MA 160 Quiz 8 Sections (5.1, 5.2, 5.3 and 6.1) Name: Solutions
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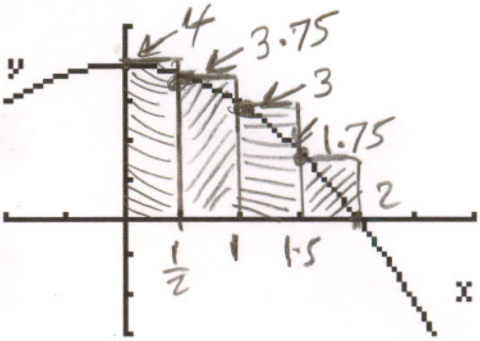
1.(a) Use 4 approximating rectangles and left endpoints to estimate the area under the graph of $f(x) = 4 - x^2$ on the interval [0, 2].

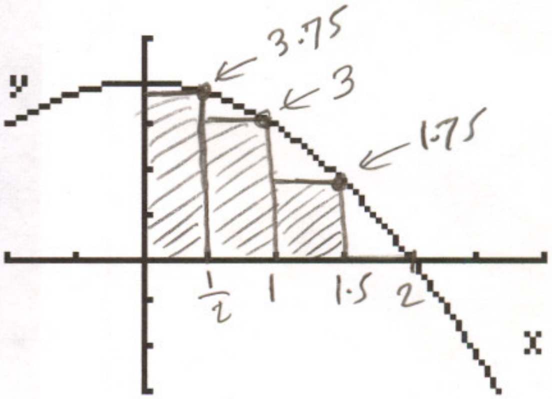


$= \frac{1}{2}(4) + \frac{1}{2}(3.75) + \frac{1}{2}(3) + \frac{1}{2}(1.75)$

$= \frac{1}{2}(4 + 3.75 + 3 + 1.75)$

$= \boxed{6.25}$

(b) Use 4 approximating rectangles and right endpoints to estimate the area under the graph of $f(x) = 4 - x^2$ on the interval [0, 2].



$= \frac{1}{2}(3.75) + \frac{1}{2}(3) + \frac{1}{2}(1.75 + \frac{1}{2}(0)$

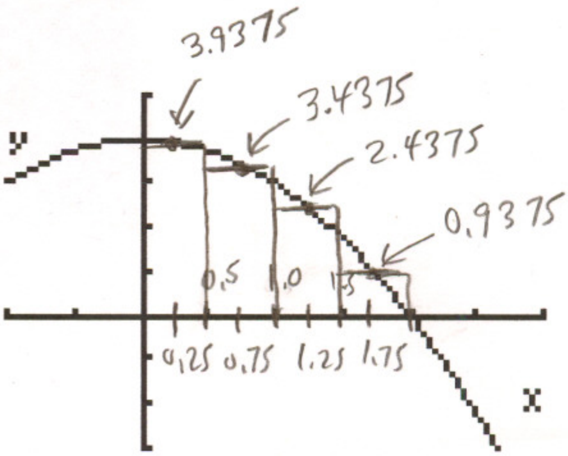
$= \frac{1}{2}(3.75 + 3 + 1.75 + 0)$

$= \boxed{4.25}$

(c) We usually get a better approximation using midpoints instead of left or right endpoints. To see this, use 4 approximating rectangles and midpoints to estimate the area under the graph of $f(x) = 4 - x^2$ on the interval [0, 2].

$= \frac{1}{2}(3.9375 + 3.4375 + 2.4375 + 0.9375)$

$= \boxed{5.375}$



2. Find the antiderivative of each function. Write answers with no negative exponents.

(a) $f(x)=12x^3-8x+5$ $F(x)=\frac{12x^4}{4}-\frac{8x^2}{2}+5x+C$ $\boxed{=3x^4-4x^2+5x+C}$	(b) $f(x)=12\sqrt{x}$ $F(x)=\frac{12x^{1.5}}{1.5}+C=\boxed{8x^{\frac{3}{2}}+C}$
(c) $f(x)=\frac{12}{x^2}=12x^{-2}$ $F(x)=\frac{12x^{-1}}{-1}+C=\boxed{\frac{-12}{x}+C}$	(d) $f(x)=\frac{12}{\sqrt{x}}=12x^{-1/2}$ $F(x)=\frac{12x^{1/2}}{1/2}+C$ $\boxed{=24x^{1/2}+C}$
(e) $f(x)=\frac{5x^4-7x^3+3x^2-4x}{x^3}=5x-7+\frac{3}{x}-4x^{-2}$ $F(x)=\frac{5x^2}{2}-7x+3\ln\lvert x\rvert-\frac{4x^{-1}}{-1}+C$ $F(x)=\boxed{\frac{5}{2}x^2-7x+3\ln\lvert x\rvert+\frac{4}{x}+C}$	(f) $f(x)=3e^{7x}$ $\boxed{F(x)=\frac{3e^{7x}}{7}+C}$
(g) $f(x)=5x^4+\frac{5}{x^4}+\frac{x^4}{5}+5\sqrt[4]{x}$ $f(x)=5x^4+5x^{-4}+\frac{1}{5}x^4+5x^{\frac{1}{4}}$ $F(x)=\frac{5x^5}{5}+\frac{5x^{-3}}{-3}+\frac{x^5}{25}+\frac{5x^{5/4}}{5/4}+C$ $\boxed{=x^5-\frac{5}{3x^3}+\frac{x^5}{25}+4x^{5/4}+C}$	(h) $f(x)=(3x+2)(x-5)$ $3x^2-15x+2x-10$ $3x^2-13x-10$ $F(x)=\frac{3x^3}{3}-\frac{13}{2}x^2-10x+C$ $\boxed{=x^3-\frac{13}{2}x^2-10x+C}$

3. Find the function $f(x)$ such that $f'(x) = \frac{4}{x}$ and $f(e) = 10$.

$F(x) = 4\ln|x| + c$

But $f(e) = 10 \Rightarrow 10 = 4\overset{1}{\cancel{\ln e}} + c$; and $\ln e = 1$

$10 = 4 + c \Rightarrow \boxed{c = 6}$

$\boxed{F(x) = 4\ln|x| + 6}$

4. In this problem, you are going to find the area of the region between the graphs of the functions $y = 9 - x^2$ and $y = x + 3$. To solve this problem,

(a) Since no boundary points are given, you must first find the points of intersection of the graphs of the functions. To do this, set the two functions equal and solve for x.

$9 - x^2 = x + 3$

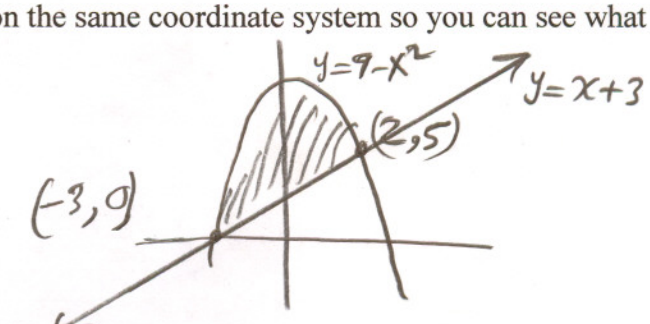
$0 = x^2 + x + 3 - 9$

$0 = x^2 + x - 6$

$0 = (x+3)(x-2)$

$\rightarrow \boxed{x = -3}$ & $\boxed{x = 2}$

(b) Sketch a graph of the two curves on the same coordinate system so you can see what the region looks like.



(c) Set up an integral which represents the value of the area of the given region and then evaluate this integral to answer the question.

$\int_{-3}^{2} ((9-x^2) - (x+3))\,dx = \int_{-3}^{2} (9 - x^2 - x - 3)\,dx = \int_{-3}^{2} (-x^2 - x + 6)\,dx$

$= -\frac{x^3}{3} - \frac{x^2}{2} + 6x \Big|_{-3}^{2} = \left(\frac{-2^3}{3} - \frac{2^2}{2} + 6(2)\right) - \left(-\frac{(-3)^3}{3} - \frac{(-3)^2}{2} + 6(-3)\right)$

$\cong \boxed{20.83}$