1. A farmer has 1800 ft of fencing and wants to
  fence off a rectangular field that borders a
  long brick wall.
  He needs no fencing along the wall.
  We are going to try to determine the dimensions
  of the field that will have the largest area.
   (5 Points)

(a)   Suppose that the width is w. If the length
         is l, write an equation relating w and l.   width
         You will have to use the fact that
         the total amount of fencing is 1800 ft.

(b)   Solve the equation for in part (a) for l.

(c)   Since the area of a rectangle is the product of width times length, $A = lw$. Use
         your answer to part (c) to rewrite this formula for area in terms of w only.

(d)   The width of a field must be a positive number, so we must have $w > 0$.   The length also
         must be positive. Use your expression for l to get another restriction on the value of w.
         These two inequalities give the domain of the area function.
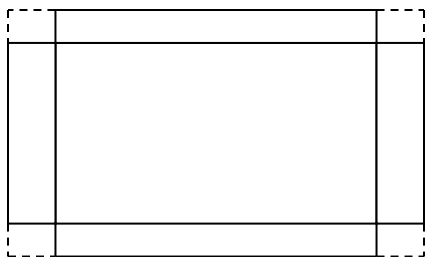
(e)   Use the techniques of Calculus to find the dimensions of the field (the values of the width
         and the length) that will result in a maximum value of the function A on the interval
         determined by your answer to part (e). Confirm that your answer gives a maximum and
         not a minimum by using the First or Second Derivative Test. What is the maximum value
   of A?
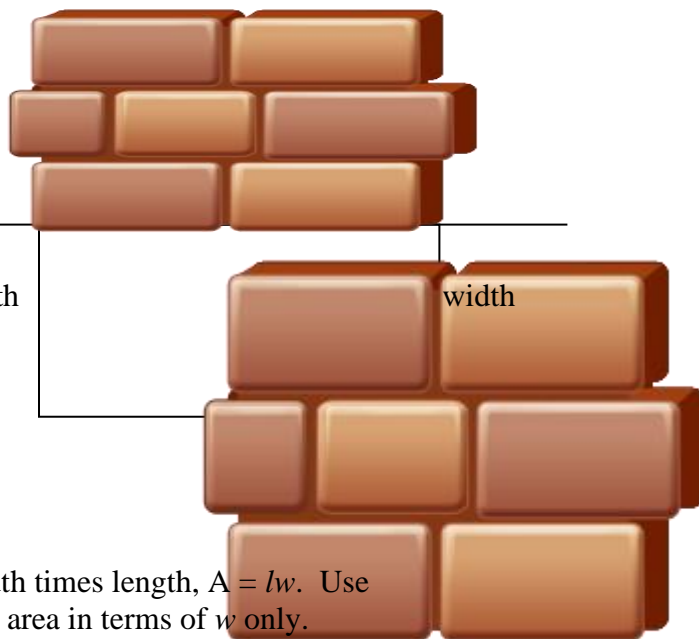
2.   You are going to make a box out of a piece of cardboard that is 12 in. by 20 in. by cutting out squares from each corner, turning up the sides and taping the sides to make a box. You want the box to hold as much as possible, so you want the volume of the box to be a maximum. What size square should be cut out of each corner to maximize the volume of the box?   (5 Points)

Here's a picture to show what's happening:

Flat piece of cardboard with corners cut out   Cardboard folded up to make a box
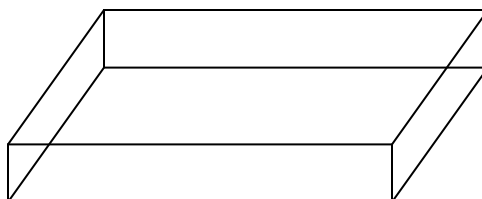
(a)   Let x be the length of each side of the square that is cut out from each corner. This will become the height of the box.

       Write expressions for the length and the width of the box.

(b)   The volume of a box is (length)(width)(height). Using the dimensions you found above, write an expression for the volume $V(x)$ of the box as a function of x.

(c)   Since each dimension of the box must be positive, use this to write the domain of $V(x)$.

(d)   Use the techniques of Calculus to find the dimensions of the box that will result in a maximum value of the volume function $V(x)$ on the interval determined by your answer to part (c). Confirm that your answer gives a maximum and not a minimum by using the First or Second Derivative Test. What is the maximum volume?

3.   A rectangular box open at the top is to be constructed so that the volume is
 20 cubic meters and the base has **length equal to four times its width**. Our goal is to
find the dimensions of the box that will minimize the surface area of the box.
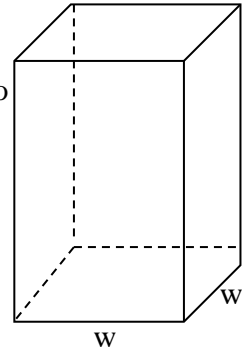
To help you in solving this problem, follow these steps:(5 Points)

(a) A picture is provided, label the dimensions of the box.
        Use w for the width and h for the height.

(b)   Write an expression in terms of h and w for the surface area S of the box. Keep in mind that
        the box has five surfaces --- the bottom plus four sides.

(c)   Use the fact that the volume of the box is 20 m^3 to relate h and w and then solve for h in
        terms of w.

(d)   Substitute the expression for h that you found in (c) into the expression for S in (b).

(e)   You should now have an expression for S in terms of w only. Use calculus to find the critical
        value of S. Then use either the First or Second Derivative Test to determine that S actually
        has a minimum as opposed to a maximum at the critical value.

(f)   What are the dimensions of the box that will result in a minimum surface area?

(g)   What is the minimum surface area?

4.   You've gotten so good at constructing boxes that you have agreed to construct a bunch of boxes for your little sister and brother and their friends to hold some of their small toys. Your brother and sister insist that the base of the box be square, and that the volume of the box be 16 cubic feet. (They're both a little strange.) You are going to make the bottom of the box out of heavy cardboard costing 25 cents per square foot and the sides out of lighter cardboard costing 10 cents per square foot. The box will not have a top. Naturally, you want to spend as little as possible on doing this good deed. Using x as the side of the square in the base and h as the height as in the box shown,

(5 Points)
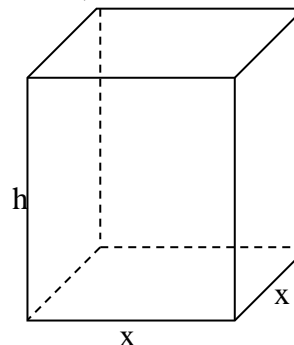
(a)   Write an expression for the cost of the cardboard for the box.

(b)   Use the fact that the volume of the box is to be 16 cubic feet to write an equation relating x and h.

(c)   Solve the equation in part (b) for h.

(d)   Substitute the expression that you got for h in part (c) into the cost expression that you found in part (a) and simplify your answer.

(e)   You should now have an expression for cost that is a function of x only. Use the techniques of Calculus to find the value of x (to the nearest tenth of a foot) that will result in a minimum cost.

(f)   What are the dimensions of the box of minimum cost and what is the minimum cost for each box?