MA 110 WORKSHEET (8.2) “EXTRA WORKSHEET 1”

Name ____________________________

1. A bag contains 40 marbles. Seven marbles are purple, eleven are pink, and twenty-two are green.

   A. A marble is selected at random, what is the probability that it is purple?

   \[
   \frac{7}{40}
   \]

   B. Two marbles are selected one after the other without replacement, what is the probability that they are both purple?

   \[
   \frac{7}{40} \cdot \frac{6}{39}
   \]

   C. Four marbles are selected at random one after the other without replacement, what is the probability that they are all purple?

   \[
   \frac{7}{40} \cdot \frac{6}{39} \cdot \frac{5}{38} \cdot \frac{4}{37}
   \]

   D. Recall there are 40 marbles in all, how many ways could a combination of 4 marbles be selected?

   \[
   C_{40,4} = 91,390
   \]

   E. There are seven purple marbles, how many ways could a combination of 4 purple marbles be selected?

   \[
   C_{7,4} = 35
   \]

   F. From the application of the formula \( C_{n,r} = \frac{n!}{(n-r)!r!} \),

   \[
   C_{7,4} = \frac{7!}{7-4 \cdot 4!} = \frac{7!}{3 \cdot 4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = \frac{7 \cdot 6 \cdot 5 \cdot 4}{4 \cdot 3 \cdot 2 \cdot 1}
   \]

   and

   \[
   C_{40,4} = \frac{40!}{40-4 \cdot 4!} = \frac{40!}{36 \cdot 4!} = \frac{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36!}{36 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{40 \cdot 39 \cdot 38 \cdot 37}{4 \cdot 3 \cdot 2 \cdot 1}
   \]

   Thus,

   \[
   \frac{C_{7,4}}{C_{40,4}} = \frac{7 \cdot 6 \cdot 5 \cdot 4}{40 \cdot 39 \cdot 38 \cdot 37} \cdot \frac{4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{7 \cdot 6 \cdot 5 \cdot 4}{40 \cdot 39 \cdot 38 \cdot 37} = \frac{7}{40} \cdot \frac{6}{39} \cdot \frac{5}{38} \cdot \frac{4}{37}
   \]
G. If seven marbles are selected at random one after the other without replacement, what is the probability that they are all purple?

\[ \frac{\binom{7}{7}}{\binom{40}{7}} = \frac{1}{18,643,560} = 0.000000054 \]

H. If ten marbles are selected at random one after the other without replacement, what is the probability that three are purple and seven are green?

\[ \frac{\binom{7}{3} \cdot \binom{22}{7}}{\binom{40}{10}} = \frac{35 \cdot 170,544}{847,660,528} = 0.007 \]

I. Three marbles are selected one after the other without replacement, what is the probability that they are all purple?

\[ \frac{\binom{7}{3}}{\binom{40}{3}} = \frac{35}{9880} = 0.0035 \]

J. Three marbles are selected one after the other without replacement, what is the probability that exactly two are purple?

\[ \frac{\binom{7}{2} \cdot \binom{33}{1}}{\binom{40}{3}} = \frac{21 \cdot 33}{9880} = \frac{693}{988} = 0.07 \]

K. Three marbles are selected one after the other without replacement, what is the probability that at least two are purple?

\[ \frac{\binom{7}{2} \cdot \binom{33}{1}}{\binom{40}{3}} + \frac{\binom{7}{3}}{\binom{40}{3}} = \frac{693}{9880} + \frac{35}{9880} = \frac{728}{9880} = 0.074 \]
2. A small college has 30 faculty members, eight from business, six from education, nine from science, and seven from math. From this group of faculty, a committee of three people will be chosen at random. What is the probability that this committee will contain

A. only faculty from science?

\[ \frac{C_9,3}{C_{30,3}} = \frac{84}{4060} = 0.0207 \]

B. no people from education?

\[ \frac{C_{24,3}}{C_{30,3}} = \frac{2024}{4060} = 0.4985 \]

C. Suppose a larger committee of ten faculty is chosen. What is the probability that five are from education and five are from math?

\[ \frac{C_{6,5} \cdot C_{7,5}}{C_{30,10}} = \frac{6 \cdot 21}{30,045,015} = 0.000042 \]

3. A bag contains 40 marbles. Seven marbles are purple, eleven are pink, and twenty-two are green.

A. A marble is selected at random, what is the probability that it is green?

\[ \frac{22}{40} \]

B. Two marbles are selected one after the other with replacement, what is the probability that they are both green?

\[ \frac{22}{40} \cdot \frac{22}{40} = \left( \frac{22}{40} \right)^2 = 0.55^2 = 0.3025 \]

C. Ten marbles are selected one after the other with replacement, what is the probability that they are all green?

\[ \left( \frac{22}{40} \right)^{10} = 0.55^{10} = 0.0025 \]

D. Ten marbles are selected one after the other with replacement, what is the probability that seven are green?

\[ C_{10,7} \left( \frac{22}{40} \right)^7 \left( \frac{18}{40} \right)^3 = 120 \cdot 0.55^7 \cdot 0.45^3 = 0.1665 \]