MATH 120 7.3 Basic Counting Principles

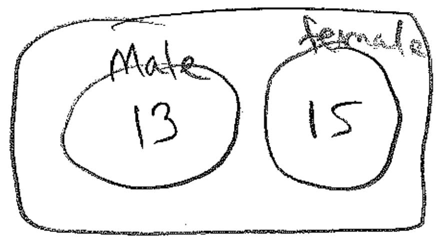
Addition Principle for Counting:
For any two sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
If A and B are disjoint (they have no intersection), $n(A \cup B) = n(A) + n(B)$.

Venn Diagrams & the Addition Principle

Examples Draw a Venn Diagram to solve.

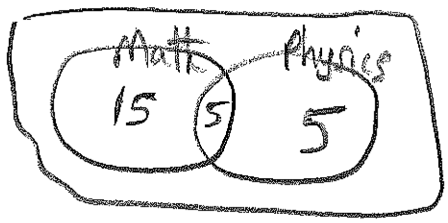
1) Suppose there are 13 males and 15 females in a class. How many students are in the class?

$= \boxed{28}$



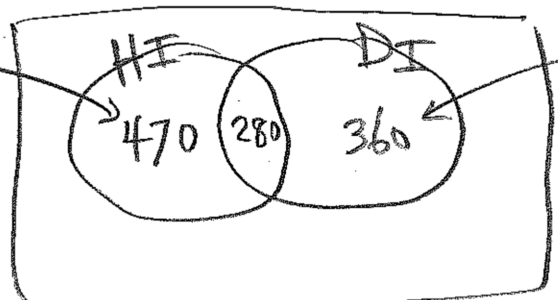
2) Suppose there are 20 math majors and 10 physics majors in a class. Also, in this class, 5 are both math and physics majors. How many students are in the class?

$15+5+5 = \boxed{25}$



3) According to a survey of business firms in a certain city, 750 firms offer their employees health insurance, 640 offer dental insurance and 280 offer both health and dental insurance. How many firms offer their employees health or dental insurance?

$750 - 280$



$640 - 280 = 360$

Health or Dental $= 470 + 280 + 360 = \boxed{1110}$

$n(H \cup D) = n(H) + n(D) - n(H \cap D)$
$= 750 + 640 - 280 = \boxed{1110}$

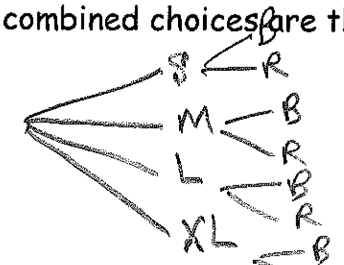
Examples: 4) Use the Addition Principle to solve problems 1, 2 and 3 above.

$$n(H \cup D) = n(H) + n(D) - n(H \cap D)$$
$$= 750 + 640 - 280 = \boxed{1110}$$

Tree Diagrams & the Multiplication Principle

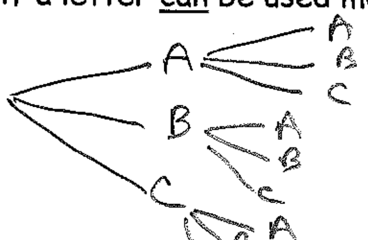
Examples Use a tree diagram to solve the problem.

5) A retail store stocks windbreaker jackets in small, medium, large and extra-large. All are available in blue or red. What are the combined choices and how many combined choices are there?



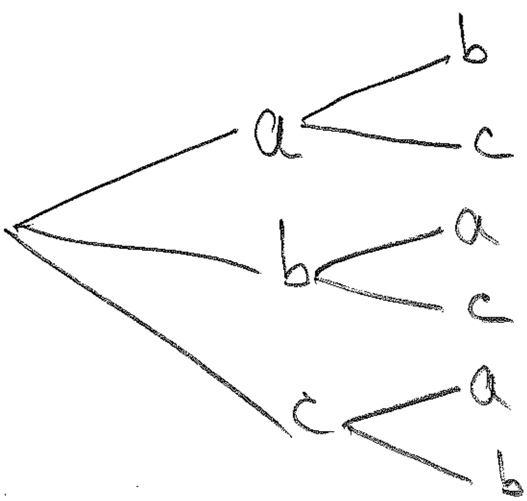
$$\frac{4}{\text{Size}} \cdot \frac{2}{\text{Color}} = \boxed{8}$$

6) How many 2 letter code words can be formed from the first 3 letters of the alphabet if a letter can be used more than once.



$$\frac{3}{\begin{matrix}a\\b\\c\end{matrix}} \cdot \frac{3}{\begin{matrix}a\\b\\c\end{matrix}} = \boxed{9}$$

7) How many 2 letter code words can be formed from the first 3 letters of the alphabet if a letter cannot be used more than once.



$$\underline{3} \cdot \underline{2} = \boxed{6}$$

Multiplication Principle for Counting:
If n operations $O_1, O_2, \ldots, O_n$ are performed in order, with possible number of outcomes $N_1, N_2, \ldots, N_n$, respectively, then there are $N_1 * N_2 * \ldots * N_n$ possible combined outcomes of the operations performed in the given order.

Example 8) Use the Multiplication Principle to solve numbers 5, 6, and 7 above.

Examples Use the Multiplication Principle to solve.

9) A college offers 4 introductory courses in history, 3 in science, 3 in mathematics, 4 in philosophy and 2 in English. a) If a student takes one course in each area, how many course selections are possible? b) If a student can only take one introductory course, how many selections are possible?

a) $\underset{\text{History}}{\underline{4}} \cdot \underset{\text{Science}}{\underline{3}} \cdot \underset{\text{Math}}{\underline{3}} \cdot \underset{\text{philosophy}}{\underline{4}} \cdot \underset{\text{English}}{\underline{2}} = \boxed{288}$

(*) b) $4+3+3+4+2 = 16$ $\boxed{\text{choices}}$

10) You would like to make a salad that consists of lettuce, tomato, cucumber and croutons. At the store there are 11 varieties of lettuce, 3 varieties of tomatoes, 5 varieties of cucumbers and 4 varieties of croutons. How many different salads can you make?

$\underset{\text{lettuce}}{\underline{11}} \cdot \underset{\text{Tomato}}{\underline{3}} \cdot \underset{\text{cucumber}}{\underline{5}} \cdot \underset{\text{croutons}}{\underline{4}} = \boxed{660}$ choices

11) A combination lock has 7 wheels each wheel having the digits 0 through 9.

a) How many 7-digit combinations are possible if no digit is repeated?

10 9 8 7 6 5 4 = 604800 $_{10}P_7$

b) How many 7-digit combinations are possible if digits can be repeated?

10 10 10 10 10 10 10 $= 10^7 = \boxed{10\,000\,000}$

12) How many different license plates are possible if the license plate contains 2 letters followed by 8 digits?

$\underbrace{26 \quad 26}_{\text{letters}} \quad \underbrace{10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10}_{\text{digits}}$

$= 676{,}000\,000\,00 = 6.76 \times 10^{10}$

13) A corporation plans to fill 2 different positions for vice-president, V_1 and V_2, from administrative officers in 2 of its manufacturing plants. Plant A has 6 officers and plant B has 8.

$\underset{V_1}{14} \cdot \underset{V_2}{13} = \boxed{182}$

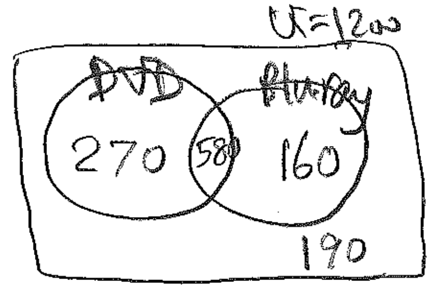
a) How many ways can these 2 positions be filled if the V_1 position is to filled from plant A and the V_2 position is to be filled from plant B?

$\underset{A}{6} \cdot \underset{B}{8} = \boxed{48}$

b) How many ways can the 2 positions be filled if the selection is made without regard to plant?

$14 \times 13 = \boxed{182}$

14) A survey of 1200 people indicates that 850 own DVD players, 740 own Blu-ray players, and 580 own both DVD players and Blu-ray players. Draw a Venn Diagram to answer the following:



a) How many people own either a DVD player or a Blu-ray player?

$270 + 580 + 160 = 1010$

b) How many own neither a DVD player or a Blu-ray player?

190

c) How many own a DVD player but not a Blu-ray player?

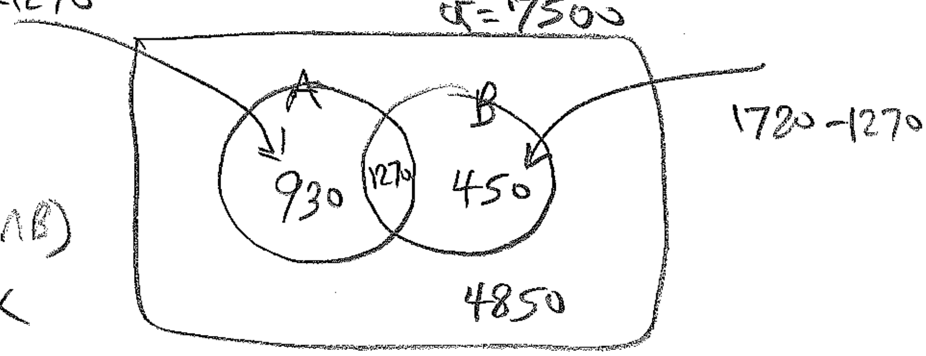
270

d) How many own a Blu-ray player but not a DVD player. [Venn Diagram]

160

15) A cable television company has 7500 subscribers in a suburban community. The company offers two premium channels, A and B. If 2200 subscribers receive channel A, 1720 receive channel B, and 4850 do not receive any premium channel, how many subscribers receive both channel A and channel B? [Venn Diagram]

$2200 - 1270$



$7500 - 4850 = 2650$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$2650 = 2200 + 1720 - X$

$X = 1270$

$\boxed{A \cap B = 1270}$