Dr. Katiraie MA160 Quiz 3 Summer 08

Name Solution

Solve the problem.

(3 Pts) 1) If the price charged for a candy bar is p(x) cents, then x thousand candy bars will be sold in a certain city, where $p(x) = 61 - \frac{x}{24}$. How many candy bars must be sold to maximize revenue?

$$R(x) = xp = 61x - \frac{x^2}{24}$$

$$R' = 61 - \frac{2x}{24} = 0$$

$$61 = \frac{2x}{24} \Rightarrow 2x = 1464 \Rightarrow \boxed{x = 732 \text{ thousand Candy Bars}}$$

(3 Pts) 2) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:
$R(x) = 40x - 0.5x^2$
$C(x) = 8x + 5.$

$$P(x) = R - C = (40x - 0.5x^2) - (8x + 5)$$

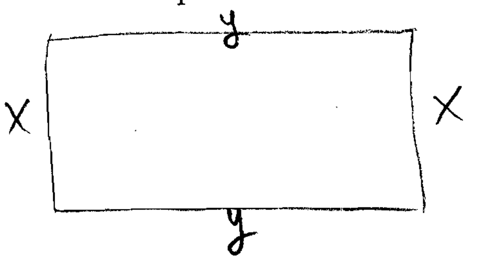
$$P(x) = 40x - 0.5x^2 - 8x - 5$$

$$P(x) = -0.5x^2 + 32x - 5$$

$$P'(x) = -x + 32 = 0$$

$$\boxed{x = 32 \text{ Units}}$$

(4 Pts) 3) A rectangular field is to be enclosed on four sides with a fence. Fencing costs \$3 per foot for two opposite sides, and \$2 per foot for the other two sides. Find the dimensions of the field of area 850 ft^2 that would be the cheapest to enclose.



$$xy = 850 \Rightarrow y = \frac{850}{x}$$

$$\text{Cost} = 3y + 3y + 2x + 2x$$

$$\text{Cost} = 6y + 4x = 6\left(\frac{850}{x}\right) + 4x$$

$$\text{Cost} = 5100x^{-1} + 4x$$

$$(\text{Cost})' = -5100x^{-2} + 4 \Rightarrow \frac{-5100}{x^2} + 4 = 0$$

$$\frac{5100}{x^2} = \frac{4}{1} \Rightarrow 4x^2 = 5100$$

$$x^2 = 1275$$

$$x = 35.71$$

$$\boxed{\begin{array}{l} x = 35.71 \text{ feet} \\ y = 23.80 \text{ feet} \end{array}}$$

(4 Pts) 4) A baseball team is trying to determine what price to charge for tickets. At a price of \$10 per ticket, it averages 40,000 people per game. For every increase of \$1, it loses 5,000 people. What price per ticket should be charged in order to maximize revenue?

$(40000, \$10)$ $\quad m = \dfrac{11-10}{35000-40000} = \dfrac{1}{-5000} = -0.0002$

$(35000, \$11)$

$y - 10 = -0.0002(x - 40000) \Rightarrow y = -0.0002x + 18$

$p = -0.0002x + 18$

Revenue $= xp = -0.0002x^2 + 18x$

$R' = -0.0004x + 18 = 0$

$x = 45000 \Rightarrow$ price $= -0.0002(45000) + 18 = \boxed{\$9}$

(3 Pts) 5) A carpenter is building a rectangular room with a fixed perimeter of 440 ft. What are the dimensions of the largest room that can be built? What is its area?

$2y + 2x = 440 \Rightarrow x + y = 220 \Rightarrow y = 220 - x$

Area $= xy = x(220 - x) = 220x - x^2$

Area$' = 220 - 2x = 0$

$x = 110$ feet

$y = 220 - 110 = 110$ feet

Area $= (110)(110) = 12100$ feet^2

Pts) 6) A company is constructing an open-top, square-based, rectangular metal tank that will have a volume of 53 ft^3. What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary.

$x^2 y = 53 \Rightarrow y = \dfrac{53}{x^2}$

Area $= x^2 + 4xy = x^2 + 4x\left(\dfrac{53}{x^2}\right) = x^2 + 212x^{-1}$

$(\text{Area})' = 2x - 212x^{-2} = 0 \qquad 2x = \dfrac{212}{x^2} \Rightarrow 2x^3 = 212$

$x = 4.7$ ft

$y = 2.4$ ft

4.7 feet by 4.7 feet by 2.4 feet

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Name____

Solve the problem.

(3pts) 1) If the price charged for a candy bar is p(x) cents, then x thousand candy bars will be sold in a certain city, where $p(x) = 125 - \frac{x}{18}$. How many candy bars must be sold to maximize revenue?

$R = x\left(125 - \frac{x}{18}\right) = 125x - \frac{x^2}{18} \Rightarrow R'(x) = 125 - \frac{2}{18}x = 0$

$125 = \frac{1}{9}x \Rightarrow \boxed{x = 1125 \text{ thousand candy bars}}$

(3pts) 2) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:

$R(x) = 50x - 0.5x^2$

$C(x) = 9x + 6.$

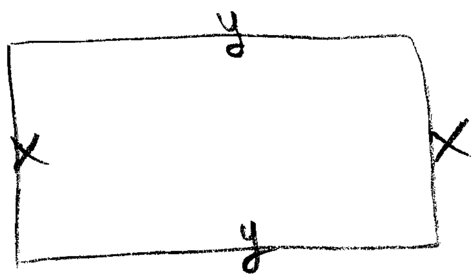
$P = R - C = (50x - 0.5x^2) - (9x + 6)$

$= 41x - 0.5x^2 - 6$

$P' = 41 - 1x = 0$

$\boxed{x = 41 \text{ units}}$

(4pts) 3) A rectangular field is to be enclosed on four sides with a fence. Fencing costs \$6 per foot for two opposite sides, and \$3 per foot for the other two sides. Find the dimensions of the field of area 690 ft^2 that would be the cheapest to enclose.



$xy = 690 \Rightarrow y = \frac{690}{x}$

$\text{Cost} = 6y + 6y + 3x + 3x$

$\text{Cost} = 12y + 6x$

$\text{Cost} = 12\left(\frac{690}{x}\right) + 6x = 8280x^{-1} + 6x$

$\text{Cost}' = \frac{-8280}{x^2} + 6 = 0$

$6x^2 = 8280$

$x^2 = 1380$

$\boxed{x = 37.15 \text{ feet}}$

$y = \frac{690}{x} \Rightarrow \boxed{y = 18.57 \text{ feet}}$

(4pts) 4) A baseball team is trying to determine what price to charge for tickets. At a price of \$10 per ticket, it averages 45,000 people per game. For every increase of \$1, it loses 5,000 people. What price per ticket should be charged in order to maximize revenue?

$(45000, \$10)$

$(40000, \$11)$

$$m = \frac{11-10}{40000-45000} = -0.0002$$

$$y - 10 = -0.0002(x - 45000) \Rightarrow y = -0.0002x + 19$$

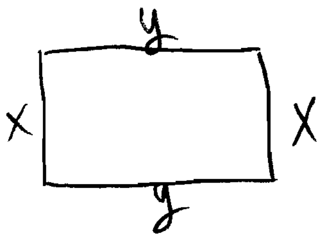
$$P = -0.0002x + 19$$

$$R(x) = xP = x(-0.0002x + 19) = -0.0002x^2 + 19x$$

$$R' = -0.0004x + 19 = 0 \Rightarrow x = 47500$$

$$\text{Price} = -0.0002(47500) + 19 = \boxed{\$9.5}$$

(3pts) 5) A carpenter is building a rectangular room with a fixed perimeter of 100 ft. What are the dimensions of the largest room that can be built? What is its area?



$$2x + 2y = 100 \Rightarrow x + y = 50$$

$$y = 50 - x$$

$$\text{Area} = xy = x(50 - x) = 50x - x^2$$

$$A' = 50 - 2x = 0 \Rightarrow x = 25 \text{ feet}$$

$$y = 25 \text{ feet}$$

$$\text{Area} = 625 \text{ ft}^2$$

(3pts) 6) A company is constructing an open-top square-based, rectangular metal tank that will have a volume of 56 ft^3. What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary.

(4.8 ft by 4.8 ft by 2.4 ft)

$$x^2 y = 56 \Rightarrow y = \frac{56}{x^2}$$

$$\text{Area} = x^2 + 4xy = x^2 + 4x\left(\frac{56}{x^2}\right) = x^2 + 224x^{-1}$$

$$\text{Area}' = 2x - 224x^{-2} = 0$$

$$2x = \frac{224}{x^2}$$

$$2x^3 = 224 \Rightarrow x \stackrel{3}{=} 112 \Rightarrow \boxed{x = 4.8 \text{ feet}}$$

$$y = \frac{56}{4.8^2} = 2.4 \text{ ft}$$