MATH 120 Section 8.4 Bayes' Formula

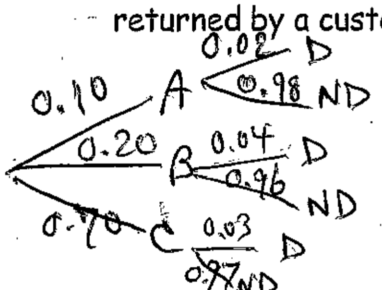
In the preceding section, we calculated the conditional probability of a later event given an earlier event. Now we will find the probability of an earlier event given a later event.

<u>Bayes' Formula</u>

$$P(U_1|E) = \frac{P(U_1 \cap E)}{P(E)} = \frac{\textit{product of branch probabilites leading E through } U_1}{\textit{sum of all branch products leading to E}}$$

where $U_1, U_2, \ldots U_n$ are n mutually exclusive events whose union is the sample space S, and E is an arbitrary event in S such that $P(E) \neq 0$.

1. A manufacturer obtains clock-radios from three different subcontractors: 10% from A, 20% from B and 70% from C. The defective rates for these subcontractors are 2%, 4% and 3% respectively. If a defective clock-radio is returned by a customer,



$$\quad AD \quad + \quad BD \quad + \quad CD$$

$$P(D) = 0.10 * 0.02 + 0.20 * 0.04 + 0.70 * 0.03$$

$$P(D) = 0.031$$

a) What is the probability that it came from subcontractor A?

$$P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{0.10 * 0.02}{0.031} = 0.0645 = \frac{2}{31}$$

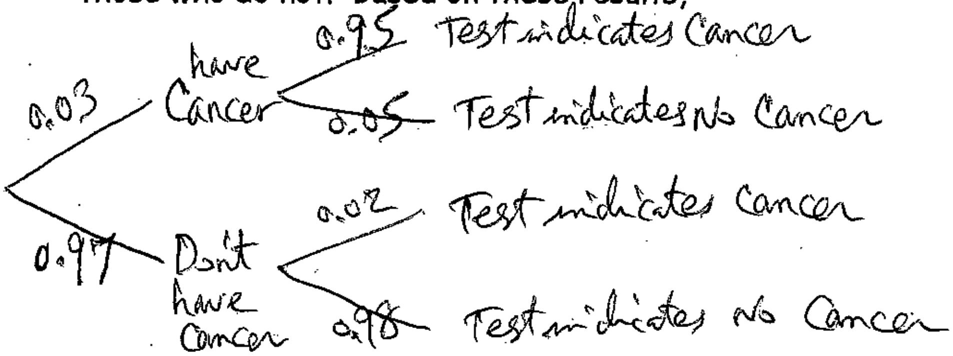
b) What is the probability that it came from subcontractor B?

$$P(B|D) = \frac{P(B \cap D)}{P(D)} = \frac{0.20 * 0.04}{0.031} = \frac{8}{31} \approx 0.258$$

c) What is the probability that it came from subcontractor C?

$$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{0.70 * 0.03}{0.031} = \frac{21}{31} \approx 0.677$$

2. A new, simple test has been developed to detect a particular type of cancer. The test must be evaluated before it is put into use. A medical researcher selects a random sample of 3000 adults and finds (by other means) that 3% have this type of cancer. Each of the 3000 adults is given the test, and it is found that the test indicates cancer in 95% of those who have it and in 2% of those who do not. Based on these results,



a) What is the probability of a randomly chosen person having cancer given the test indicates cancer?

$$P(\text{have cancer} \mid \text{test indicates cancer}) = \frac{P(\text{have cancer and test says cancer})}{P(\text{test indicates cancer})}$$

$$= \frac{0.03 * 0.95}{(0.03 * 0.95 + 0.97 * 0.02)} = \boxed{0.595}$$

b) What is the probability of a randomly chosen person having cancer given that the test does not indicate cancer?

$$P(\text{have cancer} \mid \text{test does not indicate cancer})$$

$$= \frac{P(\text{have cancer and test does not indicate})}{P(\text{test does not indicate cancer})}$$

$$= \frac{0.03 * 0.05}{(0.03 * 0.05 + 0.97 * 0.98)} = \boxed{0.0016}$$