Dr. Katiraie

MA160 Quiz 4

Sections 4.5, 4.6, 5.1, 5.2

Find the derivative of the function.

$$(Pt) \quad 1) y = \frac{\ln(x+8)}{5-6x} \\ y' = (\frac{1}{x+8})(5-6x) - (-6)\ln(x+8) \\ (5-6x)^2 = \frac{1}{(x+8)(5-6x)} + \frac{6\ln(x+8)}{(5-6x)^2}$$

Solve the problem.

2) Assume the total revenue from the sale of x items is given by $R(x) = 39 \ln (8x + 1)$, while the total cost to produce x items is C(x) = x/5. Find the approximate number of items that should be manufactured so that profit, R(x) = R(x) - C(x), is maximum.

$$P(x) = 39 \ln (8X+1) - \frac{X}{5}$$

$$P'(x) = 39 \left(\frac{1}{8X+1}\right) \cdot 8 - \frac{1}{5} = 0$$

$$\frac{312}{8X+1} = \frac{1}{5} \implies 1560 = 8X+1 \implies X = 194.88 \% 195$$

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$$\frac{3}{8} \text{ Suppose that the demand function for x units of a certain item is } p = 110 + \frac{200\ln(x+5)}{x}, \text{ where } p \text{ is the price per unit, in dollars. Find the marginal revenue. } \left(R(X) = X \cdot P\right)$$

$$R(x) = x \cdot p = x(110 + \frac{200 \ln(x+5)}{x}) = 110 \times + 200 \ln(x+5)$$

$$R'(x) = 110 + \frac{200}{x+5}$$

4) The population of coyotes in the northwestern portion of Alabama is given by the formula $p(t) = (t^2 + 100) \ln(t + 2)$, where t represents the time in years since 2000 (the year 2000 corresponds to t = 0). Find the rate of change of the coyote population in 2008 (t = 8).

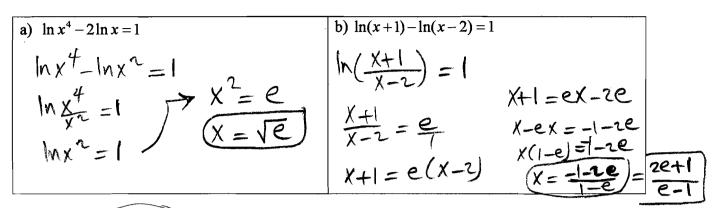
$$P'(t) = 2t \ln(t+2) + (t^{2} + 100) \frac{1}{t+2}$$

$$P'(8) = 2(8) \ln(8+2) + (8^{2} + 100) \frac{1}{8+2} = 53.24 \approx 53 \frac{\text{Coyster}}{\text{year}}$$

A-1

5) Solve the following equations for x.

(1 Point Each)



6) A sample of 8 grams of radioactive material is placed in a vault. Let P(t) be the amount remaining after t years. And suppose that P(t) satisfies the differential equation P'(t) = -.021P(t) (1 point Each)

a) Find the formula for P(t) = 8 e

b) What is
$$P(0)$$
? = 8 grams

- c) What is the decay constant? 0.09
- d) How much of the material will remain after 10 years?

$$P(10) = 8 e^{-0.021(10)} = 6.48 \text{ Gramg}$$

e) Use the differential equation to determine how fast the sample is disintegrating when just one gram remains.

$$P' = -0.021(1) = -0.021$$
 grams
year

f) What amount of radioactive material remains when it is disintegrating at the rate of 105 grams per year?

$$-0.105 = -0.021 P(t) \implies P(t) = \frac{-0.105}{-0.021} = 5 grams$$

g) The radioactive material has a half life of 33 years. How much will remain after 66 years 2 grams

Initially there was 8 grams of radio active material After 33 years 4 grams of material (1 of 8). After 66 years 2 grams of material (1/2 of 4).

7) Four thousand dollars is deposited into a saving account at 3.5% interest compounded continuously. (1 point Each)

a) What is the formula for A(t), the balance after t years?

b) What differential equation is satisfied by A(t), the balance after t years.

$$(A'(t) = 0.035 A(t))$$

c) How much money will be in the account after 2 years?

$$A(2) = 4000 e^{0.035(2)} = #4290.03$$

d) When will the balance reach \$5000?

$$5000 = 4000 e$$

$$5000 = 4000 e$$

$$5000 = 0.035t$$

$$\frac{5000}{4000} = e$$

$$0.035t$$

$$\ln(5/4) = he$$

$$\ln(5/4) = t \implies t = 6.38 \text{ years}$$

e) How fast is the balance growing when it reaches \$5000?

$$A' = 0.035 A(t)$$

= 0.035 (500)=^{\$} 175 per year