MATH 120 Practice Test 4

Slope-Intercept Form	Point-Slope Form	Slope Formula
$y = mx + b$	$y - y_1 = m(x - x_1)$	$m = \frac{y_2 - y_1}{x_2 - x_1}$

Universal Set, Intersection & Union

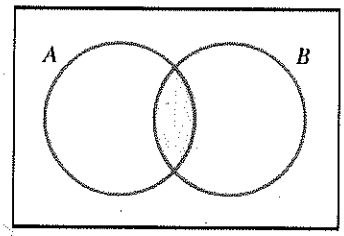
U: The universal set is the set of all elements under consideration.

$A \cap B = \{x | x \in A \text{ and } x \in B\}$: A intersect B is a set of all elements in A AND B.

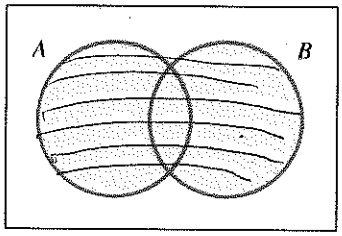
$A \cup B = \{x | x \in A \text{ or } x \in B\}$: A union B is a set of all elements in A OR B (or both).

$A' = \{x \notin A\}$: The complement of A is the set of elements in the universal set that are not in A.

Venn Diagrams

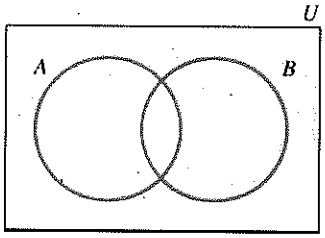


$A \cap B$



$A \cup B$

A or B



A'

For any two sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

If A and B are disjoint (they have no intersection), $n(A \cup B) = n(A) + n(B)$.

Multiplication Principle for Counting:

If n operations $O_1, O_2, \ldots, O_n$ are performed in order, with possible number of outcomes $N_1, N_2, \ldots, N_n$, respectively, then there are $N_1 * N_2 * \ldots * N_n$ possible combined outcomes of the operations performed in the given order.

Probability of an Event P(E): The likelihood that an event will occur.

$$P(E) = \frac{n(E)}{n(S)} = \frac{the\ number\ of\ elements\ in\ E}{the\ number\ of\ elements\ in\ S}$$

Certain Event: P(E) = 1 Impossible Event: P(E) = 0 Note: $0 \leq P(E) \leq 1$

Recall the probability of an event E: $P(E) = \frac{n(E)}{n(S)}$

The probability of the complement of an event: $P(E') = 1 - P(E)$

Odds: This is a comparison of happening: not happening for example, winning: losing

Odds for an Event = $P(E):P(E')$ **Odds Against an Event** = $P(E'):P(E)$

Odds for an Event = $\frac{P(E)}{P(E')}$ **Odds Against an Event** = $\frac{P(E')}{P(E)}$

Conditional Probability

For events A and B in a sample space S, we define the conditional probability of A given B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

1) Examples: Let $A = \{-3, -1, 1, 3\}$, $B = \{3, -3, 1, -1\}$, $C = \{-3, -2, -1, 0, 1, 2, 3\}$. Determine if the following are true or false.

2) $3 \in A$ True	7) $A \subset C$ True
3) $0 \in A$ False	8) $C \subset A$ False
4) $A \subset B$ True	9) $A \neq C$ True
5) $B \subset A$ True	10) $\emptyset \subset A$ True
6) $A = B$ True	11) $\emptyset \subset C$ True
	12) $\emptyset \in A$ False

13) List all subsets of the set {a, b, c}.

$\{\}, \{a, b\}, \{a, c\}, \{b, c\}$ $\{a, b, c\}$

$\{a\}, \{b\}, \{c\}$

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{3, 6, 9\}$ and $B = \{3, 4, 5, 6, 7\}$. Determine the following sets. Write using the listing method.

14) $A \cap B = \{3, 6\}$

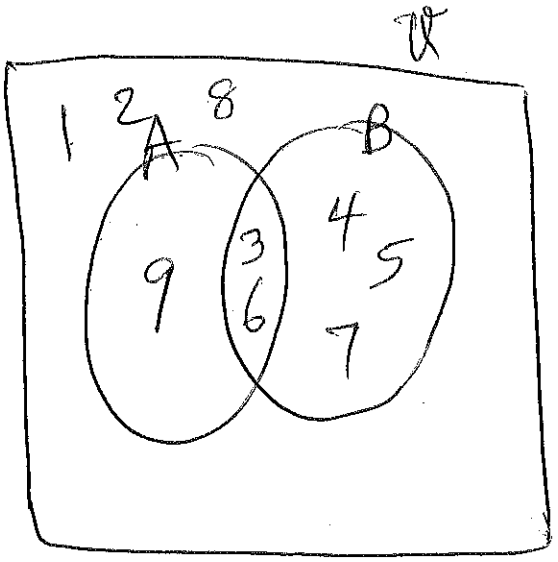
15) $A \cup B = \{3, 4, 5, 6, 7, 9\}$

16) $A' = \{1, 2, 4, 5, 7, 8\}$

17) $B' = \{1, 2, 8, 9\}$

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {3, 6, 9} and B = {3, 4, 5, 6, 7}. Draw a Venn diagram and then answer the following:

18) $A \cap B = \{3, 6\}$



19) $A \cup B = \{3, 4, 5, 6, 7, 9\}$

20) $A' = \{1, 2, 4, 5, 7, 8\}$

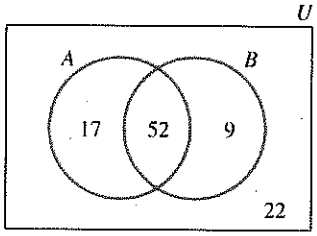
21) $B' = \{1, 2, 8, 9\}$

22) $(A \cap B)' = \{1, 2, 4, 5, 7, 8, 9\}$

23) $(A \cup B)' = \{1, 2, 8\}$

Use the Venn Diagram to determine the indicated <u>number of elements</u>.

24) $n(U) = 17 + 52 + 9 + 22 = 100$



25) $n(A \cap B) = 52$

26) $n(A) = 17 + 52 = 69$

27) $n(B) = 52 + 9 = 61$

28) $n(A \cup B) = 17 + 52 + 9 = 78$

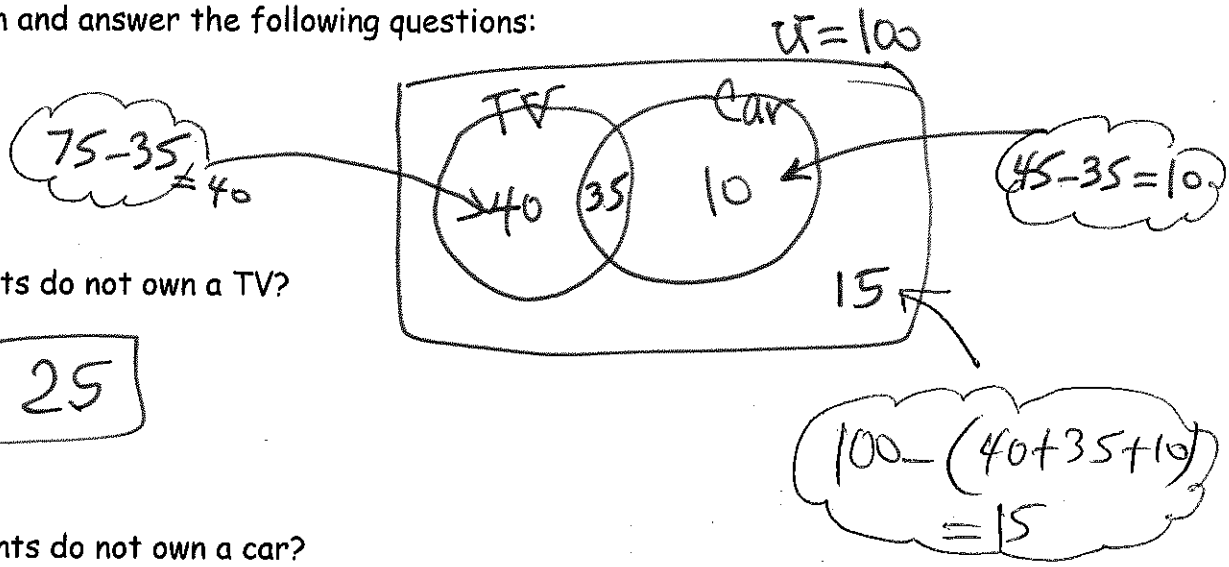
29) $n(A') = 9 + 22 = 31$

30) $n(B') = 17 + 22 = 39$

31) A survey was given to 100 randomly chosen students which included the following three questions and responses.

Do you own a TV?	Do you own a car?	Do you own a TV and a car?
75 said yes	45 said yes	35 said yes

Draw a Venn diagram and answer the following questions:



a) How many students do not own a TV?

40+15 = 25

b) How many students do not own a car?

40+15 = 55

cc) How many students own a car or a TV? (But Not Both) = 40+10 = 50

c) How many students do not own a car or a TV?

15

d) How many students own a TV but not a car? 40

e) How many students own a car but not a TV? 10

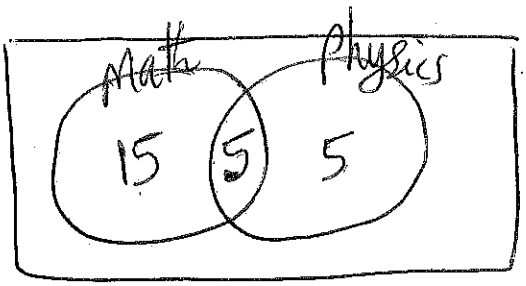
f) How many students own either a TV or a car?

40+35+10 = 85

32) Suppose there are 13 males and 15 females in a class. How many students are in the class?

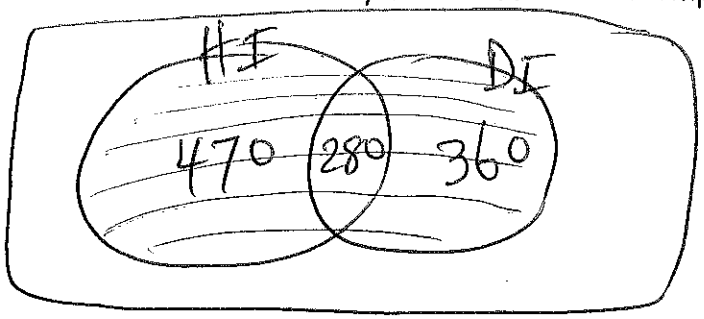
$$13 + 15 = 28$$

33) Suppose there are 20 math majors and 10 physics majors in a class. Also, in this class, 5 are both math and physics majors. How many students are in the class?



$$15 + 5 + 5 = 25$$

34) According to a survey of business firms in a certain city, 750 firms offer their employees health insurance, 640 offer dental insurance and 280 offer both health and dental insurance. How many firms offer their employees health or dental insurance?



$$470 + 280 + 360 = 1110$$

35) A retail store stocks windbreaker jackets in small, medium, large and extra-large. All are available in blue or red. What are the combined choices and how many combined choices are there?

$$\frac{4 \text{ choices}}{\text{size}} \cdot \frac{2 \text{ choices}}{\text{color}} = 8$$

Use Multiplication Principle

36) How many 2 letter code words can be formed from the first 3 letters of the alphabet if a letter can be used more than once.

$$\underset{\substack{a\\b\\c}}{3}\quad\underset{\substack{a\\b\\c}}{3}=9$$

37) How many 2 letter code words can be formed from the first 3 letters of the alphabet if a letter cannot be used more than once.

$$\underset{\substack{\cancel{a}\\b\\c}}{3}\quad\underset{\substack{b\\c}}{2}=6$$

38) A college offers 4 introductory courses in history, 3 in science, 3 in mathematics, 4 in philosophy and 2 in English.

a) If a student takes one course in each area, how many course selections are possible?

$$\underset{\text{History}}{4}\quad\underset{\text{Science}}{3}\quad\underset{\text{Math}}{3}\quad\underset{\text{philosophy}}{4}\quad\underset{\text{English}}{2}=288$$

b) If a student can only take one introductory course, how many selections are possible?

$4+3+3+4+2 = 16$

39) You would like to make a salad that consists of lettuce, tomato, cucumber and croutons. At the store there are 11 varieties of lettuce, 3 varieties of tomatoes, 5 varieties of cucumbers and 4 varieties of croutons. How many different salads can you make?

$\underset{\text{lettuce}}{11} \cdot \underset{\text{Tomatoes}}{3} \cdot \underset{\text{cucumbers}}{5} \cdot \underset{\text{Croutons}}{4} = 660$

40) A combination lock has 7 wheels each wheel having the digits 0 through 9.

a) How many 7-digit combinations are possible if no digit is repeated?

$10 \quad 9 \quad 8 \quad 7 \quad 6 \quad 5 \quad 4 = 604800$

b) How many 7-digit combinations are possible if digits can be repeated?

$10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 = 10^7$

$= 10\,000\,000$

41) How many different license plates are possible if the license plate contains 2 letters followed by 8 digits?

$\underbrace{26 \quad 26}_{\text{letters}} \quad \underbrace{10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10}_{\text{digits}}$

$= 676\,000\,000\,00$

6 officers (Plant A) | 8 officers (Plant B)

42) A corporation plans to fill 2 different positions for vice-president, V_1 and V_2, from administrative officers in 2 of its manufacturing plants. Plant A has 6 officers and plant B has 8.

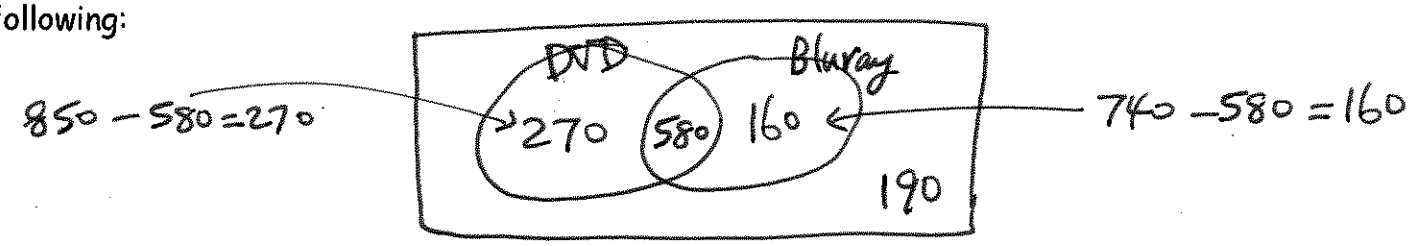
a) How many ways can these 2 positions be filled if the V_1 position is to filled from plant A and the V_2 position is to be filled from plant B?

$$\frac{6 \text{ choices}}{V_1} \cdot \frac{8 \text{ choices}}{V_2} = 48$$

b) How many ways can the 2 positions be filled if the selection is made without regard to plant?

$$\frac{14}{V_1} \cdot \frac{13}{V_2} = 182$$

43) A survey of 1200 people indicates that 850 own DVD players, 740 own Blu-ray players, and 580 own both DVD players and Blu-ray players. Draw a Venn Diagram to answer the following:



a) How many people own either a DVD player or a Blu-ray player?

$$270 + 580 + 160 = 1010$$

b) How many own neither a DVD player nor a Blu-ray player? 190

c) How many own a DVD player but not a Blu-ray player? 270

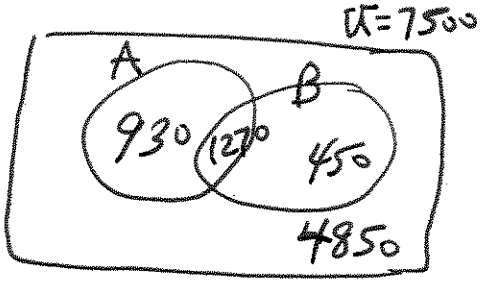
d) How many own a Blu-ray player but not a DVD player. [Venn Diagram] 160

44) A cable television company has 7500 subscribers in a suburban community. The company offers two premium channels, A and B. If 2200 subscribers receive channel A, 1720 receive channel B, and 4850 do not receive any premium channel, how many subscribers receive both channel A and channel B? [Venn Diagram]

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$2650 = 2200 + 1720 - n(A \cap B)$

$n(A \cap B) = 1270$



45. Experiment: Rolling 1 die. Find the probability of the following events: $\{1,2,3,4,5,6\}$

a) rolling a 2 $= \frac{1}{6}$

b) rolling an even number $= \frac{3}{6} = \frac{1}{2}$

c) rolling a 7 $= 0$

d) rolling a number that is divisible by 3 $= \frac{2}{6} = \frac{1}{3}$

e) rolling a number less than 7 $= \frac{6}{6} = 1$

46. Experiment: Flipping 2 coins. Find the probability of the events:

a) getting exactly one head $= \frac{2}{4} = \frac{1}{2}$

b) getting two heads $= \frac{1}{4}$

c) getting at least one head $= \frac{3}{4}$

H H
H T
T H
T T

47. Experiment: Rolling two dice. Find the probability of getting:

a) sum of 7 $= \frac{6}{36} = \frac{1}{6}$

b) sum of 11 $= \frac{2}{36} = \frac{1}{18}$

c) sum less than 5 $\frac{6}{36} = \frac{1}{6}$

d) sum of 7 or 11 $\frac{6+2}{36} = \frac{8}{36} = \frac{2}{9}$

e) a double $= \frac{6}{36} = \frac{1}{6}$

f) sum greater than 5 $= \frac{26}{36} = \frac{13}{18}$

g) snake eyes $= \frac{1}{36}$
two ones

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

48. Experiment: Drawing 1 card from a deck of cards. Find the probability of drawing :

a) an 8 $\frac{4}{52} = \frac{1}{13}$

b) a queen $\frac{4}{52} = \frac{1}{13}$

c) a face card $\frac{12}{52} = \frac{3}{13}$

d) a spade or an ace $\frac{13+3}{52} = \frac{16}{52} = \frac{4}{13}$

e) a spade and an ace $= \frac{1}{52}$

f) a red and a spade $= \frac{0}{52} = 0$

49. A box of 20 gumdrops was found to contain the number of gumdrops listed in the table. Assume that each gumdrop has an equal chance of being selected. If one gumdrop is drawn from the box, what is the probability of drawing

Lemon	Orange	Cherry	Grape
6	8	3	3

total = 20

a) an orange gumdrop $= \frac{8}{20} = \frac{2}{5}$

b) a cherry gumdrop $= \boxed{\frac{3}{20}}$

c) a lemon or orange gumdrop

$$\frac{6+8}{20} = \frac{14}{20} = \boxed{\frac{7}{10}}$$

50. In a family with two children,

G G
G B
B G
B B

find the probability of having:

a) exactly one girl $= \frac{2}{4} = \frac{1}{2}$

b) two girls $= \boxed{\frac{1}{4}}$

c) at least one girl $= \boxed{\frac{3}{4}}$

d) two boys $= \boxed{\frac{1}{4}}$

51. In a family of 3 children, $2^3 = 8$

find the probability of having:

a) one girl $= \frac{3}{8}$

b) two girls $= \frac{3}{8}$

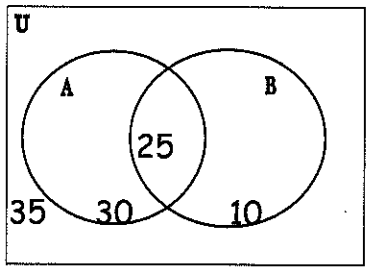
c) 3 girls $= \frac{1}{8}$

d) having at least one girl $= \frac{7}{8}$

G G G
G G B
G B G
G B B
B G G
B G B
B B G
B B B

$U = 100$

52) Refer to the Venn diagram. Determine the following:



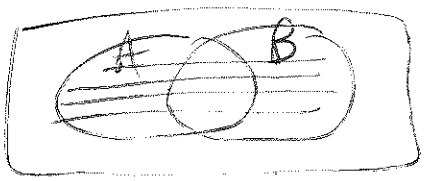
a) $P(A) = \frac{55}{100} = \frac{11}{20}$

b) $P(B) = \frac{35}{100} = \frac{7}{20}$

c) $P(A') = \frac{45}{100} = \frac{9}{20}$

d) $P(B') = \frac{35+30}{100} = \frac{65}{100} = \frac{13}{20}$

e) $P(A \cap B) = \frac{25}{100} = \frac{1}{4}$



f) $P(A \cup B) = \frac{65}{100} = \frac{13}{20}$

g) $P((A \cap B)') = \frac{75}{100} = \frac{3}{4}$

53) If the chance of rain is 40%, what is the chance of no rain? $1 - 40\% = 60\%$

$100\% - 40\% = 60\%$

$S = \{1, 2, 3, 4, 5, 6\}$

54) Consider the experiment of rolling a single die. Let the event be rolling a 6.

a) What is P(E)? $= \frac{1}{6}$

b) What is $P(E')$? $= \frac{5}{6}$

55) The probability that a candidate wins an election is 0.81.

a) What is the probability that he loses? $1 - 0.81 = 0.19$

b) What are the odds that he wins? $\frac{0.81}{0.19} = \frac{81}{19}$

c) What are the odds that he loses? $= \frac{19}{81}$

56) Compute the odds in favor of $S = \{1, 2, 3, 4, 5, 6\}$

$P(E) = \frac{3}{6} = \frac{1}{2}$ $P(E') = \frac{3}{6} = \frac{1}{2}$

a) Obtaining an even number in a single roll of a die.

$\frac{P(E)}{P(E')} = \frac{1/2}{1/2} = \frac{1}{1}$

b) Obtaining a sum of 7 in a single roll of two dice.

$P(\text{Sum is } 7) = \frac{6}{36} = \frac{1}{6}$

$P(\text{Sum is NOT } 7) = \frac{5}{6}$

odds in favor $= \frac{1/6}{5/6} = \boxed{\frac{1}{5}}$

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

57) Given the following probabilities for an event E, find the odds for and against E.

a) $P(E) = \frac{6}{11}$ $P(E') = \frac{5}{11} \Rightarrow$ odds in favor $\frac{6/11}{5/11} = \frac{6}{5}$ odds against $= \frac{5}{6}$

b) $P(E) = \frac{7}{12}$ $P(E') = \frac{5}{12} \Rightarrow$ odds in favor $= \frac{7/12}{5/12} = \frac{7}{5}$; odds against $= \frac{5}{7}$

c) P(E) = 37% $= 0.37$ $P(E') = 0.63 \Rightarrow$ odds in favor $= \frac{0.37}{0.63} = \frac{37}{63}$; odds against $= \frac{63}{37}$

d) P(E) = 0.95 $P(E') = 0.05 \Rightarrow$ odds in favor $\frac{0.95}{0.05} = \frac{19}{1}$; odds against $= \frac{1}{19}$

58)Given that you have drawn a red card, what is the probability that it is a heart?

$$\frac{13 \text{ hearts}}{26 \text{ Red cards}} = \frac{13}{26} = \frac{1}{2}$$

59) Given that an odd number turns up after rolling one die, what is the probability

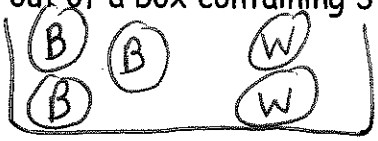
a) it is a 3? $1/3$

b) it is not a 3? $\frac{2}{3}$

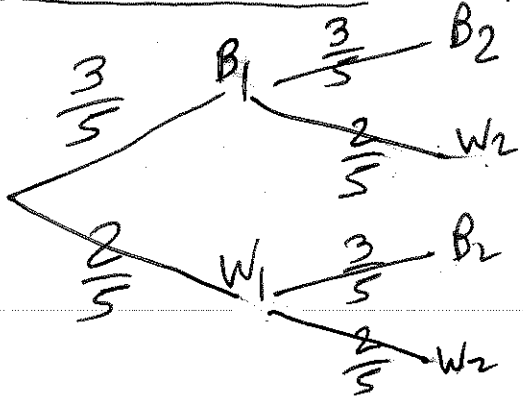
~~{1, 2, 3, 4, 5, 6}~~

odds {1, 3, 5}

60) Two marbles are drawn in succession out of a box containing 3 blue and 2 white marbles.



a) Find the probability that the second marble was white, given that the first marble was replaced before the second draw. Draw a probability tree-diagram to write the sample space.

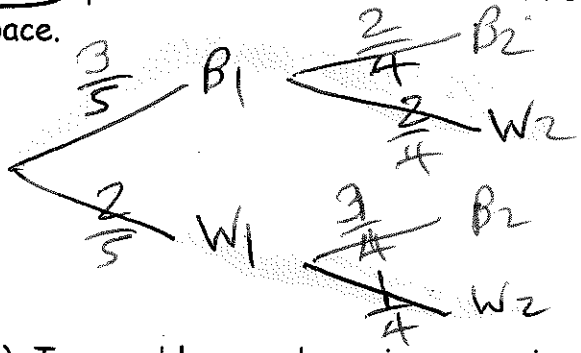


B_1W_2 OR W_1W_2

$\frac{3}{5}\frac{2}{5} + \frac{2}{5}\frac{2}{5}$

$= \frac{6}{25} + \frac{4}{25} = \frac{10}{25} = \frac{2}{5}$

b) Find the probability that the second marble was white, given that the first marble was NOT replaced before the second draw. Draw a probability tree-diagram to write the sample space.



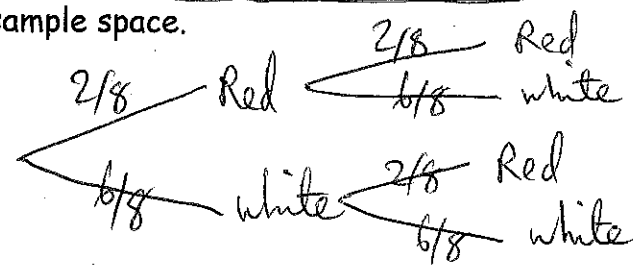
$B_1W_2 + W_1W_2$

$\frac{3}{5}\frac{2}{4} + \frac{2}{5}\frac{1}{4} = \frac{6}{20} + \frac{2}{20} = \frac{8}{20}$

$= \frac{2}{5}$

61) Two marbles are drawn in succession out of a box containing 2 red and 6 white marbles.

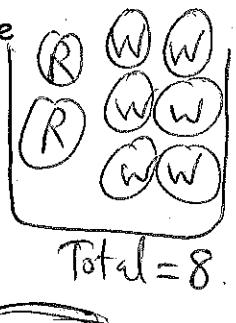
a) Find the probability that at least 1 marble was red, given that the first marble was replaced before the second draw. Draw a probability tree diagram to write the sample space.



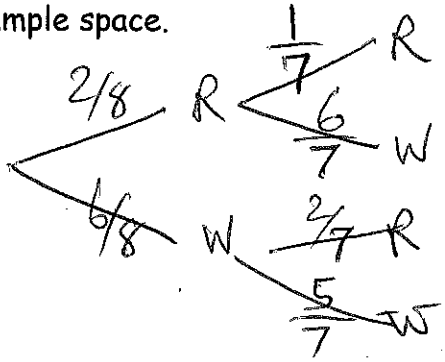
$RR + RW + WR$

$\frac{2}{8}\cdot\frac{2}{8} + \frac{2}{8}\frac{6}{8} + \frac{6}{8}\cdot\frac{2}{8}$

$= \frac{7}{16}$

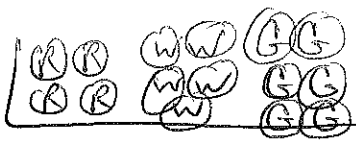


b) Find the probability that at least 1 marble was red, given that the first marble was NOT replaced before the second draw. Draw a probability tree diagram to write the sample space.

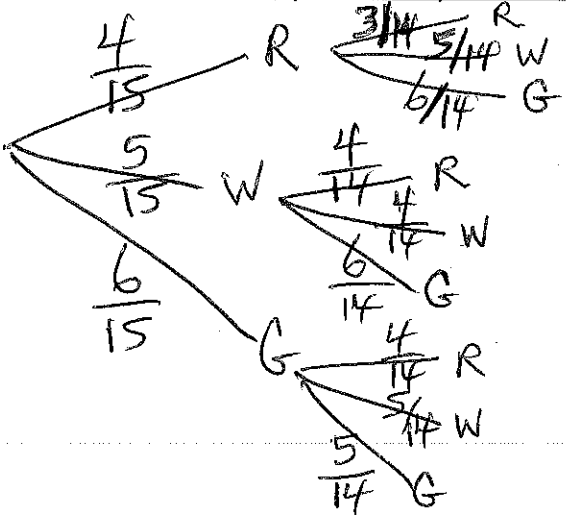


$RR + RW + WR$

$\frac{2}{8}\cdot\frac{1}{7} + \frac{2}{8}\frac{6}{7} + \frac{6}{8}\cdot\frac{2}{7} = \frac{13}{28}$



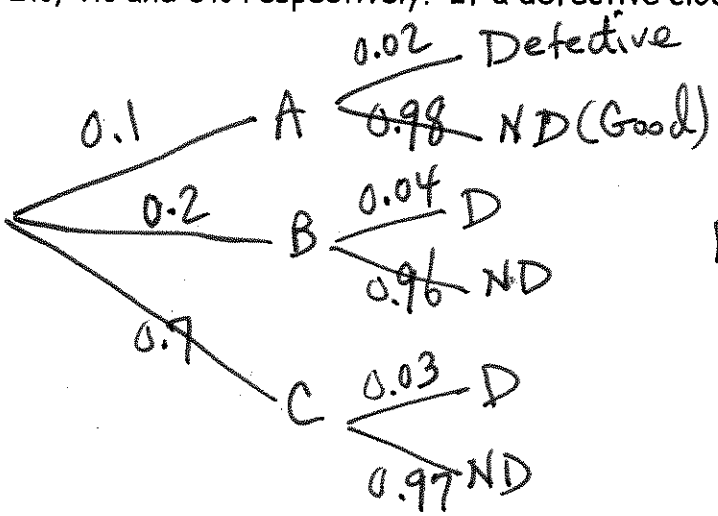
62) A box contains 4 red, 5 white and 6 green marbles. Two marbles are drawn out of the box in succession without replacement. What is the probability that both marbles are the same color? Draw a probability tree diagram to write the sample space.



$RR + WW + GG$

$\frac{4}{15} \cdot \frac{3}{14} + \frac{5}{15} \cdot \frac{4}{14} + \frac{6}{15} \cdot \frac{5}{14} = \frac{31}{105}$

63. manufacturer obtains clock-radios from three different subcontractors: 10% from A, 20% from B and 70% from C. The defective rates for these subcontractors are 2%, 4% and 3% respectively. If a defective clock-radio is returned by a customer,



$P(\text{Defective}) = 0.1(0.02) + 0.2(0.04) + 0.7 * 0.03 = 0.031$ (OR)

a) What is the probability that it came from subcontractor A?

$$P(A \mid D) = \frac{P(A \wedge D)}{P(D)} = \frac{0.1 * 0.02}{0.031} \cong 0.0645$$

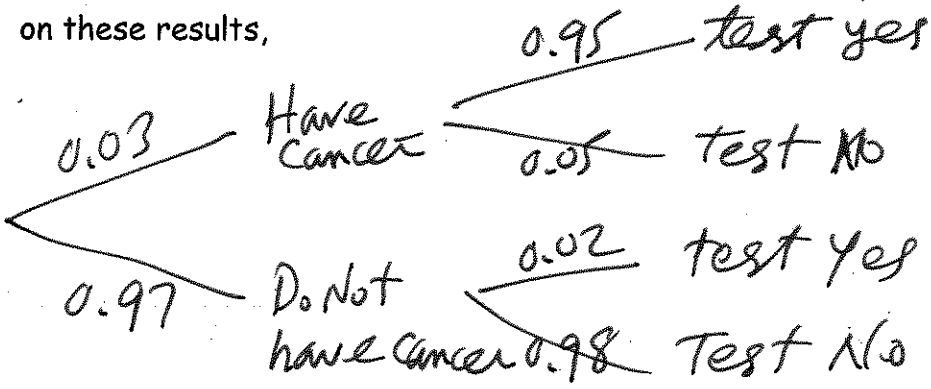
b) What is the probability that it came from subcontractor B?

$$P(B \mid D) = \frac{P(B \wedge D)}{P(D)} = \frac{0.2 * 0.04}{0.031} \cong 0.258$$

c) What is the probability that it came from subcontractor C?

$$P(C \mid D) = \frac{P(C \wedge D)}{P(D)} \qquad \frac{0.7 * 0.03}{0.031} \cong 0.677$$

64. A new, simple test has been developed to detect a particular type of cancer. The test must be evaluated before it is put into use. A medical researcher selects a random sample of 3000 adults and finds (by other means) that 3% have this type of cancer. Each of the 3000 adults is given the test, and it is found that the test indicates cancer in 95% of those who have it and in 2% of those who do not. Based on these results,



a) What is the probability of a randomly chosen person having cancer given the test indicates cancer?

$$\frac{0.03(0.95)}{0.03(0.95)+0.97(0.02)} = 0.59$$

b) What is the probability of a randomly chosen person having cancer given that the test does not indicate cancer?

$$\frac{0.03*0.05}{(0.03*0.05+0.97*0.98)} = 0.0016$$

65) In a horse race, how many different finishes among the first 3 places are possible if 17 horses are running? (Exclude ties) order is important

$17\,nPr\,3 = 4080$; $17 \cdot 16 \cdot 15 = 4080$

66) How many ways can a 4-person subcommittee be selected from a committee of 8 people? order is Not important

$8\,nCr\,4 = 70$

67) How many ways can a president, vice-president, secretary, and treasurer be chosen from a committee of 8 people?

$8\,nPr\,4 = 1680$; $8 \cdot 7 \cdot 6 \cdot 5 = 1680$