MA 160 Practice Quiz #4 – Summer 2008

(20 points) Name Solutions

Show all of your work on the quiz paper. Full credit is not given unless the answer follows from the work shown.

Be sure to use appropriate units in all of your answers.

1. (10 points) The United States federal budget submitted by President Clinton for 2000 was $1.8 trillion. The United States federal budget submitted by President Bush for 2005 was $2.4 trillion. Assume that the United States federal budget grows at a rate proportional to its size and let the year 2000 correspond to t = 0.

(a) Find the growth constant k **correct to four decimal places** for this situation.

	t	
2000	0	1.8
2005	5	2.4

$P(t) = P_0 e^{kt}$

$P(t) = 1.8e^{kt}$

$P(5) = 1.8e^{k5} = 2.4$

$e^{5k} = 2.4/1.8$

$\ln e^{5k} = \ln(2.4/1.8)$

$5k = \ln(2.4/1.8)$ *so* $k = \ln(2.4/1.8)/5 \approx .0575$

(b) Write the exponential model.

$$P(t) = 1.8e^{.0575t}$$

(c) According to the model, what is the amount of the United States federal budget submitted for 2009?

$P(9) = 1.8e \wedge (.0575 * 9) \approx \$3.02$ *trillion*

OVER⟶

2. (10 points) A sample of 12 grams of a radioactive substance is placed in a vault. Let $P(t)$ be the amount remaining after t years, and suppose that $P(t)$ satisfies the differential equation

$$P'(t) = -.037P(t).$$

(a) Use the differential equation to determine how fast the substance is decaying when there are 8 grams present.

$P'(t) = -.037P(t)$

$P'(t) = -.037(8) = -.296$

The radioactive substance is decaying at a rate of .296 grams/year.

(b) Find the formula for $P(t)$. $P(t) = 12e^{-.037t}$

(c) Find the half-life of the substance. Round your answer to the nearest whole year.

$12e^{-.037t} = 6$

$e^{-.037t} = .5$

$\ln e^{-.037t} = \ln(.5)$

$-.037t = \ln(.5)$

$t = \ln(.5)/-.037 \approx 19\ \ years$

3. (5 points) How much money must you invest now at 4.8% interest compounded continuously in order to have $15000 at the end of 5 years?

$P(t) = P_0\, e^{rt}$

$P(5) = P_0\, e \wedge (.048 * 5) = 15000$

$P_0 = 15000 / e \wedge (.048 * 5)$

$P_0 = \$11799.42$