

### MATH 120 Section 8.3 Conditional Probability

For events A and B in a sample space S, we define the conditional probability of A given B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

#### Examples

1) Given that you have drawn a red card, what is the probability that it is a heart?

$$\frac{13}{26} = \frac{1}{2}$$

2) Given that an odd number turns up after rolling one die, what is the probability

a) it is a 3?  $= \frac{1}{3}$

$$S = \{1, \cancel{2}, 3, \cancel{4}, 5, \cancel{6}\}$$

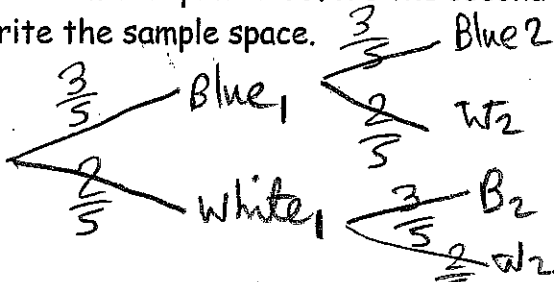
b) it is not a 3?  $= \frac{2}{3}$

3) Two marbles are drawn in succession out of a box containing 3 blue and 2 white marbles.



Total 5

a) Find the probability that the second marble was white, given that the first marble was replaced before the second draw. Draw a probability tree-diagram to write the sample space.

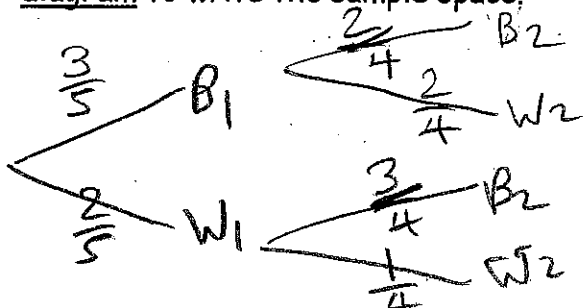


$$B_1 W_2 + W_1 W_2$$

OR

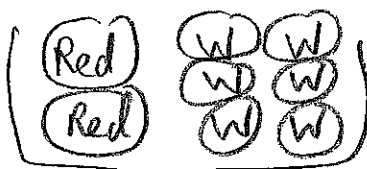
$$= \frac{3}{5} \frac{2}{5} + \frac{2}{5} \frac{2}{5} = \frac{6}{25} + \frac{4}{25} = \frac{10}{25} = \frac{2}{5}$$

b) Find the probability that the second marble was white, given that the first marble was NOT replaced before the second draw. Draw a probability tree-diagram to write the sample space.



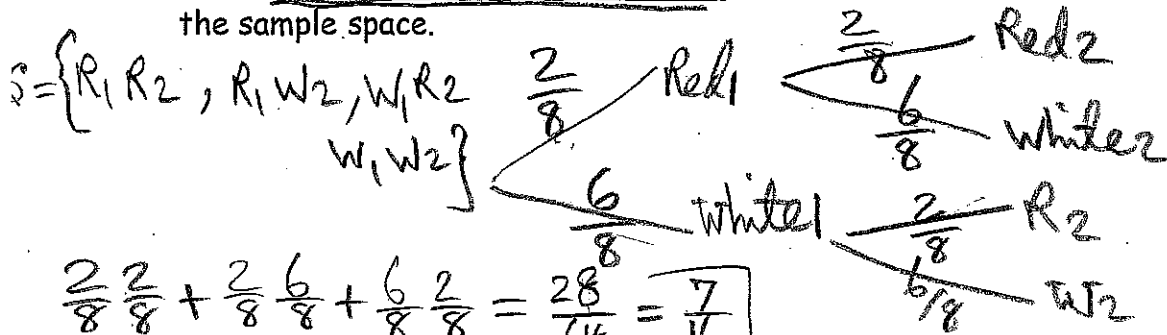
$$B_1 W_2 + W_1 W_2$$

$$\left(\frac{3}{5}\right)\left(\frac{2}{4}\right) + \left(\frac{2}{5}\right)\left(\frac{1}{4}\right) = \frac{6}{20} + \frac{2}{20} = \frac{8}{20} = \frac{2}{5}$$



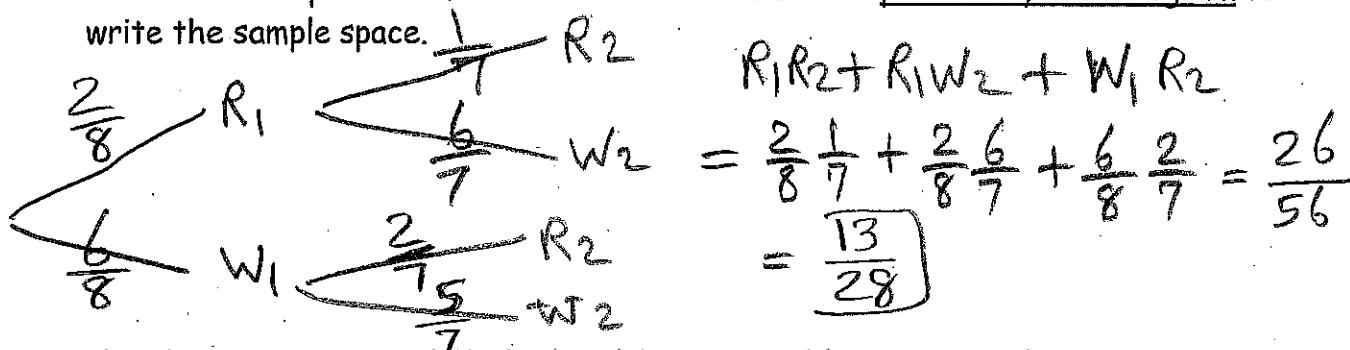
4) Two marbles are drawn in succession out of a box containing 2 red and 6 white marbles.

a) Find the probability that at least 1 marble was red, given that the first marble was replaced before the second draw. Draw a probability tree diagram to write the sample space.

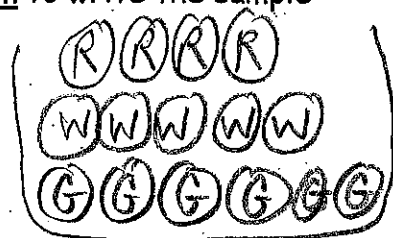
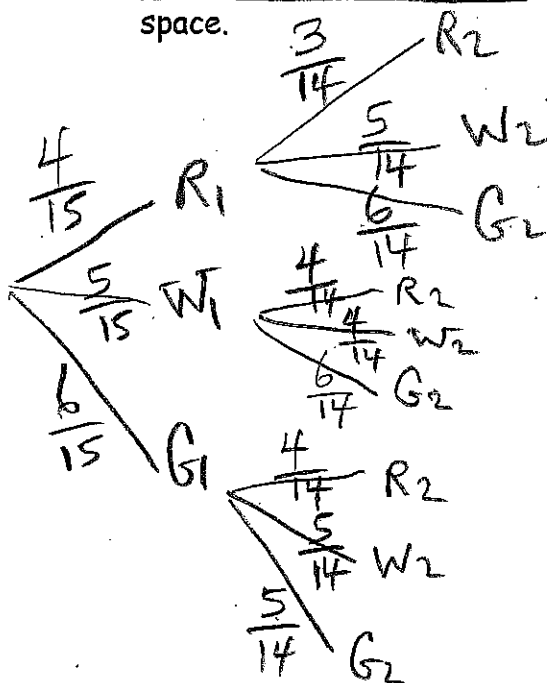


$$\frac{2}{8} \frac{2}{8} + \frac{2}{8} \frac{6}{8} + \frac{6}{8} \frac{2}{8} = \frac{28}{64} = \frac{7}{16}$$

b) Find the probability that at least 1 marble was red, given that the first marble was NOT replaced before the second draw. Draw a probability tree diagram to write the sample space.



5) A box contains 4 red, 5 white and 6 green marbles. Two marbles are drawn out of the box in succession without replacement. What is the probability that both marbles are the same color? Draw a probability tree diagram to write the sample space.



Total = 15

$$R_1 R_2 + W_1 W_2 + G_1 G_2$$

$$\frac{4}{15} \frac{3}{14} + \frac{5}{15} \frac{4}{14} + \frac{6}{15} \frac{5}{14} = \frac{62}{210} = \frac{31}{105}$$