Dr. Katiraie MA160 Test I Form A (Sections 0.1---1.5) Summer 08

Name Solution Total Possible Points = 100 Points

Show all your work.

1. Let $g(x) = x^2 - 10x + 9$ and $f(x) = \frac{-1}{2}x + 4$ (8 points)

a. Find x when $f(x) = 6$	b. Find x when $g(x) = 0$	c. Find $g(a+1)$
$-\frac{1}{2}x + 4 = 6$ $-\frac{1}{2}x = 2$ $\boxed{x = -4}$	$x^2 - 10x + 9 = 0$ $(x-9)(x-1) = 0$ $\boxed{x = 9}$ $\boxed{x = 1}$	$(a+1)^2 - 10(a+1) + 9$ $= a^2 + 2a + \cancel{1} - 10a - \cancel{10} + 9$ $= \boxed{a^2 - 8a}$

2. Solve the following equations: (8 points)

a. $\frac{2x}{x-5} = 3$

$3x - 15 = 2x$

$\boxed{x = 15}$

b. $\frac{3x-1}{7} = \frac{2x+1}{2}$

$6x - 2 = 14x + 7$

$-9 = 8x$

$\boxed{x = \frac{-9}{8}}$

3. Solve the following: (8 Points)

a) $x^2 - 16 = 0$	b) $y^2 - 6y = 7$	c) $25x^2 - 64 = 0$	$x^2 - 3x = 40$
$(x+4)(x-4) = 0$ $\boxed{x = \pm 4}$	$y^2 - 6y - 7 = 0$ $(y-7)(y+1) = 0$ $\boxed{y = 7 \quad y = -1}$	$(5x+8)(5x-8) = 0$ $\boxed{x = \frac{-8}{5} \quad x = +\frac{8}{5}}$	$x^2 - 3x - 40 = 0$ $(x-8)(x+5) = 0$ $x = 8$ $x = -5$

4. Perform the indicated operations. Simplify your answers. **(4 pts)**

a) $\left(\frac{x^2}{y^6}\right)^{\frac{3}{2}}$ $= \frac{x^3}{y^9}$	b) $\sqrt{x}\cdot\sqrt[3]{x}$ $= x^{\frac{1}{2}} x^{\frac{1}{3}}$ $\boxed{= x^{\frac{5}{6}}}$
c) $\sqrt[3]{x^7}$ $= x^{\frac{7}{3}}$	d) $-5x^{-3}$ $= \frac{-5}{x^3}$

5. Find the derivative of each function below and write your answers with no negative exponents. **(16 pts)**

a) $f(x)=x^2-81$ $\boxed{f'(x) = 2x}$	b) $f(x)=x\sqrt{x}$ $f(x) = x\,x^{\frac{1}{2}}$ $f(x) = x^{\frac{3}{2}}$ $\boxed{f'(x) = \frac{3}{2}x^{\frac{1}{2}}}$	c) $f(x)=\frac{1}{x^2}$ $f(x) = x^{-2}$ $f'(x) = -2x^{-3}$ $\boxed{f'(x) = \frac{-2}{x^3}}$	d) $f(x)=\frac{x}{7}$ $f(x) = \frac{1}{7}x$ $\boxed{f'(x) = \frac{1}{7}}$
e) $f(x)=-\pi^2+81$ $\boxed{f'(x) = 0}$	f) $f(x)=-81x$ $\boxed{f'(x) = -81}$	g) $f(x)=-81+x$ $\boxed{f'(x) = 1}$	h) $f(x)=\frac{x}{\sqrt{x}}$ $f(x) = x^1 x^{-\frac{1}{2}} = x^{\frac{1}{2}}$ $\boxed{f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}}$

6. Find the point on the curve $f(x)=\sqrt{x}$

where the tangent line is parallel to the line $f(x)=\frac{x}{8}$ **(8 pts)**

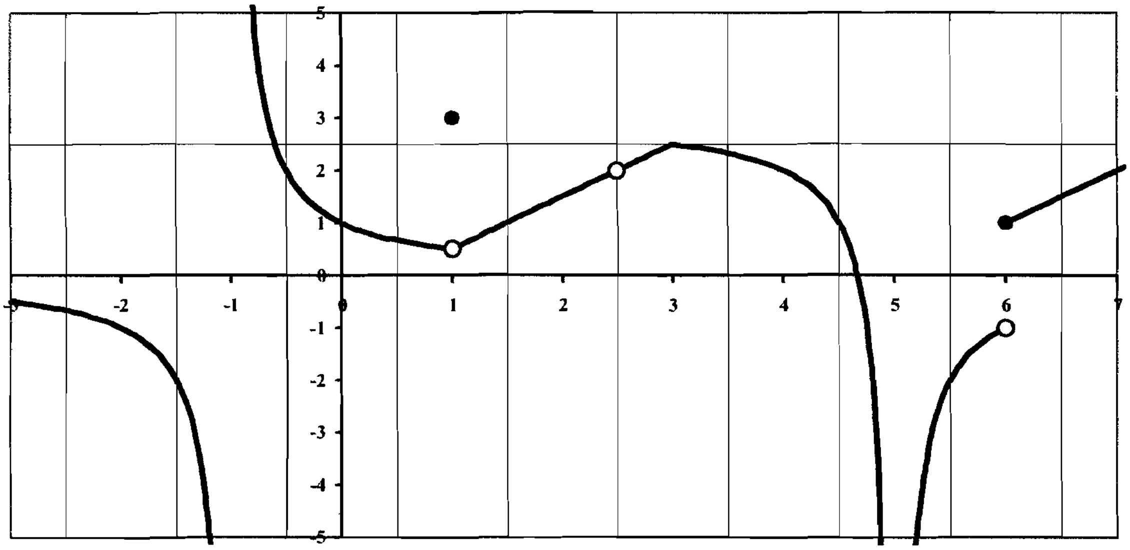
$\boxed{(16, 4)}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ $\qquad f'(x) = \frac{1}{8}$

$\frac{1}{2\sqrt{x}} = \frac{1}{8} \Longrightarrow 8 = 2\sqrt{x} \Longrightarrow \sqrt{x} = 4 \Longrightarrow \boxed{x = 16}$

$y = \sqrt{16} \Longrightarrow \boxed{y = 4}$

7. Use the graph of a function $y = f(x)$ given below to determine whether or not each limit exists. If the limit exists, find the limit. (6 Points)



(a) $\lim_{x\to -1} f(x)$	(b) $\lim_{x\to 0} f(x)$	(c) $\lim_{x\to 1} f(x)$	(d) $\lim_{x\to 2.5} f(x)$	(e) $\lim_{x\to 5} f(x)$	(f) $\lim_{x\to 6} f(x)$
= DNE	$=1$	$=\frac{1}{2}$	$=2$	$-\infty$ OR DNE	$\boxed{\text{DNE}}$

8. Find each limit that exists: (8 Points)

(a) $\lim_{x\to -2}(3x^2 - 5x + 1)$

$= 3(-2)^2 - 5(-2) + 1$

$= 12 + 10 + 1$

$\boxed{= 23}$

(b) $\lim_{x\to 4}\frac{2x-3}{x+5} = \frac{2(4)-3}{4+5} = \boxed{\frac{5}{9}}$

(c) $\lim_{x \to 3} \frac{x^2+2x-15}{x-3}$

(d) $\lim_{x \to 3} \frac{x^2+2x}{x-3}$ $= \frac{9+6}{0} = \frac{15}{0} = \text{DNE}$

$$\lim_{x \to 3} \frac{(x+5)\cancel{(x-3)}}{\cancel{(x-3)}} = \boxed{8}$$

Let $f(x) = x^2 + 7x - 5$.

(9) Use the limit definition of the derivative to find $f'(x)$ for the given function $f(x)$. (6 Points)

$$\text{limit}_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 + 7(x+h) - 5 - (x^2 + 7x - 5)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{7x} + 7h \cancel{-5} \cancel{-x^2} \cancel{-7x} \cancel{+5}}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x+h+7)}{\cancel{h}} = \boxed{2x+7}$$

10) Write the equation of the tangent line to $f(x) = x^2 + 7x - 5$ when x = -2 on the curve. (6 points)

$f'(-2) = 2(-2) + 7 = 3$

$f(-2) = (-2)^2 + 7(-2) - 5 = 4 - 14 - 5 = -15$

$y - -15 = 3(x - -2)$

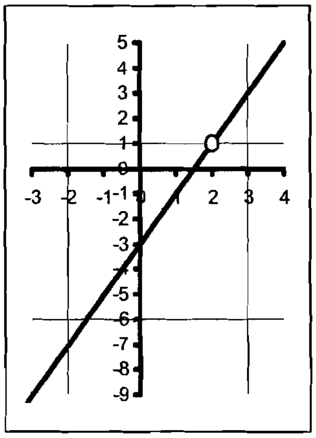
$y + 15 = 3x + 6$

$\boxed{y = 3x - 9}$

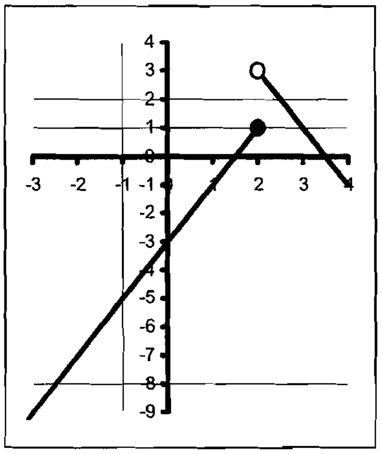
Informally, a function f is continuous at x = a if the function does not have a hole, jump or break of some kind at x = a.

11. There are four functions pictured below. Which of these four do you think is continuous at x = 2? (2 Points)

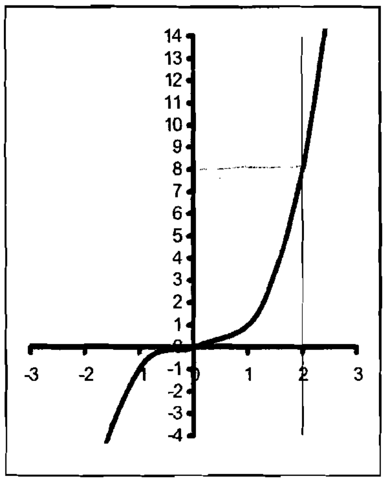
(a)



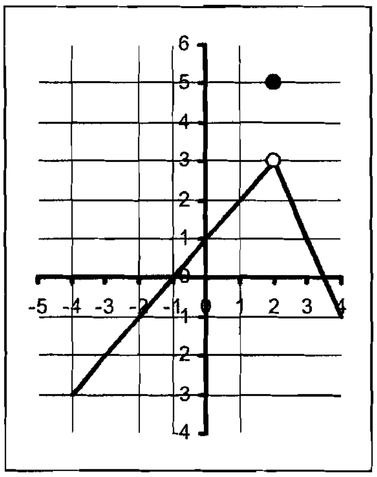
(b)



(c)



(d)



12. For each function above, find $f(2)$ and $\lim_{x\to 2} f(x)$. (10 Points)

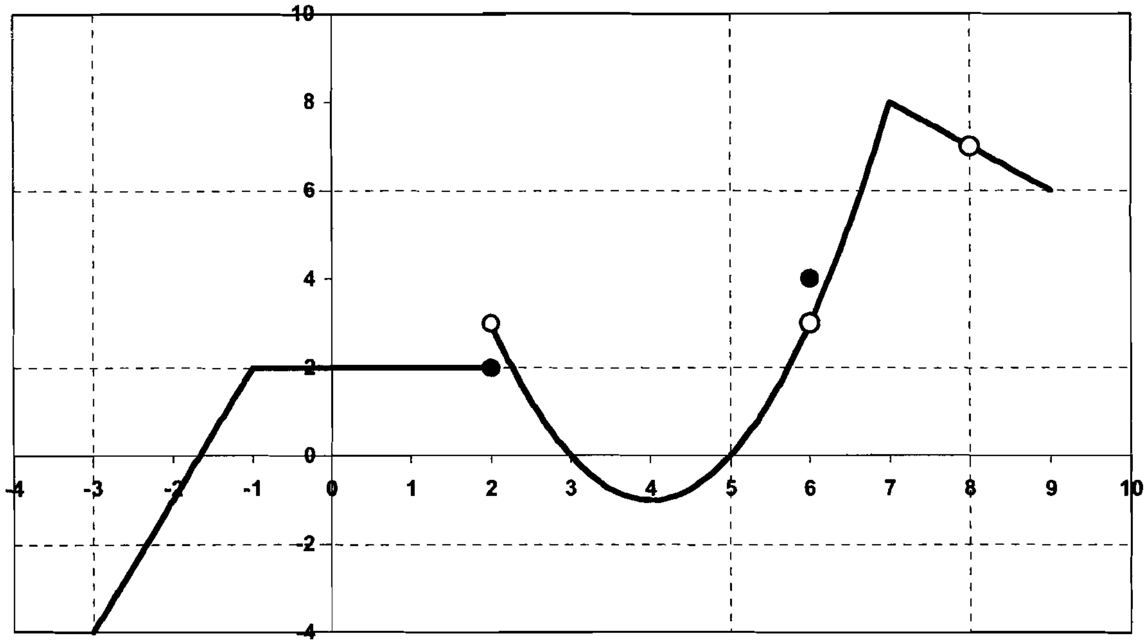
(a) $f(2) =$ DNE $\lim_{x\to 2} f(x) = 1$ (b) $f(2) = 1$ $\lim_{x\to 2} f(x) =$ DNE

(c) $f(2) = 8$ $\lim_{x\to 2} f(x) = 8$ (d) $f(2) = 5$ $\lim_{x\to 2} f(x) = 3$

Look at your results for questions 11 and 12 for each function. What condition or conditions involving limits do you think must be satisfied for a function to be continuous at x = a?

$$\lim_{x\to a} f(x) = f(a)$$

13. A function f is said to be **differentiable** at $(a, f(a))$ iff $f'(a)$ exists. Graphically, this means that the function must have a tangent line with a defined slope at $(a, f(a))$. (10 Points)



Consider the function y = f(x) whose graph is given. For each value of x = a, determine whether or not

i. the function is continuous at $x = a$.

ii. the function is differentiable at x = a.

Complete the chart by *answering "yes" or "no"*

	x_0	Is f continuous at x = a?	Is f differentiable at x = a?
a.	-1	Yes	NO
b.	1	yes	yes
c.	2	No	NO
d.	4	Yes	Yes
e.	6	NO	No