Dr. Katiraie MA160 Test I (Sections 1.1---2.2) Spring 2013

Name **(1 point):** Solutions

Total Possible Points = 100 Points + 10 Extra Credit for WebAssign Homework (IF > 95% COMPLETED)

Show all your work.

1. Solve the following Algebraically: (12 Points)

a) $y^2-10y=-9$	b) $25x^2-64=0$	c) $x^2-3x=40$
$y^2-10y+9=0$ $(y-9)(y-1)=0$ $\boxed{y=9}$ $\boxed{y=1}$	$(5x-8)(5x+8)=0$ $x=\frac{8}{5}$ $x=\frac{-8}{5}$	$x^2-3x-40=0$ $(x-8)(x+5)=0$ $x=8$ $x=-5$

2. Perform the indicated operations. Simplify your answers. **(8 pts)**

a) $\left(\frac{x^2}{y^6}\right)^{\frac{3}{2}} = \boxed{\frac{x^3}{y^9}}$	b) $\sqrt{x}\cdot\sqrt[3]{x}$ $x^{\frac{1}{2}}x^{\frac{1}{3}} = \boxed{x^{\frac{5}{6}}}$
c) $\sqrt[3]{x^{18}}$ $=x^{18/3} \boxed{=x^6}$	d) $\frac{-5x^{-3}}{x^{-9}}$ $-5x^{-3+9} = \boxed{-5x^6}$

3. Let $f(x)=(0.2)^x$ Evaluate the following (8 points)

a. $f^{-1}(390625)$	b. Find x when $f(x)=\frac{1}{625}$
$390625=0.2^x$ $\log(390625)=x\log(0.2)$	$\frac{1}{625}=(0.2)^x$ $\log(\frac{1}{625})=x\log(0.2)$

$x=\frac{\log(390625)}{\log(0.2)} = \boxed{-8}$

$\therefore f^{-1}(390625) = \boxed{-8}$

$x=\frac{\log(1/625)}{\log(0.2)} = \boxed{4}$

4. Solve the following algebraically. (12 points)

a) $5(3)^x = 10945$ $3^x = 2189 \to x = \frac{\log(2189)}{\log 3}$ $\boxed{x \cong 7.00}$	b) $6(5)^x - 660 = 0$ $5^x = 110$ $x = \log(110) \div \log(5) \boxed{\cong 2.92}$
c) $5\ln(x) = 10$ $\ln x = 2$ $x = e^2 \boxed{\cong 7.39}$	d) $2e^x - 8 = 0$ $e^x = 4$ $x = \ln(4) \boxed{\cong 1.39}$

6. Given $f(x) = x^2 - 10x + 9$, find the following and simplify your answer (12 points)

a) $f(x+h)$ $= (x+h)^2 - 10(x+h) + 9$

$= x^2 + 2xh + h^2 - 10x - 10h + 9$

b) $f(x+h) - f(x)$

$= \cancel{x^2} + 2xh + h^2 - \cancel{10x} - 10h + \cancel{9} - (\cancel{x^2} - \cancel{10x} + \cancel{9})$

$= 2xh + h^2 - 10h$

c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{2xh + h^2 - 10h}{h} = \dfrac{\cancel{h}(2x + h - 10)}{\cancel{h}} = 2x + h - 10$

d) $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} 2x + h - 10 = \boxed{2x - 10}$

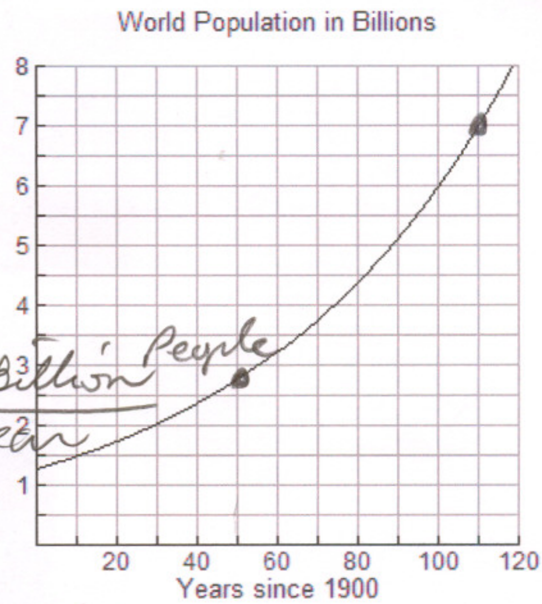
5. Find the following limits: (8 Points)

a) $\lim_{t\to 4} \dfrac{t^2-16}{t-4} = \lim\limits_{t\to 4} \dfrac{\cancel{(t-4)}(t+4)}{\cancel{(t-4)}} \boxed{= 4 + 4 = 8}$

b) $\lim_{x\to -5} \dfrac{\frac{1}{5}+\frac{1}{x}}{5+x} = \dfrac{\frac{x+5}{5x}}{5+x} = \lim\limits_{x\to -5} \dfrac{\cancel{x+5}}{5x} \cdot \dfrac{1}{\cancel{x+5}} = \boxed{\dfrac{-1}{25}}$

$(1950, 2.75)\ (2010, 7)$

7. The graph shown models $W(t)$, the world population (in billions), where t is measured in years since 1900. Use this graph to estimate the average rate of change in the world population from 1950 to 2010 and write a sentence interpreting your result. Be sure to use appropriate units in your answer. (5 points)



$$\text{Avg Rate} = \frac{7-2.75}{2010-1950} = 0.070833 \frac{\text{Billion People}}{\text{year}}$$

The average rate of change for the population between 1950 to 2010 was 0.0708 Billion people per year.

8. If a ball is projected vertically upward from the surface of the moon with a speed of 64 ft/s, its height in feet after t seconds is given by $h(t) = -2.6t^2 + 64t$. (9 points)

A) Find the average speed of the ball during each of the following time intervals. Write your answers correct to at least two decimal places.
Please use appropriate units in your answers.

(i) [5, 5.1]

x	$y = -2.6t^2 + 64t$
5	255
5.1	258.774

$$V_{avg} = \frac{258.774 - 255}{5.1 - 5} = \boxed{37.74 \frac{ft}{sec}}$$

(ii) [5, 5.001]

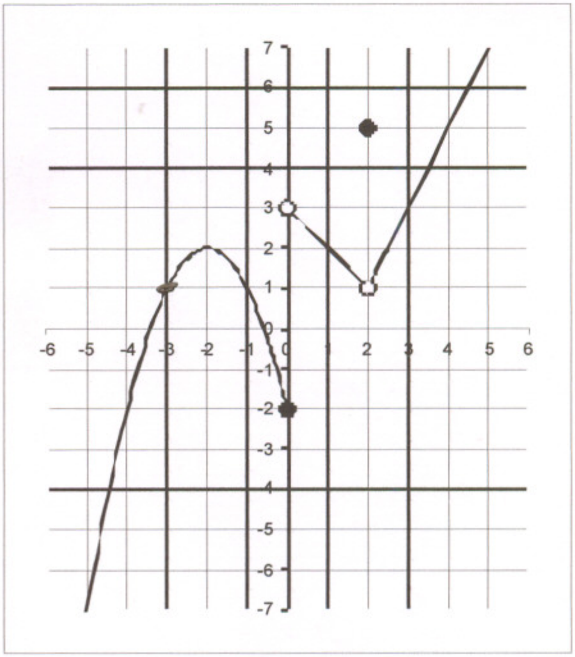
x	y
5	255
5.001	255.038

$$\Rightarrow V_{avg} = \frac{255.038 - 255}{5.001 - 5} = \boxed{38\ ft/sec}$$

B) Estimate the speed of the ball when $t=5$ seconds
Please use appropriate units in your answers

Speed at 5 seconds is $\boxed{38\ ft/sec}$

9. Let f(x) be the graph to the right. Use this function to answer each of the questions below. (15 points)



(a) $f(-3) =$ 1 $\lim_{x \to -3^-} f(x) =$ 1

$\lim_{x \to -3^+} f(x) =$ 1 $\lim_{x \to -3} f(x) =$ 1

(b) Is f(x) continuous or discontinuous at x = -3? Why or why not?

Yes, because $\lim_{x \to -3} f(x) = f(-3)$

(c) $f(0) =$ -2 $\lim_{x \to 0^-} f(x) =$ -2

$\lim_{x \to 0^+} f(x) =$ 3 $\lim_{x \to 0} f(x) =$ DNE

(d) Is f(x) continuous or discontinuous at x = 0? Why or why not?

No, Because $\lim_{x \to 0} f(x)$ DNE and $\lim_{x \to 0} f(x) \neq f(0)$

(e) $f(2) =$ 5 $\lim_{x \to 2^-} f(x) =$ 1

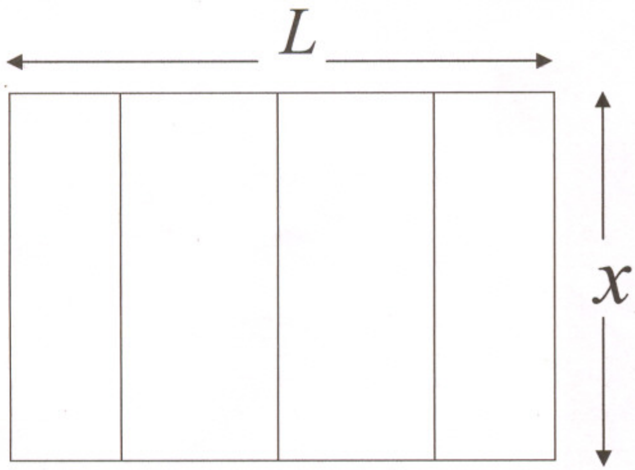
$\lim_{x \to 2^+} f(x) =$ 1 $\lim_{x \to 2} f(x) =$ 1

(f) Is f(x) continuous or discontinuous at x = 2? Why or why not?

No Because $\lim_{x \to 2} f(x) \neq f(2)$

10. A farmer wants to enclose a rectangular area and then divide it into four smaller rectangular regions by putting up fencing parallel to one side of the rectangle. The total amount of fencing available is 1000 feet. An illustration is shown below.

A) Express the total area of the enclosed region as a function of x and simplify your answer. (8 points)



$$5X + 2L = 1000 \Rightarrow 2L = 1000 - 5X$$

$$L = 500 - 2.5X$$

$$A = XL = X(500 - 2.5X)$$

$$A(X) = 500X - 2.5X^2 \quad (\text{feet}^2)$$

B) State the Domain of the above function. (Hint: what are the possible values of x) (2 points)

$$0 < X < 200 \quad (\text{feet})$$