MATH 120 Section 7.4 Permutations & Combinations

Factorial: \( n! = n \cdot (n - 1) \cdot (n - 2) \cdots 2 \cdot 1 \) Note: By definition, \( 0! = 1 \)

Examples Compute the following Factorials

1) \( 5! = 120 \) 2) \( 4! = 24 \) 3) \( 3! = 6 \) 4) \( 2! = 2 \) 5) \( 1! = 1 \) 6) \( 0! = 1 \)

\[ \frac{7!}{6!} = 7 \quad \frac{8!}{5!} = 8 \cdot 7 \cdot 6 \cdot 5 \]

\[ \frac{5!}{3!} = \frac{120}{6} = 20 \]

A permutation is an arrangement of objects in a specific order. [Order Matters]

Example: Awarding 1st place, 2nd place and 3rd to a group of 10 students in a spelling contest.

\[ _{10}P_3 = 720 \]

A combination is an arrangement of objects in any order. [Order Doesn’t Matter]

Example: Choosing 3 students from a group of 10 students to participate on a team.

\[ _{10}C_3 = \frac{10!}{3!(12-3)!} = \frac{10!}{3!7!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120 \]

Examples Permutation or Combination?

11) Suppose that 7 people enter a swim meet. Assuming that there are no ties, in how many ways could the gold, silver, and bronze medals be awarded?

\[ 7P_3 = 210 \]

12) In a horse race, how many different finishes among the first three places are possible if 22 horses are running?

\[ 22P_3 = 9240 \]

13) John bought a machine to make fresh juice. He has five different fruits: strawberries, oranges, apples, pineapples, and lemons. If he only uses two fruits, how many different juice drinks can John make?

\[ \leq _5C_2 = 10 \]

14) How many different committees of 3 people can be chosen to work on a special project from a group of 9 people?

\[ 9C_3 = 84 \]

15) From a committee of 10 people, how many ways can we choose a chair, a vice-chair and a secretary?

\[ 10P_3 = 720 \]

16) From a committee of 10 people, how many ways can we choose a subcommittee of 3 people?

\[ _{10}C_3 = 120 \]
Calculating Permutations & Combinations

<table>
<thead>
<tr>
<th>Permutation: The number of permutations of n objects taken r at a time (no repetition):</th>
<th>Combination: The number of combinations of n objects taken r at a time (no repetition):</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{n,r} = \frac{n!}{(n-r)!} )</td>
<td>( C_{n,r} = \frac{n!}{r!(n-r)!} )</td>
</tr>
</tbody>
</table>

Examples

16. Calculate the following. a) \( P_{9,2} \)  
   b) \( C_{8,3} \)

17. Calculate the above problems 11 - 16

\[
\begin{align*}
P_{9,2} &= 9 \text{nP}r \ 2 = 72 \\
C_{8,3} &= 8 \text{nCr} \ 3 = 56 
\end{align*}
\]