

MA 110 WORKSHEET (8.2) "EXTRA WORKSHEET 1"

Name \_\_\_\_\_

1. A bag contains 40 marbles. Seven marbles are purple, eleven are pink, and twenty-two are green.

- A. A marble is selected at random, what is the probability that it is purple?

$$\frac{7}{40}$$

- B. Two marbles are selected one after the other without replacement, what is the probability that they are both purple?

$$\frac{7}{40} \cdot \frac{6}{39}$$

- C. Four marbles are selected at random one after the other without replacement, what is the probability that they are all purple?

$$\frac{7}{40} \cdot \frac{6}{39} \cdot \frac{5}{38} \cdot \frac{4}{37}$$

- D. Recall there are 40 marbles in all, how many ways could a combination of 4 marbles be selected?

$$C_{40,4} = 91,390$$

- E. There are seven purple marbles, how many ways could a combination of 4 purple marbles be selected?

$$C_{7,4} = 35$$

- F. From the application of the formula  $C_{n,r} = \frac{n!}{n-r!r!}$ ,

$$C_{7,4} = \frac{7!}{7-4!4!} = \frac{7!}{3!4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{7 \cdot 6 \cdot 5 \cdot 4}{4 \cdot 3 \cdot 2 \cdot 1}$$

$$\text{and } C_{40,4} = \frac{40!}{40-4!4!} = \frac{40!}{36!4!} = \frac{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36!}{36! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{40 \cdot 39 \cdot 38 \cdot 37}{4 \cdot 3 \cdot 2 \cdot 1}$$

$$\text{Thus, } \frac{C_{7,4}}{C_{40,4}} = \frac{\frac{7 \cdot 6 \cdot 5 \cdot 4}{4 \cdot 3 \cdot 2 \cdot 1}}{\frac{40 \cdot 39 \cdot 38 \cdot 37}{4 \cdot 3 \cdot 2 \cdot 1}} = \frac{7 \cdot 6 \cdot 5 \cdot 4}{40 \cdot 39 \cdot 38 \cdot 37} = \frac{7}{40} \cdot \frac{6}{39} \cdot \frac{5}{38} \cdot \frac{4}{37}$$

- G. If seven marbles are selected at random one after the other without replacement, what is the probability that they are all purple?

$$\frac{C_{7,7}}{C_{40,7}} = \frac{1}{18,643,560} = 0.000000054$$

- H. If ten marbles are selected at random one after the other without replacement, what is the probability that three are purple and seven are green?

$$\frac{C_{7,3} \cdot C_{22,7}}{C_{40,10}} = \frac{35 \cdot 170,544}{847,660,528} = 0.007$$

- I. Three marbles are selected one after the other without replacement, what is the probability that they are all purple?

$$\frac{C_{7,3}}{C_{40,3}} = \frac{35}{9880} = 0.0035$$

- J. Three marbles are selected one after the other without replacement, what is the probability that exactly two are purple?

$$\frac{C_{7,2} C_{33,1}}{C_{40,3}} = \frac{21 \cdot 33}{9880} = \frac{693}{988} = 0.07$$

- K. Three marbles are selected one after the other without replacement, what is the probability that at least two are purple?

$$\frac{C_{7,2} C_{33,1}}{C_{40,3}} + \frac{C_{7,3}}{C_{40,3}} = \frac{693}{9880} + \frac{35}{9880} = \frac{728}{9880} = 0.074$$

2. A small college has 30 faculty members, eight from business, six from education, nine from science, and seven from math. From this group of faculty, a committee of three people will be chosen at random. What is the probability that this committee will contain

- A. only faculty from science?

$$C_{9,3}/C_{30,3} = 84/4060 = 0.0207$$

- B. no people from education?

$$C_{24,3}/C_{30,3} = 2024/4060 = 0.4985$$

- C. Suppose a larger committee of ten faculty is chosen. What is the probability that five are from education and five are from math?

$$\frac{C_{6,5} \cdot C_{7,5}}{C_{30,10}} = \frac{6 \cdot 21}{30,045,015} = 0.0000042$$

3. A bag contains 40 marbles. Seven marbles are purple, eleven are pink, and twenty-two are green.

- A. A marble is selected at random, what is the probability that it is green?

$$\frac{22}{40}$$

- B. Two marbles are selected one after the other with replacement, what is the probability that they are both green?

$$\frac{22}{40} \cdot \frac{22}{40} = \left(\frac{22}{40}\right)^2 = 0.55^2 = 0.3025$$

- C. Ten marbles are selected one after the other with replacement, what is the probability that they are all green?

$$\left(\frac{22}{40}\right)^{10} = 0.55^{10} = 0.0025$$

- D. Ten marbles are selected one after the other with replacement, what is the probability that seven are green?

$$C_{10,7} \left(\frac{22}{40}\right)^7 \left(\frac{18}{40}\right)^3 = 120 \cdot 0.55^7 \cdot 0.45^3 = 0.1665$$