrix} \begin{matrix} \text{New} & \text{Old} \end{matrix} \\ \begin{bmatrix} .7 & .3 \\ .2 & .8 \end{bmatrix} \end{matrix}$$

B. If during grand opening week, 60% of the people went to the new grocery store. Set up the initial-state matrix for this situation.

$$S_0 = \begin{matrix} \begin{matrix} \text{New} & \text{Old} \end{matrix} \\ \begin{bmatrix} .6 & .4 \end{bmatrix} \end{matrix}$$

C. What percentage of the people will be going to the new grocery store 3 weeks after the grand opening? Clearly show what matrices you multiplied. Give answer to one decimal place.

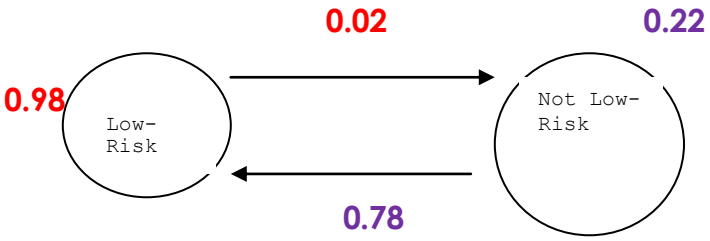
$$S_0P^3 = \begin{matrix} \begin{matrix} \text{New} & \text{Old} \end{matrix} \\ \begin{bmatrix} .425 & .575 \end{bmatrix} \end{matrix} \rightarrow 42.5\%$$

D. What percentage of people will be going to the new grocery store in the long run?

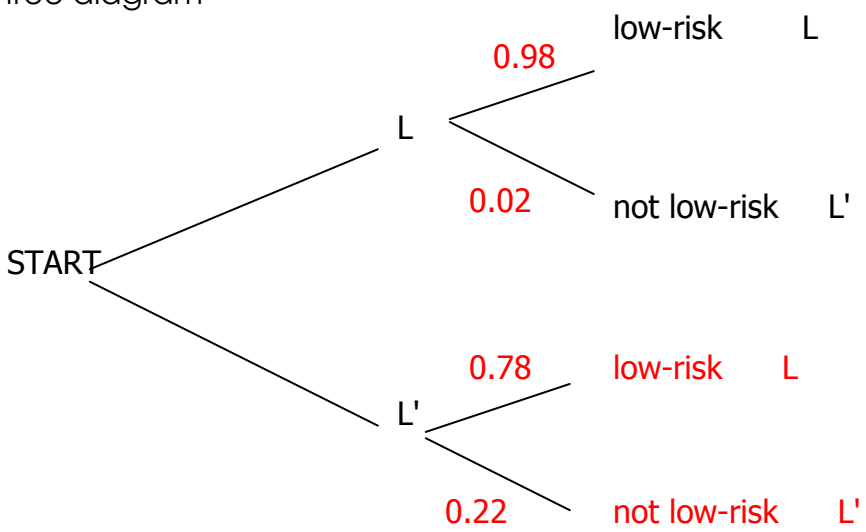
$$S_0P^{100} = \begin{matrix} \begin{matrix} \text{New} & \text{Old} \end{matrix} \\ \begin{bmatrix} .4 & .6 \end{bmatrix} \end{matrix} \rightarrow 40\% \text{ go to the new grocery store in the long run.}$$

2) An insurance company classifies drivers as low – risk if they are accident – free for 1 year. Past records indicate that 98% of the drivers in the low – risk category (L) on year will remain in that category the next year, and 78% of the drivers who are not in low – risk category (L') one year will be in the low – risk category the next year.

a) Draw the transition diagram



b) Draw the tree diagram



c) Write the transition matrix

$$P = \begin{array}{c} \begin{matrix} L & L' \end{matrix} \\ \begin{bmatrix} .98 & .02 \\ .78 & .22 \end{bmatrix} \end{array}$$

d) If 90% of the drivers in the community are in the Low-Risk category this year, what is the probability that a driver chosen at random from the community will be in the low-risk category next year?

$$S_0 = \begin{matrix} L & L' \end{matrix} \begin{bmatrix} .9 & .1 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} .9 & .1 \end{bmatrix} \begin{bmatrix} .98 & .02 \\ .78 & .22 \end{bmatrix} = \begin{bmatrix} .96 & .04 \end{bmatrix}$$

In conclusion, the probability that a driver chosen at random from the community will be in the low-risk category next year is 0.96 or 96%.

e) If 90% of the drivers in the community are in the Low-Risk category this year, what is the probability that a driver chosen at random from the community will be in the low-risk category in the Year after that next year?

$$S_2 = \begin{bmatrix} .9 & .1 \end{bmatrix} \begin{bmatrix} .98 & .02 \\ .78 & .22 \end{bmatrix}^2 = \begin{bmatrix} .972 & .028 \end{bmatrix}$$

In conclusion, the probability that a driver chosen at random from the community will be in the low-risk category the year after next year is 0.972 or 97.2%.

3) Mice in a certain experiment involving a choice between path A and B are observed to have the following pattern:
Of those who choose path A one day, 30% choose path A the next day and 70% choose path B, while of those who choose path B one day, 80% choose path B the next day and 20% choose path A.

a) Set up the transition matrix for this situation.

$$P = \begin{matrix} A & B \end{matrix} \begin{bmatrix} .3 & .7 \\ .2 & .8 \end{bmatrix}$$

b) If 50% of mice choose path A on Monday and 50% of mice choose path B, what percentages choose each path on Tuesday?

$$S_0 = \begin{matrix} A & B \end{matrix} \begin{bmatrix} .5 & .5 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} .5 & .5 \end{bmatrix} \begin{bmatrix} .3 & .7 \\ .2 & .8 \end{bmatrix} = \begin{bmatrix} .25 & .75 \end{bmatrix}$$

In conclusion: On Tuesday 25% of mice will choose path A and 75% of mice will choose path B .

c) If 50% of mice choose path A on Monday and 50% of mice choose path B, what percentages choose each path on Wednesday?

$$S_2 = \begin{bmatrix} .5 & .5 \end{bmatrix} \begin{bmatrix} .3 & .7 \\ .2 & .8 \end{bmatrix}^2 = \begin{bmatrix} .225 & .775 \end{bmatrix}$$

In conclusion: On Wednesday 22.5% of mice will choose path A and 77.5% of mice will choose path B.

d) If 50% of mice choose path A on Monday and 50% of mice choose path B, what percentages choose each path on Sunday?

$$S_6 = \begin{bmatrix} .5 & .5 \end{bmatrix} \begin{bmatrix} .3 & .7 \\ .2 & .8 \end{bmatrix}^6 = \begin{bmatrix} .2222 & .7778 \end{bmatrix}$$

In conclusion: On Sunday 22.22% of mice will choose path A and 77.78% of mice will choose path B.

e) In the long run, what portion of the mice choose path A each day and what portion of mice choose path B?

$$S_{100} = \begin{bmatrix} .5 & .5 \end{bmatrix} \begin{bmatrix} .3 & .7 \\ .2 & .8 \end{bmatrix}^{100} = \begin{bmatrix} .2222 & .7778 \end{bmatrix} = \begin{bmatrix} \frac{2}{9} & \frac{7}{9} \end{bmatrix}$$

In conclusion: in a long run, 2/9 of mice choose path A each day and 7/9 of mice choose path B