Dr. Katiraie MA160 Quiz 4 Sections 4.5, 4.6, 5.1, 5.2

Name Solución

Find the derivative of the function.

(1 pt) 1) $y = \dfrac{\ln(x+8)}{5-6x}$

$$y' = \frac{\left(\frac{1}{x+8}\right)(5-6x) - (-6)\ln(x+8)}{(5-6x)^2} = \boxed{\frac{1}{(x+8)(5-6x)} + \frac{6\ln(x+8)}{(5-6x)^2}}$$

Solve the problem.

(1 pt) 2) Assume the total revenue from the sale of x items is given by $R(x) = 39 \ln(8x + 1)$, while the total cost to produce x items is $C(x) = x/5$. Find the approximate number of items that should be manufactured so that profit, $P(x) = R(x) - C(x)$, is maximum.

$$P(x) = 39\ln(8x+1) - \frac{x}{5}$$

$$P'(x) = 39\left(\frac{1}{8x+1}\right)\cdot 8 - \frac{1}{5} = 0$$

$$\frac{312}{8x+1} = \frac{1}{5} \implies 1560 = 8x+1 \implies 8x = 1559$$

$$x = 194.88 \approx \boxed{195 \text{ items}}$$

(2 pts) 3) Suppose that the demand function for x units of a certain item is $p = 110 + \dfrac{200\ln(x + 5)}{x}$, where p is the price per unit, in dollars. Find the marginal revenue. $(R(x) = x \cdot p)$

$$R(x) = x \cdot p = x\left(110 + \frac{200\ln(x+5)}{x}\right) = 110x + 200\ln(x+5)$$

$$\boxed{R'(x) = 110 + \frac{200}{x+5}}$$

(2 pts) 4) The population of coyotes in the northwestern portion of Alabama is given by the formula $p(t) = (t^2 + 100)\ln(t + 2)$, where t represents the time in years since 2000 (the year 2000 corresponds to $t = 0$). Find the rate of change of the coyote population in 2008 ($t = 8$).

$$P'(t) = 2t\ln(t+2) + (t^2+100)\frac{1}{t+2}$$

$$P'(8) = 2(8)\ln(8+2) + (8^2+100)\frac{1}{8+2} = 53.24 \approx \boxed{53 \frac{\text{Coyotes}}{\text{year}}}$$

5) Solve the following equations for x. (1 Point Each)

a) $\ln x^4 - 2\ln x = 1$	b) $\ln(x+1) - \ln(x-2) = 1$
$\ln x^4 - \ln x^2 = 1$ $\ln \frac{x^4}{x^2} = 1$ $\ln x^2 = 1$ $\to x^2 = e$ $x = \sqrt{e}$	$\ln\left(\frac{x+1}{x-2}\right) = 1$ $\frac{x+1}{x-2} = \frac{e}{1}$ $x+1 = e(x-2)$ $x+1 = ex - 2e$ $x - ex = -1 - 2e$ $x(1-e) = -1 - 2e$ $x = \frac{-1-2e}{1-e} = \frac{2e+1}{e-1}$

6) A sample of 8 grams of radioactive material is placed in a vault. Let P(t) be the amount remaining after t years. And suppose that P(t) satisfies the differential equation $P'(t) = -.021P(t)$ (1 point Each)

a) Find the formula for P(t)

$P(t) = 8e^{-0.021t}$

b) What is P(0)? $= 8$ grams

c) What is the decay constant?

0.021

d) How much of the material will remain after 10 years?

$P(10) = 8e^{-0.021(10)} = 6.48$ Grams

e) Use the differential equation to determine how fast the sample is disintegrating when just one gram remains.

$P' = -0.021(1) = -0.021 \ \frac{\text{grams}}{\text{year}}$

f) What amount of radioactive material remains when it is disintegrating at the rate of .105 grams per year?

$-0.105 = -0.021\,P(t) \Longrightarrow P(t) = \frac{-0.105}{-0.021} = 5$ grams

g) The ~~radioactive~~ material has a half life of 33 years. How much will remain after 66 years? 2 grams

Initially there was 8 grams of radioactive material
After 33 years 4 grams of material ($\frac{1}{2}$ of 8).
After 66 years 2 grams of material ($\frac{1}{2}$ of 4).

7) Four thousand dollars is deposited into a saving account at 3.5% interest compounded continuously. (1 point Each)

a) What is the formula for A(t), the balance after t years?

$A(t) = 4000\, e^{0.035t}$

b) What differential equation is satisfied by A(t), the balance after t years.

$\boxed{A'(t) = 0.035\, A(t)}$

c) How much money will be in the account after 2 years?

$A(2) = 4000\, e^{0.035(2)} = \boxed{\$4290.03}$

d) When will the balance reach $5000?

$5000 = 4000\, e^{0.035t}$

$\frac{5000}{4000} = e^{0.035t}$

$\ln(5/4) = \cancel{\ln e}^{0.035t}$

$\frac{\ln(5/4)}{0.035} = t \implies \boxed{t = 6.38 \text{ years}}$

e) How fast is the balance growing when it reaches $5000?

$A' = 0.035\, A(t)$

$= 0.035(5000) = \boxed{\$175 \text{ per year}}$